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STATE ESTIMATION PROBLEMS IN PRF-SHIFT MAGNETIC RESONANCE THERMOMETRY

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ABSTRACT

The calculation of temperature shifts *via* PRF-Shift MR Thermometry is herein performed under a Bayesian framework. This formulation aims to result in more robust temperature measurements, accounting for known uncertainties in some important parameters.

Key Words: *Heat Transfer, MR Thermometry, State Estimation Problems.*

1. INTRODUCTION

Non-intrusive quantification of temperature in living tissue has become a very important feature in developing more robust and reliable thermal therapies. Several methodologies designed for this purpose are available, wherein the PRF-Shift Magnetic Resonance (MR) Thermometry is the most widely used due to its robustness and near tissue-independency [1]. In this technique, the temperature values are calculated using phase mapping. These values, acquired by MR equipment, are usually noisy and propagate uncertainties to the indirect temperature measurements. In this work, these uncertainties are quantified by recasting this problem as a state estimation problem, which enables the use of techniques that have been successfully used in other areas [2], [3].

2. FORWARD AND INVERSE PROBLEM

The calculation of temperature shifts via phase mapping in the PRF-Shift MR Thermometry is given by Eq. (1) [1]. Here, $\Phi(T_2) - \Phi(T_1)$ stands for the phase difference measured within a GRE-type pulse sequence, α represents the linear relationship between temperature and chemical shift, γ is the gyromagnetic ratio, t_{TE} is the echo time and B_0 is the externally imposed magnetic field.

$$\Delta T = T_2 - T_1 = \frac{\Phi(T_2) - \Phi(T_1)}{\alpha \gamma t_{TE} B_0} \tag{1}$$

In order to process the noisy measurements of phase shift we recast this problem in the form of a state estimation problem, given by an evolution and an observation model, respectively given by

$$\mathbf{x}_{n+1} = \mathbf{F}\mathbf{x}_n + \mathbf{w}_{n+1} \tag{2}$$

$$\mathbf{y}_n = \mathbf{H}\mathbf{x}_n + \mathbf{v}_n \tag{3}$$

In these equations, $\mathbf{x}$ is the state vector, $\mathbf{y}$ is the observation vector, $\mathbf{w}$ is the evolution noise and $\mathbf{v}$ is the observation noise. Both of these noise vectors are Gaussian with zero mean and known covariance matrices $\mathbf{Q}$ and $\mathbf{R}$, respectively. Within the proposed approach, the state vector is the temperature shift, while the observation vector contains the phase shift.

$$\mathbf{x}_n = \mathbf{\Delta T}_n \quad \text{and} \quad \mathbf{y}_n = \mathbf{\Delta \Phi}_n \tag{4}$$

By noticing that the model given by Eq. (1) is linear, it might be possible to estimate the sought variables sequentially by using the Kalman filter (KF) [2]–[4]. An evolution model based on the heat conduction equation, presented by Eq. (5), is also considered. This partial differential equation is a particular case of the well-known bioheat equation [5], without perfusion terms. In this model, all boundaries are considered as thermally insulated. The initial condition is discussed below.

$$\rho c_p \partial_t \Delta T = k \nabla^2 (\Delta T) \tag{5}$$

After the assembling of the required matrices for the KF, it follows that the resulting system is linear and time-invariant (LTI). In this particular case, an approximation of the KF equations can be performed, resulting in the *steady-state Kalman filter* (SSKF) [4]. In comparison with the original KF, the SSKF is orders of magnitude faster and can easily provide the required temporal resolution for situations like monitoring and controlling of thermal therapies. The equations for the SSKF are

$$\mathbf{P}_\infty = \mathbf{F}\mathbf{P}_\infty\mathbf{F}^T - \mathbf{F}\mathbf{P}_\infty\mathbf{H}^T\left(\mathbf{H}\mathbf{P}_\infty\mathbf{H}^T + \mathbf{R}\right)^{-1}\mathbf{H}\mathbf{P}_\infty\mathbf{F}^T + \mathbf{Q} \tag{6}$$

$$\mathbf{K}_\infty = \mathbf{P}_\infty\mathbf{H}^T\left(\mathbf{H}\mathbf{P}_\infty\mathbf{H}^T + \mathbf{R}\right)^{-1} \tag{7}$$

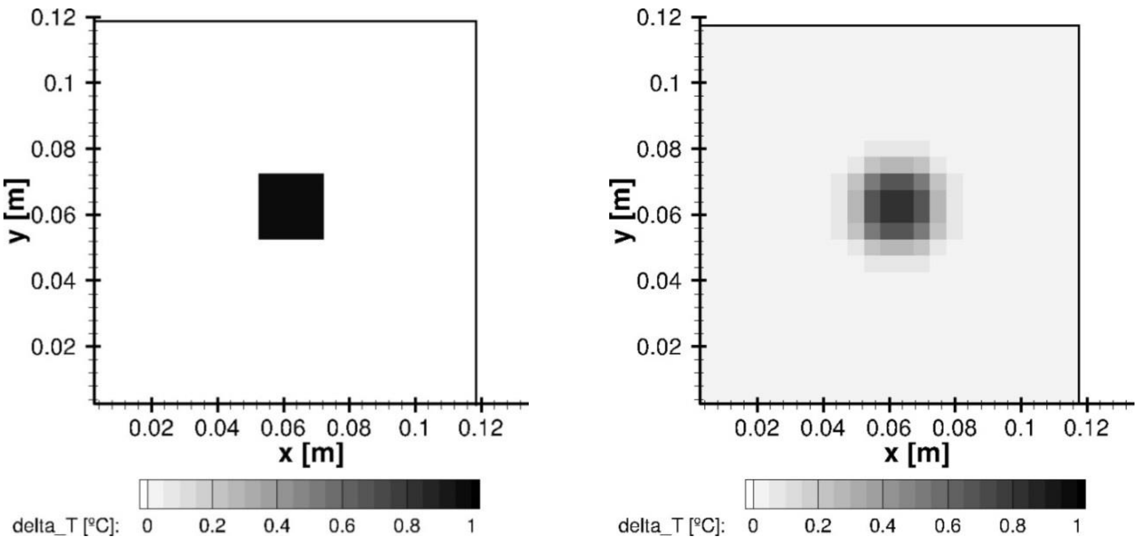
$$\hat{\mathbf{x}}_n^+ = \left(\mathbf{I} - \mathbf{K}_\infty\mathbf{H}\right)\mathbf{F}\hat{\mathbf{x}}_{n-1}^+ + \mathbf{K}_\infty\mathbf{y} \tag{8}$$

3. RESULTS

In this work, the synthetic phase measurements were simulated by solving a 2D heat conduction problem, similar to the one given by Eq. (5). The phase measurements were calculated through Eq. (1). Gaussian noise with zero mean and standard deviation of 10% of the maximum phase shift observed was added to the measurements. These synthetic measurements were considered available throughout a 24 x 24 uniform grid with domain dimensions 12 cm x 12 cm. The MR [6] and thermal [7] parameters used both in the measurements simulation and in the SSKF are presented in Eqs. (9) and (10), respectively. Figure 1 shows the initial condition and the temperature distribution at t = 100 s.

$$\gamma = 42.57 \text{ MHz/T}, \quad t_{TE} = 18 \text{ ms}, \quad \alpha = -0.01 \text{ ppm/°C}, \quad B_0 = 1.5 \text{ T} \tag{9}$$

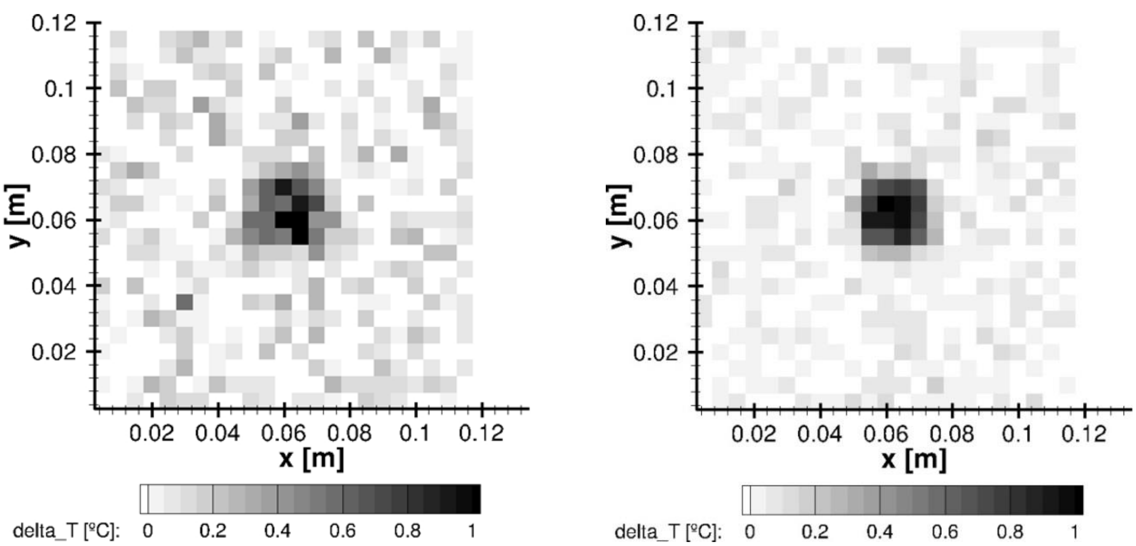
$$\rho c_p = 4180 \text{ kJ/m}^3\text{°C}, \quad k = 0.60 \text{ W/m°C} \tag{10}$$



(a) (b)

FIGURE 1. Contour plots for exact temperature at (a) t = 0 s, and (b) t = 100 s.

Figure 2 presents the temperature shift estimates at t = 100 s, obtained by directly inverting Eq. (1) (Figure 2a) and by applying the proposed approach (Figure 2b). Comparing these results with Figure 1b clearly shows the improved performance of the proposed approach. The time evolution of a voxel within the central region is tracked and shown in Figure 3a. Once again, one can observe that the proposed approach produces estimates with better agreement with the reference temperature values. This improvement is better observed in Figure 3b, where the temperature errors for both approaches are shown. The SSKF method outperformed the direct inversion throughout the simulation time, resulting in temperature erros significantly smaller than the ones obtained *via* direct inversion.



(a) (b)

FIGURE 2. Contour plots for temperature at t = 100 s calculated through (a) direct inversion, and (b) proposed approach using SSKF.

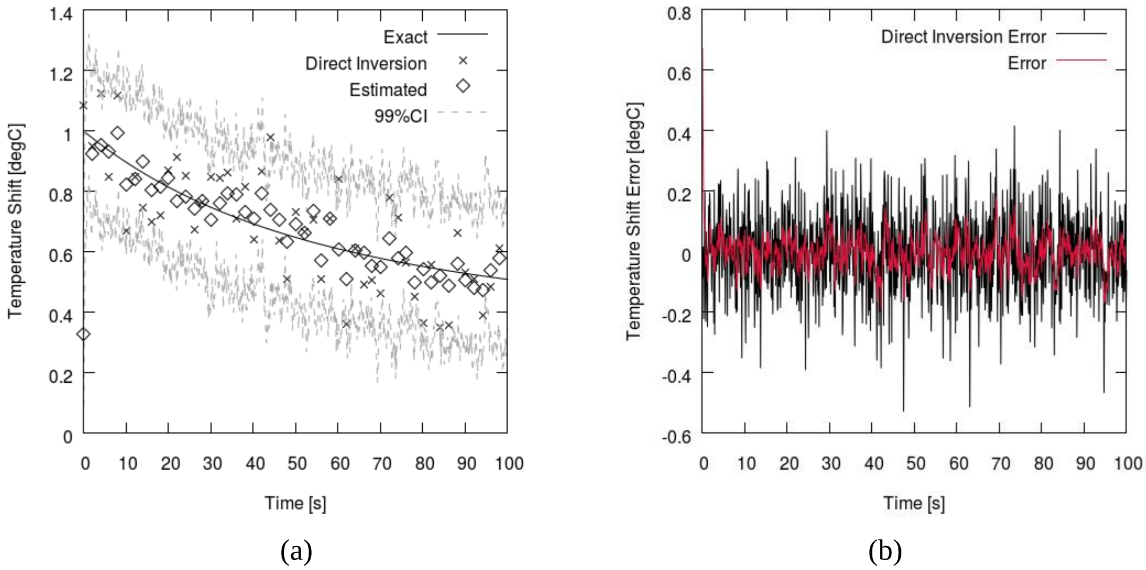


FIGURE 3. Time evolution of (a) temperature at the central region and (b) temperature errors at the heated region for both methods.

4. CONCLUSIONS

The proposed approach resulted in temperature shift estimates that are in excellent agreement with the reference values and that have better quality in comparison with the temperature shift values obtained through direct inversion, with the added benefit of quantifying the uncertainty of the estimates.

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