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Inverse Design of Cooling Arrays of Micro Pin-Fins Subject to Specified Coolant Inlet Temperature and Hot Spot Temperature

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ABSTRACT

Given a micro pin-fin array cooling scheme with these constraints: (a) given maximum allowable temperature of the material (the hot spot temperature), (b) given inlet cooling fluid temperature, (c) given total pressure loss (pumping power affordable), and (d) given overall thickness of the entire micro pin-fin cooling array, find the maximum possible average heat flux on the hot surface and find the maximum possible heat flux at the hot spot under the condition that the entire amount of the inputted heat is removed by the cooling fluid. The goal was to create an optimum performance map for a cooling micro array having specified inlet coolant temperature and maximum temperature. Fully 3D conjugate heat transfer analysis was performed on each of the randomly created candidate configurations. Response surfaces based on Radial Basis Functions were coupled with a genetic algorithm to arrive at a Pareto set of best trade-off solutions. These Pareto optimized configurations indicate the maximum physically possible heat fluxes for specified material and constraints. Detailed off-design performance maps of such Pareto-optimized cooling arrays of micro pin-fins were calculated that demonstrate superior on-design and off-design performance of pin-fins having symmetric convex cross sections as opposed to the commonly used circular cross sections.

Introduction

High heat flux thermal loads on electronic components, having multiple hot spots with heat fluxes at least an order of magnitude larger than the average heat flux values, present a serious challenge to designers of electronics cooling schemes. The main issue is the extreme temperature that results at the hot spots. It is known that next generation electronic chips are expected to produce around 500 W cm^{-2} as the background and over 1000 W cm^{-2} at the hot spots [1, 2]. Special application microelectronics already need adequate efficient cooling for background heat fluxes as high as several kilowatts per square centimeter with hot spots having an order of magnitude higher heat fluxes. Consequently, maximum temperature of such electronics components will be much higher than silicon can endure. Specifically, silicon carbide is an up-and-coming material which can be used for electronic components reaching temperatures as high as 250°C . Several materials research teams are already working on developing electronic materials that can operate at temperatures as high as 800°C . Thus, it would be of interest to any electronics cooling system designer to know *a priori* how high

background heat flux and how high hot spot heat flux can possibly be handled with a particular cooling scheme if electronic material is given with its maximum allowable temperature. Various forced convection cooling schemes have been reported in the literature for their attempted efforts to handle such extreme heat fluxes, while keeping hot spot temperatures below the limit dictated by the physical properties of the material. For example, Abdoli *et al.* [3] investigated the cooling capability of two-floor microchannels under the background heat flux of 1000 W cm^{-2} and a hot spot heat flux of 2000 W cm^{-2} . Siu-Ho *et al.* [4] investigated the performance of micro pin-fin heat sinks under single-phase and two-phase regimes. The current methodology for obtaining an efficient cooling scheme employs a multi-objective optimization to find a configuration capable of cooling a specified heat flux. This approach has been applied by John *et al.* [5] and Tullius *et al.* [6] to optimize micro pin-fin heat sinks in a staggered arrangement, as is the case in this paper. An alternative constructal design approach by Bello-Ochende *et al.* [7] was employed by Adewumi *et al.* [8] to design a combined microchannel with micro pin-fins for electronics cooling.

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Cooling arrays of micro pin-fins with a 4×3 mm footprint and a 0.5×0.5 mm hot spot having circular, airfoil, and convex cross-sections were analyzed by Abdoli *et al.* [9] and optimized by Reddy *et al.* [10]. Only one half of the configuration was actually computationally analyzed, because of geometric symmetry. In their work, the exterior surface of the top wall was subjected to a uniform heat flux of 500 W cm^{-2} , while a small area on this surface represented a hot spot with heat flux of 2000 W cm^{-2} . Fabbri and Dhir [11] optimized arrays of micro-jets to cool microelectronic devices for high heat fluxes, but also requiring considerably higher pumping power for the coolant. Prasher and Chang [12] numerically analyzed the use of microchannels and micro pin-fin heat exchangers to cool a heat flux of 1250 W cm^{-2} . AlWaaly *et al.* [13] presented a liquid cooling system utilizing extremely small channels.

It should be noted that all of these cooling configurations were designed to cool a specified heat flux, and that very little is known of their performance under different thermal maps. That is, the current design approaches only allow for a cooling scheme to cool electronics up to a specified heat flux and do not provide information on what operating conditions are required to keep the optimum performance at different maximum allowable temperature of the solid material.

Thus, the question that should be asked is: What is the maximum possible average heat flux and what is the maximum possible local heat flux at the hot spot that will still not exceed the maximum specified temperature? That is, what is the maximum possible achievable performance of a given cooling scheme with specified inlet coolant temperature and specified properties of the coolant and the

material of the chip that will remove all the heat while requiring minimal pumping power?

This is an inverse design approach that can be solved as a constrained multi-objective optimization problem. The design variables include coolant inlet velocity and the geometric design parameters for the array of micro pin-fins. The simultaneous objectives in this study were to minimize maximum temperature and pumping power, while maximizing the background and hot spot heat fluxes. A constraint was imposed on the inlet temperature and maximum allowable temperature. Figure 1 shows the typical configurations used in this study.

Fully 3D conjugate heat transfer was carried out on each micro-pin-fin configuration to obtain the objective function values which were then used to create response surfaces. These response surfaces were coupled with a multi-objective genetic algorithm to create a Pareto set of the best trade-off solutions.

3D Conjugate heat transfer analysis

The three proposed geometries (Figure 1) for the pin-fins (having circular, symmetric airfoil, and symmetric convex cross sections) have been optimized by Reddy *et al.* [10] for a specified heat flux. The electronic chip in the current study has the same dimensions as those used in [10]. The proposed configurations include channeled outlets to suppress backflow at the outlet. The thickness of the channeled outlets was the same as the thickness of the pin-fins. Water was used as the cooling fluid. A computational grid was created for each of the initial candidate designs using ANSYS Meshing® [14] and 3D conjugate heat transfer analysis was performed using ANSYS Fluent®.

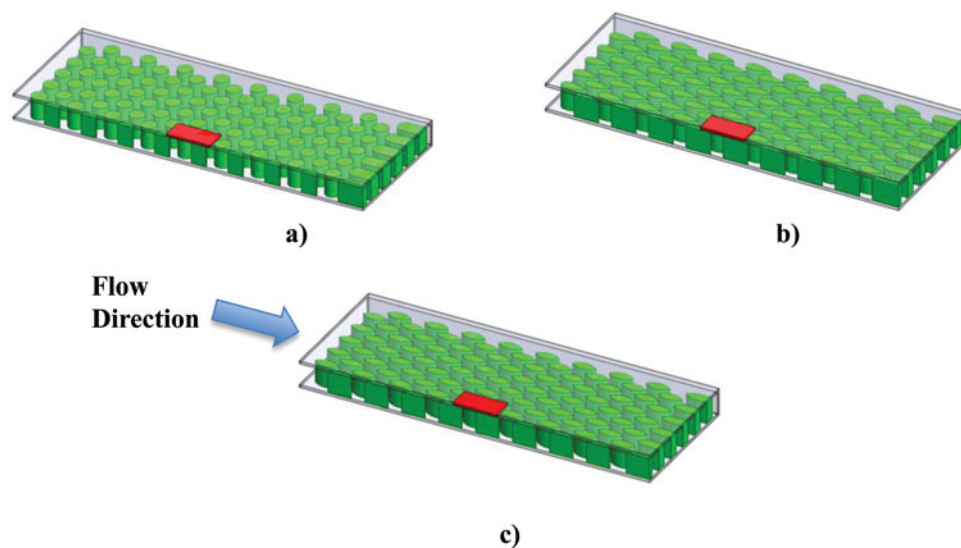


Figure 1. An array of micro pin-fins having: (a) circular, (b) symmetric airfoil, and (c) symmetric convex cross sections. Only one half of the entire array is shown and analyzed because of symmetry. Top, bottom, and side walls are not shown for clarity.

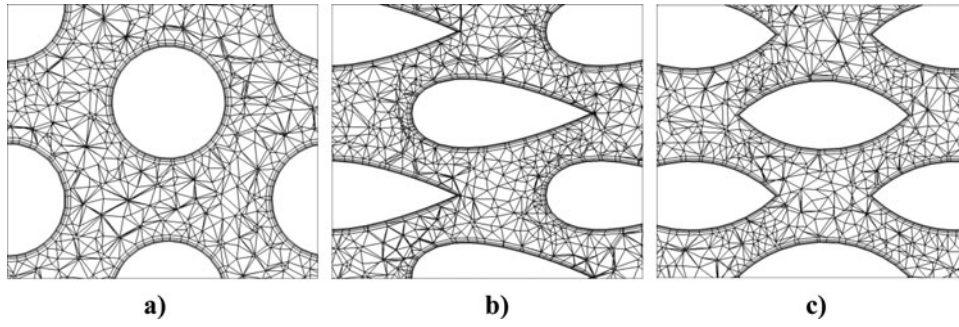


Figure 2. Typical hybrid structured/unstructured computational grid for arrays of micro pin-fins having: (a) circular, (b) symmetric airfoil, and (c) symmetric convex cross sections.

Hybrid computational grids of approximately 7 million 3D grid cells were used for the analysis of each configuration in this study. The grid consisted of four structured layers of hexahedral cells smoothly clustered towards each solid boundary, with tetrahedral cells filling the rest of the 3D space. Figure 2 shows such computational grid for each of the three configurations.

Reynolds Averaged Navier-Stokes (RANS) [14] equations with low intensity $k-\varepsilon$ turbulence model were used to simulate the turbulent flow and convective heat transfer in all cases.

$$\nabla \cdot \vec{V} = 0 \quad (1)$$

$$\rho (\vec{V} \cdot \nabla) \vec{V} = \nabla \cdot \left[-p \vec{I} + \mu \left(\nabla \vec{V} + (\nabla \vec{V})^T \right) \right] \quad (2)$$

$$\rho C_p (\vec{V} \cdot \nabla) T = (\vec{V} \cdot \nabla) p + \nabla \cdot (k \nabla T) \quad (3)$$

Notice that Eq. (3) reduces to the heat conduction equation that was solved in the solid domain. The conjugate heat transfer solver was validated again in experimental results by Balakrishna and Gangadhar Praveen Ketha [15].

It can be reported that the Reynolds number ranged from 80 to 180. Although this is in the laminar regime, Alfieri *et al.* [16] and Dennis and Dulikravich [17] demonstrated, it can still lead to vortex shedding. The steady-state heat conduction equation was solved in the 3D solid domain. The fluid and solid domains were coupled through their shared boundary and the iteration process was stopped when the residuals for both domains, and on the shared boundary, converged to six orders of magnitude below their initial values. A grid convergence study was performed utilizing a grid with 7 million and a grid with 12 million cells. The results obtained using these two grids deviated by less than 0.5% confirming grid independence beyond 7 million grid cells.

Geometry definition

The performances of all three proposed micro pin-fin shapes were optimized allowing for a fair comparison.

The geometric parameters for circular cross section pin-fins were the diameter and the height of the pin-fin. The geometric parameters for the airfoil and convex cross-section shapes were defined using the pin-fin height, chord length, and thickness. The airfoil configuration was defined using symmetric, four series NACA 00XX airfoils. The thickness of all walls of the electronic chip (top, bottom, and side walls) was kept constant at $30 \mu\text{m}$. The bottom external surface and the external sides of the chip were thermally insulated, thus, assuring that 100% of the heat is removed by the fluid. The outlet pressure was kept constant at 120 kPa. Water was used as the heat removing fluid where the inlet temperature was considered as a constraint. Table 1 shows the range for each of the geometric variables, inlet fluid conditions, and thermal loads that were used to construct the response surfaces.

Inverse design approach

The multi-objective constrained optimization was performed using a commercial software package mode FRONTIER [18]. The optimization methodology utilizes a response surface coupled with a genetic algorithm to arrive at a Pareto frontier of the best trade-off solutions. The optimization process of this scale requires analysis of typically several thousand candidate solutions. Because the computational time for each 3D conjugate heat transfer analysis was over 8 hours on the parallel computer that we used, directly coupling the optimizer with the 3D conjugate heat transfer analysis code would be too costly and time consuming. For this reason, a metamodel in

Table 1. Range of design parameters and increment sizes.

Design variable	Range	Step size
Pin-fin diameter (μm)	100–200	10
Pin-fin chord length (μm)	200–300	10
Pin-fin thickness (μm)	80–160	5
Pin-fin height (μm)	100–250	50
Coolant inlet speed (m s^{-1})	1–12	0.1
Background heat flux (W cm^{-2})	650–2000	5
Hot spot heat flux (W cm^{-2})	2300–5000	10

the form of a multi-dimensional interpolation (response surface) that can calculate the interpolated objective function values for any set of design variables in less than one second is very appealing. Several response surface generation methods, including Multiquadrics, Gaussian and Inverse Quadric Radial Basis Functions, Kriging and Gaussian Process, were tested with Multiquadric Radial Basis Functions [19] outperforming the rest. Values of the objective functions obtained from the Multiquadric Radial Basis Functions interpolation differed by less than 2% from those obtained by the full 3D conjugate heat transfer analysis.

It is known that the accuracy of the multi-dimensional response surface is greatly dependent on both the number and the distribution of its high-fidelity support points. For this reason, sets of design variables defining initial population of 120 candidate micro pin-fin arrays were created using Sobol's [20] algorithm capable of uniformly distributing points throughout the multi-dimensional design variables' space.

It should be noted that Table 1 not only lists the range for geometric parameters, but also inlet conditions and thermal loads. This allows various constraints to be placed on inlet temperature while utilizing the same response surface rather than constructing an entirely different response surface using a separate population of candidate designs. It also allows a single response surface to be used to predict the effects of various magnitudes of the applied heat fluxes on maximum temperature. This allows a constraint to be effectively placed on the maximum allowable temperature without having to construct a separate response surface for it. A constraint can also be enforced on the heat flux to mimic the traditional approach of optimizing the configuration for specified externally applied heat flux. Finally, constraint can also be placed on total pressure drop (pumping power) thus offering an effective comparison of maximum temperature and maximum heat flux possible among micro pin-fin arrays having different pin-fin cross section shapes.

The three response surfaces (one for each objective function) were coupled with the NSGA-II evolutionary optimization algorithm [21] which then searched the response surfaces in a sequence to arrive at a Pareto frontier of non-dominated (the best trade-off) solutions.

In summary, the simultaneous objectives in this paper are:

1. Maximize background heat flux
2. Maximize hot spot heat flux
3. Minimize inlet pressure

subject to specified maximum temperature equality constraint $T_{\max} = 85^\circ\text{C}$, specified inlet coolant temperature of 30.85°C and specified exit pressure of 120 kPa.

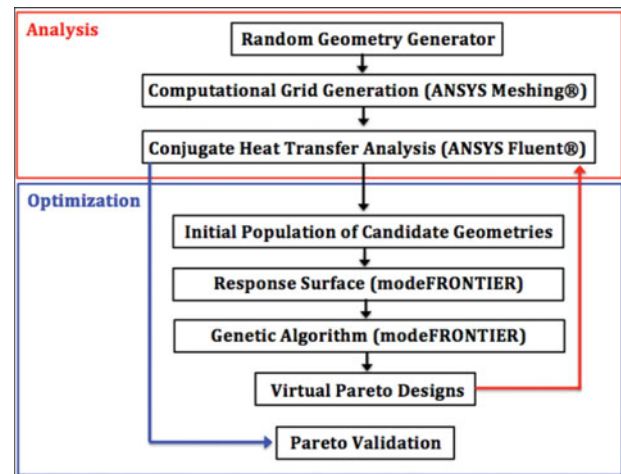


Figure 3. Workflow of different stages and software used.

Figures 3 and 4 show the workflow of design variables, objectives, and constraints implemented for the multi-objective optimization process utilizing the response surfaces.

Multi-objective optimization results

This optimization approach incorporates constraints on the inlet temperature and the electronic chip overall thickness (a sum of bottom plate thickness, height of the pin-fins connecting the bottom and the top plates, and the top plate thickness), allowing for the optimization of the configuration for different restrictions placed. The multi-objective constrained optimization was run for 100 generations and again for 200 generations. The Pareto-optimal solutions did not differ noticeably. The results presented in Figure 5 and Table 2 are after 200 optimization generations.

Figure 5a shows the Pareto optimized designs for arrays of pin-fins having circular cross sections. Figure 5b

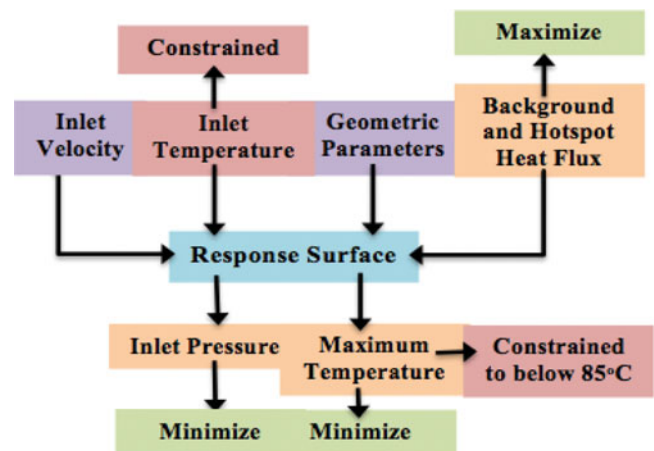


Figure 4. Formulation of the optimization problem.

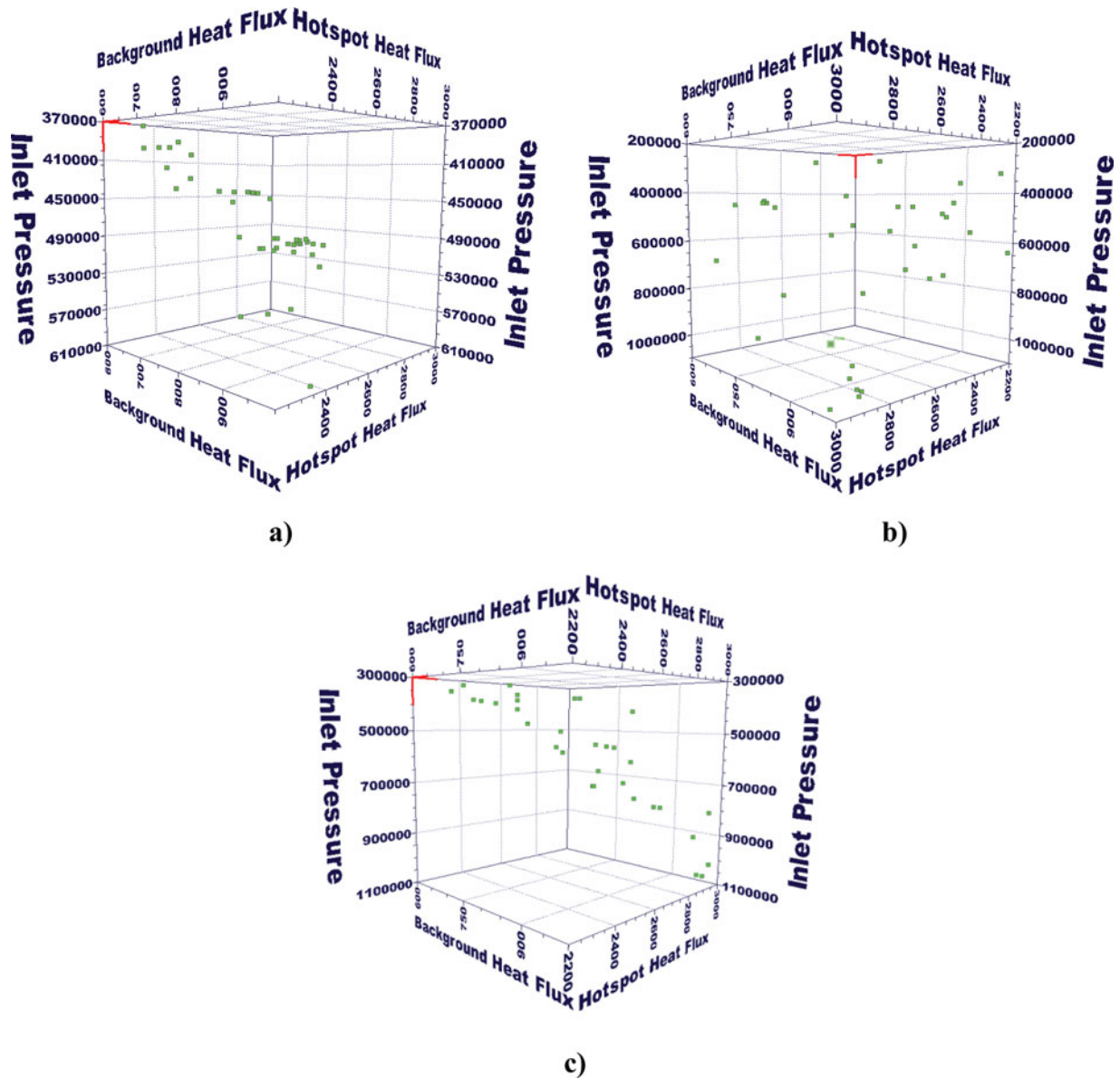


Figure 5. Three-dimensional Pareto frontiers for arrays of pin-fins having: (a) circular, (b) symmetric airfoil, and (c) symmetric convex cross sections. Pressure is measured in Pascals and heat flux is measured in W cm^{-2} .

shows the Pareto-optimized designs for arrays of pin-fins having symmetric NACA00XX airfoil cross-section

Table 2. An example of Pareto-optimized design parameters and objectives for three cross-section shapes of micro pin-fins subject to specified maximum temperature of 85°C , inlet coolant temperature of 30.85°C , and exit static pressure of 120 kPa. The three designs were chosen with similar inlet static pressures.

Pin-fin cross section	Circular	Airfoil	Convex
Pin-fin diameter (μm)	120		
Pin-fin chord length (μm)		280	220
Pin-fin thickness (μm)		120	130
Pin-fin height (μm)	250	200	250
Coolant inlet speed (m s^{-1})	6.1	4.7	4.6
Background heat flux (W cm^{-2})	805	760	995
Hot spot heat flux (W cm^{-2})	2200	2540	2240
Coolant inlet pressure (kPa)	395.171	409.746	393.390

shapes. Pareto front in this case is not well defined, indicating that more shape defining parameters should be used as design variables in this case to define more geometric complexity of the symmetric airfoil shapes. It also shows that there are very few designs that can cope with such high thermal loads, while operating at low pumping powers. It is shown that this design methodology is capable of identifying such cooling configurations.

Table 2 shows the Pareto-optimized design parameters of the cooling schemes for each of the three pin-fin cross section shapes. As one of the possible relevant bases for comparison of the performance of the arrays of micro pin-fins with three pin-fin shapes, examples in Table 2 were chosen from the respective Pareto-optimized sets so that these particular Pareto designs have similar optimized

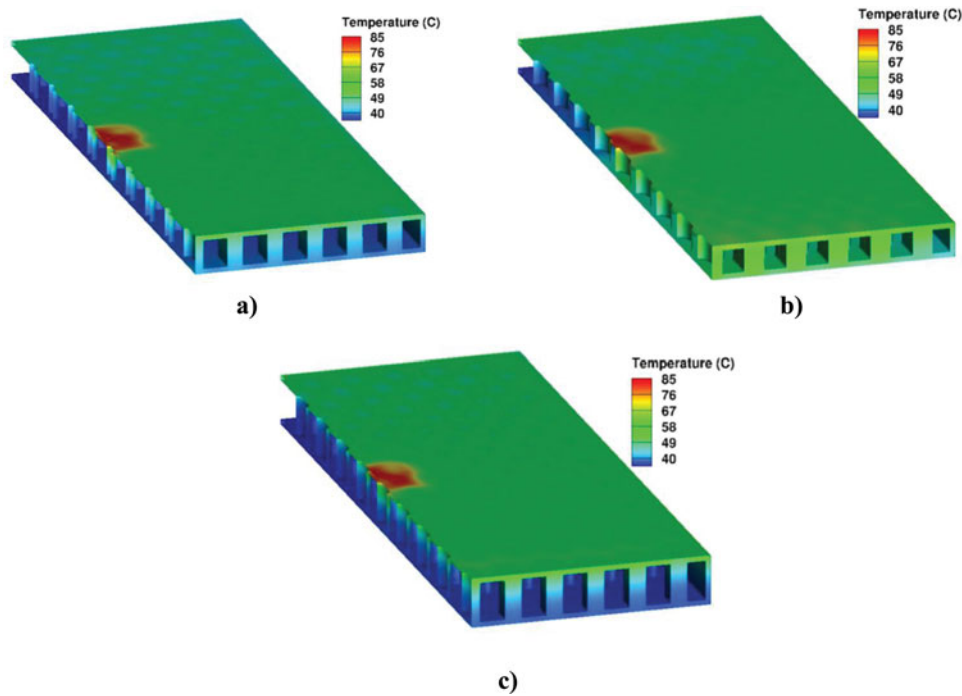


Figure 6. Temperature distributions for Pareto-optimized arrays of micro pin-fins having: (a) circular, (b) symmetric airfoil, and (c) symmetric convex cross sections.

inlet coolant pressures (with exit pressure constrained at 120 kPa).

On this basis, micro pin-fins having symmetric convex cross sections appear to have superior performance allowing for highest background heat flux and the lowest mass flow rate.

Figure 6 shows the temperature distribution on the three Pareto-optimized micro pin-fin array cooling configurations each having pin-fins with different cross-section shapes. It can be seen that arrays with airfoil and convex pin-fins resulted in a lower hot spot temperature even when a large heat flux was applied to both the background and the hot spot area. The airfoil and convex designs are known [10] to eliminate the flow recirculation at the trailing edge of the pin-fins, leading to improved heat transfer.

It should be noted that each of the three configurations presented is a randomly selected Pareto design from the corresponding Pareto frontiers. Another designs can be selected from the Pareto frontiers that best achieve the required performance. For example, the three designs could have been chosen with similar background heat fluxes.

Previous research [10] has shown that, under similar conditions, the convex shape requires less pumping power followed by the airfoil and circular configurations. The conditions under which these Pareto designs were analyzed are not the same, making a fair comparison more difficult. Although the airfoil and convex configurations here require more pumping power (Table 2), they have

been shown to handle a higher applied heat flux while resulting in lower maximum temperature. Since any other Pareto design can be selected from the Pareto optimal set, an optimized design can be easily found that best satisfies the needed performance with a pressure drop similar to the value offered by the pin-fins with circular cross section.

It can be reported that nowhere in the fluid domain did the pressure drop to the vaporization pressure of water thus assuring a single phase liquid cooling.

Off-design performance maps for optimized arrays of pin-fins

A visual perception of the interaction among inlet fluid temperature, inlet fluid pressure, and resulting maximum temperature is also a valuable tool when predicting performance of a given array of micro pin-fins with given values for applied heat fluxes. Results in Figures 7 and 8 were created for a 10 mm × 10 mm electronic chip and assuming symmetry. The entire external surface of the top wall was exposed to the uniform heat flux. A hot spot was centrally located on the top surface with a footprint of 0.25 × 0.25 mm. All optimized geometric and thermal parameters are given in Table 2.

Figure 7 gives an example of a performance surface when using an already optimized array of micro pin-fins having circular cross sections. Figure 8 shows the performance surface when using an already optimized array of

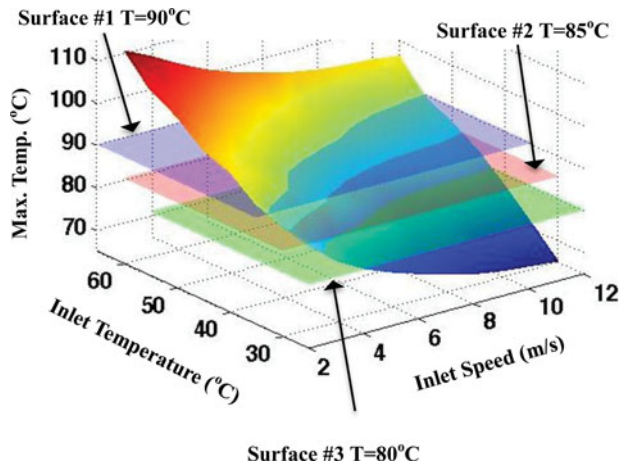


Figure 7. Performance map for a larger array of micro pin-fins having circular cross sections.

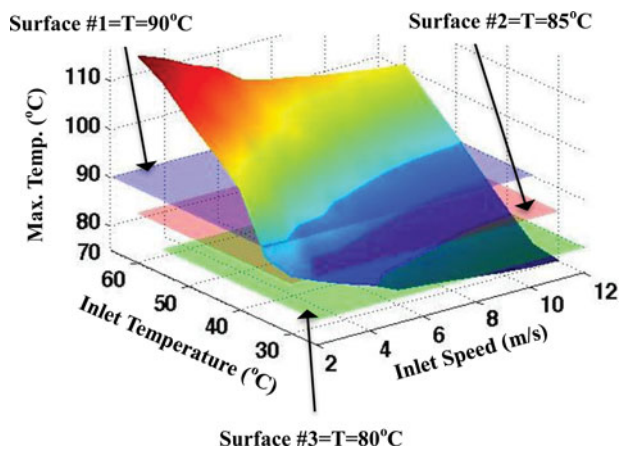
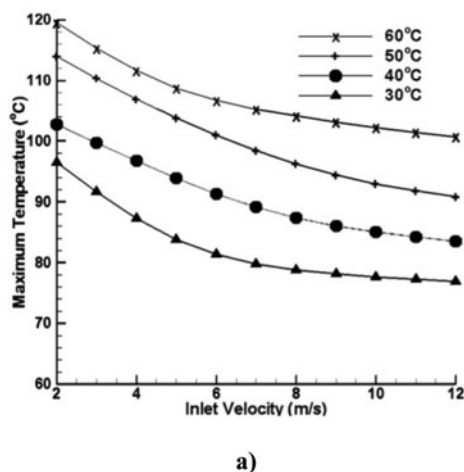


Figure 8. Performance map for a larger array of micro pin-fins having symmetric convex cross sections.

micro pin-fins having symmetric convex cross sections. For example, if inlet water temperature increases from 30°C to 40°C, inlet water speed will have to increase from



approximately 4 m s^{-1} to 10 m s^{-1} (see Figure 11) to keep the maximum temperature below 85°C (Surface #2).

Figure 9a and 9b shows the effect of various inlet velocity and inlet temperature on the maximum temperature in the Pareto-optimized arrays of micro pin-fins having circular and symmetric convex cross sections, respectively. From Figures 9a and 9b it is apparent that micro pin-fins with circular cross sections have much tighter range of inlet fluid temperature and pressure when trying to maintain the maximum temperature below a specified value.

Comparison of Figures 10a and 10b vividly demonstrates the advantage of using optimized symmetric cross-section pin-fins instead of the optimized circular cross-section pin-fins. That is, for the same inlet coolant temperature and inlet coolant speed (thus, mass flow rate) the maximum achieved temperature at the hot spot will be definitely lower when using symmetric convex cross-section pin-fins instead of circular cross section pin-fins.

Figure 11 shows the influence of inlet velocity on the pressure drop for the arrays with micro pin-fins having circular cross section and symmetric convex cross section. It should be noted that for this cooling performance analysis, the energy conservation equation was decoupled from the momentum and mass conservation equations since viscosity of water was assumed not to depend on temperature in the range between 80°C and 90°C. Thus, for a given inlet coolant speed, the coolant inlet temperature does not influence the pressure drop.

The absolute exit pressure was enforced to be 120 kPa = 120,000 Pa = 1.20 atm. The computationally obtained absolute inlet pressure was between 200 kPa = 2.0 atm (for inlet speed of 2 m s^{-1}) and 3000 kPa = 30.0 atm for inlet speed of 12 m s^{-1} when using an optimized array of micro pin-fins having symmetric convex cross sections. The required inlet pressure increases to an

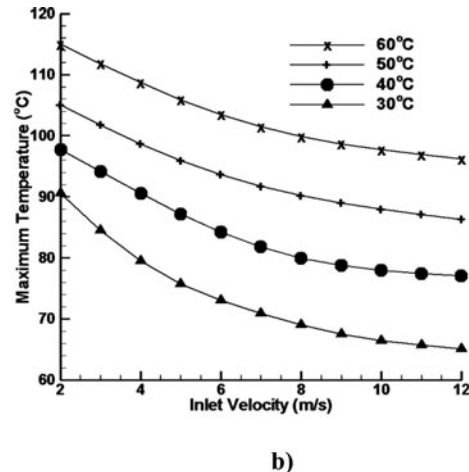


Figure 9. Influence of inlet speed of the coolant on maximum temperature subject to varying inlet coolant temperatures for Pareto-optimized arrays of micro pin-fins having cross sections of: (a) circular, and (b) symmetric convex shape.

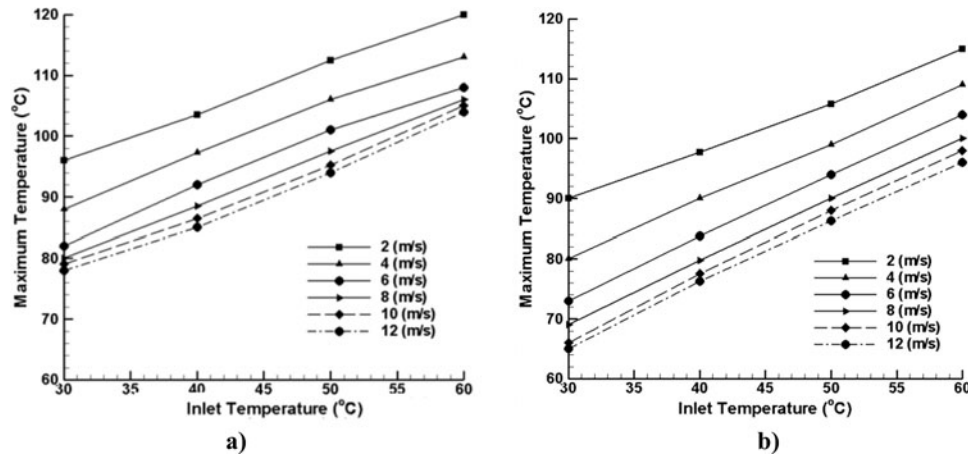


Figure 10. Influence of inlet coolant temperature on maximum temperature when varying inlet coolant speed for optimized cooling arrays having pin-fins of: (a) circular, and (b) symmetric convex cross sections.

exorbitant 9000 kPa = 90.0 atm for inlet speed of 12 m s⁻¹ when using an optimized array of micro pin-fins having circular cross sections so that 100 percent of input heat is removed, while keeping maximum temperature at 85°C.

pin-fins, the adiabatic conditions on the bottom and the side walls of the array guaranteed that the entire amount of heat entering through the top surface of the top wall is fully removed by the moving coolant. Then, thermal cooling efficiency can be expressed as

$$\eta = \frac{\dot{q}_{in/background} (A_{top} - A_{hotspot}) + \dot{q}_{in/hotspot} A_{hotspot}}{\dot{q}_{in/background} (A_{top} - A_{hotspot}) + \dot{q}_{in/hotspot} A_{hotspot} + V_{in} A_{in} (p_{in} - p_{exit})} \quad (4)$$

The coefficient of cooling efficiency, η , can be formulated as the ratio of total amount of heat removed by the coolant divided with the sum of the total amount of heat inputted through the top surface of the top wall plus the idealized pumping power [22, 23]. During the entire process of optimizing the cooling arrays of micro

pin-fins, the adiabatic conditions on the bottom and the side walls of the array guaranteed that the entire amount of heat entering through the top surface of the top wall is fully removed by the moving coolant. Then, thermal cooling efficiency can be expressed as

From Equation (4) and Figure 11, it is evident that increasing inlet coolant speed and coolant inlet pressure will rapidly decrease the cooling efficiency of the micro pin-fin array cooling concept. However, these increases are required in order to keep the maximum temperature below a specified level when operating the micro pin-fin array cooling system in the environments where the inlet coolant temperature is already high.

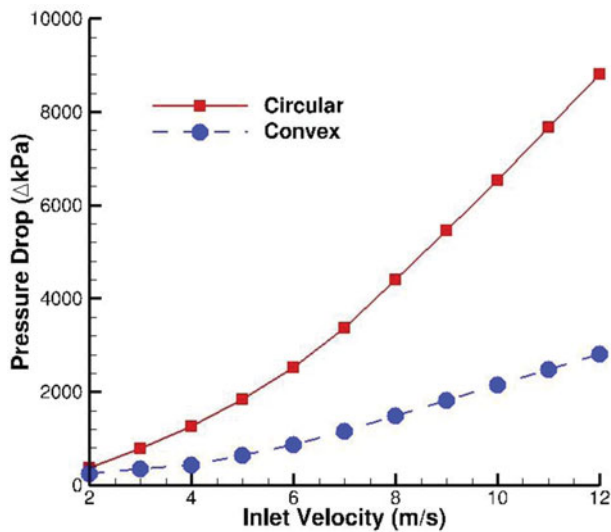


Figure 11. Influence of inlet coolant speed on pressure drop for optimized arrays of micro pin-fins having circular (a) and symmetric convex (b) cross-section shapes of pin-fins in order to keep the maximum temperature below 85°C.

Conclusions

A novel inverse approach to design of cooling schemes for high heat flux cooling micro pin-fin array based devices is proposed when they contain a hot spot. It combines geometric parameters and operating conditions to formulate a multi-objective optimization problem. This approach not only allows for the optimization of cooling scheme for a specified heat flux, but also allows for the design of a cooling configuration that can maximize the allowable applied heat flux for a given material temperature limit and the available inlet coolant temperature. A multi-objective optimization was carried out for arrays of micro pin-fins having circular, symmetric airfoil and symmetric convex cross sections to attain maximum allowable heat flux, while simultaneously reducing the pumping power and maximum temperature at the hot spot. The input

parameters used to define the cooling scheme included the size and height of the pin-fins and inlet coolant temperature and speed. A constraint was placed on the maximum allowable temperature and can also be applied to the inlet temperature. The multi-objective optimization was efficiently carried out using response surfaces coupled with a genetic algorithm to arrive at a Pareto frontier of the best trade-off solutions. All configurations obtained using the inverse approach in this study, show that efficient thermal management is possible for heat fluxes higher than those reported in the literature in the case of silicon based arrays of micro pin-fins. Requirements on inlet pressure and inlet speed of the coolant have been shown to rapidly increase with the increase of the inlet coolant temperature in order to keep the hot spot temperature below a specified value. Micro pin-fin arrays with symmetric convex cross-section shapes of the pin-fins were shown to require lower inlet pressures and inlet speeds for the same inlet coolant temperatures in order to keep the maximum temperature below a specified value, thus, offering better overall off-design performance of the optimized arrays of micro pin-fins.

Nomenclature

$A_{hotspot}$	hot spot surface area on the top surface of the top wall, m^{-2}
A_{in}	surface area of the cross section of the inlet of the cooling array of micro pin-fins, m^2
A_{top}	surface area of the top surface of the top wall exposed to heating, m^2
C_p	specific heat at constant pressure per unit volume, $J m^{-3} K^{-1}$
\vec{I}	identity tensor (unit tensor)
k	thermal conductivity, $W m^{-1} K^{-1}$
p	fluid pressure, Pa
\dot{q}	heat flux per unit area, $W m^{-2}$
T	absolute temperature, K
\vec{V}	velocity vector of the cooling fluid, $m s^{-1}$
V	average speed of the cooling fluid at inlet of the micro array, $m s^{-1}$

Greek symbols

η	thermal cooling efficiency
ρ	fluid density, $kg m^{-3}$
μ	viscosity of the cooling fluid, $kg m^{-1} s^{-1}$

Superscripts

T	transpose of a matrix
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