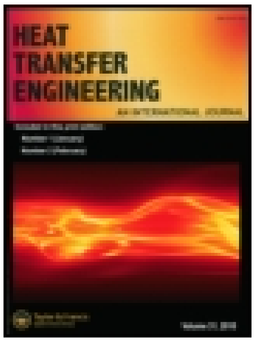

Heat Transfer Engineering

ISSN: 0145-7632 (Print) 1521-0537 (Online) Journal homepage: http://www.tandfonline.com/loi/uhte20

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To cite this article: Sohail R. Reddy, Abas Abdoli, George S. Dulikravich, Cesar C. Pacheco, Genesis Vasquez, Rajesh Jha, Marcelo J. Colaco & Helcio R. B. Orlande (2016): Multi-Objective Optimization of Micro Pin-Fin Arrays for Cooling of High Heat Flux Electronics with a Hot Spot, Heat Transfer Engineering, DOI: 10.1080/01457632.2016.1242953

To link to this article: http://dx.doi.org/10.1080/01457632.2016.1242953

Accepted author version posted online: 29 Sep 2016.
Published online: 29 Sep 2016.

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HEAT TRANSFER ENGINEERING
2017, VOL. 0, NO. 0, 1–12
http://dx.doi.org/10.1080/01457632.2016.1242953

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Multi-Objective Optimization of Micro Pin-Fin Arrays for Cooling of High Heat Flux Electronics with a Hot Spot

Sohail R. Reddy[a], Abas Abdoli[a], George S. Dulikravich[a], Cesar C. Pacheco[b], Genesis Vasquez[a], Rajesh Jha[a], Marcelo J. Colaco[b], and Helcio R. B. Orlande[b]

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ABSTRACT

This article presents fully three-dimensional conjugate heat transfer analysis and a multi-objective, constrained optimization to find sizes of pin-fins, inlet water pressure, and average speed for arrays of micro pin-fins used in the forced convection cooling of an integrated circuit with a uniformly heated 4×3 mm footprint and a centrally located 0.5×0.5 mm hot spot. Sizes of micro pin-fins having cross sections shaped as circles, symmetric airfoils, and symmetric convex lenses are optimized to completely remove heat due to a steady, uniform heat flux of 500 W cm^{-2} imposed over the entire footprint (background heat flux) and a steady, uniform heat flux of 2000 W cm^{-2} imposed on the hot spot area only (hot spot heat flux). The two simultaneous objectives are to minimize maximum substrate temperature and minimize pumping power, while keeping the maximum temperature constrained below 85°C and removing all of input thermal energy by convection. The design variables are the inlet average velocity and size of the pin-fins. A response surface is generated for each of the objectives and coupled with a genetic algorithm to arrive at a Pareto frontier of the best trade-off solutions. Numerical results show that, for a specified maximum temperature, optimized arrays with pin-fins having symmetric convex lens shapes create the lowest pressure drop, followed by the symmetric airfoil and circular cross-section pin-fins. An a posteriori three-dimensional stress–deformation analysis incorporating hydrodynamic and thermal loads shows that Von-Mises stress for each pin-fin array is significantly below the yield strength of silicon, thus, confirming structural integrity of such arrays of micro pin-fins.

Introduction

The performance of cooling systems is a major limiting factor in modern Integrated Circuit (IC) devices [1]. Various methods of cooling have been investigated by numerous authors. For example, Abdoli and his co-workers [2,3] has carried out conjugate heat transfer analysis on two-floor, single-phase flow in micro channels to study their effects in cooling chips with hot spots. Sahu et al. [4] applied a hybrid cooling scheme, combining microfluidic and solid-state cooling for cooling of a hot spot with heat flux of 250 W cm^{-2}. Alfieri et al. [5] numerically investigated cooling of three-dimensional (3D) stacked electronic chips with 50 W cm^{-2} background and 125 W cm^{-2} hot spot heat fluxes. Dembla et al. [6] also studied the fine pitch TSV (Through Silicon Vias) integration in silicon micro pin-fin heat sinks for three-dimensional ICs with 100 W cm^{-2} heat load. Kosar and Peles [7] and Ndao et al. [8] experimentally evaluated the performances of sparse arrays of micro pin-fins having circular, symmetric airfoil, oval and square cross sections. A parametric type optimization was performed by Tullius et al. [9] performed on a minichannel with cooling arrays of short micro pin-fins having six cross-section shapes. Their study resulted in important correlations between geometric parameters of the pin-fins, Nusselt number, and heat transfer coefficients. However, their study did not use a fully 3D conjugate heat transfer analysis, it did not include a hot spot, it did not perform a true multi-objective optimization, and it did not account for turbulent character of the coolant flow.

The next generation of ICs is expected to produce heat fluxes up to 500 W cm^{-2} as the background and more than 1000 W cm^{-2} at hot spots [10]. Abdoli et al. [11] performed detailed 3D conjugate heat transfer analyses of single floor and double floor dense arrays of micro pin-fins with circular, symmetric airfoil and symmetric

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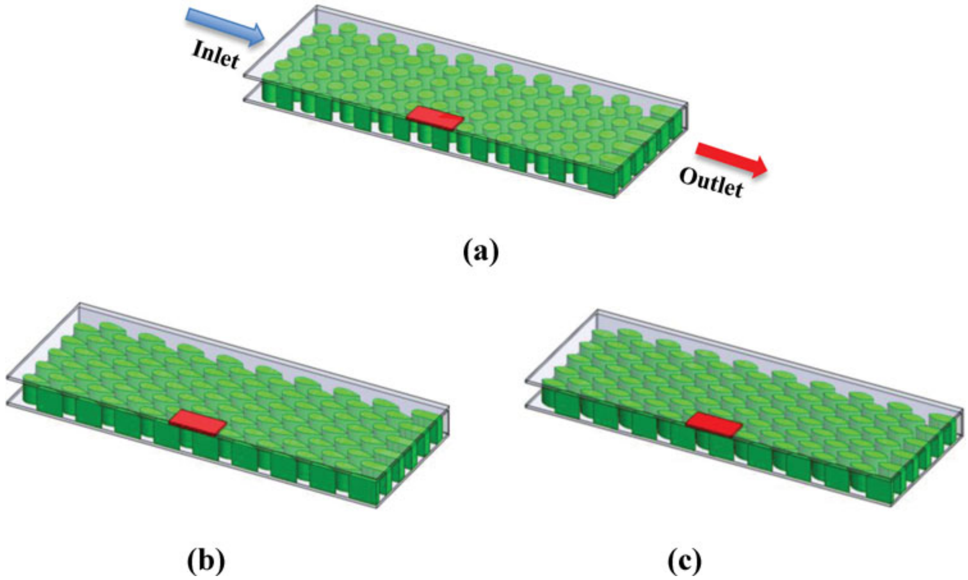

Figure 1. Cooling arrays with micro pin-fins having: (a) circular, (b) symmetric airfoil, and (c) symmetric convex cross sections. Only one half of the entire array is shown and analyzed because of symmetry. Top, bottom, and side walls are not shown for clarity.

convex lens cross sections and their effects on temperature and coolant pumping power requirements. Their analysis involved influence of a hot spot, but did not involve any optimization. Reddy and Dulikravich [12] performed a true multi-objective design optimization of the three dense arrays of micro pin-fins, but without including a hot spot. The hot spots have extremely small footprints and exceedingly high local heat fluxes causing temperatures at the hot spots to spike significantly higher than temperatures on the rest of the IC surface.

The objective of the current paper is to present a fully integrated approach to optimizing micro pin-fin cooling arrays with a hot spot, while insisting that 100 percent of the heat input is removed by the moving cooling fluid and that maximum temperature at the hot spot is constrained.

Conjugate heat transfer analysis

The conjugate heat transfer effects of the three arrays of micro pin-fins have previously been numerically analyzed in detail by Abdoli et al. [11] for nonoptimized arrays of micro pin-fins having a footprint of 2.45 × 2.45 mm with a centrally located hot spot.

In the present study, the cooling arrays of micro pin-fins (Figure 1) had a footprint of 4 × 3 mm and a centrally located hot spot measuring 0.5 × 0.5 mm (Figure 2).

This study utilized fully 3D conjugate heat transfer analyses and consequent multi-objective design optimizations of three types of arrays of micro pin-fins (Figure 1). Each array included special outlets to increase heat transfer at the outlet and to suppress backflow [11,12]. The widths of the channeled outlets were the same as the diameter or the thickness of the circular or the symmetric airfoil or the symmetric convex cross section shapes of the pin-fins, respectively. The airfoil cross-section shape for micro pin-fins was defined using a symmetric, NACA 00XX series airfoil. In a given array, all pin-fins had the same size and shape and they connected top and bottom walls of the cooling micro-array device. Top, bottom, and side walls were 30 μm thick in each of the arrays. Solid material used for pin-fins and walls was silicon.

A hybrid structured/nonstructured computational grid of approximately seven million cells was created using ANSYS Meshing® [13] for all of configurations analyzed in this study.

Four layers of structured clustered hexahedral grid cells (using inflation formulation normal to each solid boundary) were placed on each solid-fluid interface with tetrahedral grid cells generated inside solid parts of the cooling arrays and inside the fluid domain. The minimum grid cell size allowed for this inflation was one micron to satisfy the continuum assumption.

Each cooling array of micro pin-fins was exposed to a uniform background heat flux of 500 W cm^{-2} and a hot spot heat flux of 2000 W cm^{-2}. Background uniform

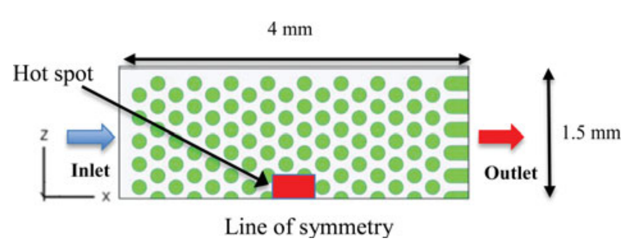

Figure 2. Top view of one half of an array of micro pin-fins and dimensions used in this study.

heat flux and hot spot heat flux were applied on the top surface of the top wall of the array. The bottom surface of the bottom wall and the outside surfaces of the side walls of the cooling arrays were treated as adiabatic to insure that the entire amount of heat intake trough the top wall was removed by the moving fluid. The cooling fluid in this work was water at an inlet temperature of 30°C, although other fluids could be used easily by changing physical properties of the coolant in the input file. Gage pressure of 20 kPa was applied to the outlets in all test cases since most electronic cooling configurations are pressurized to prevent cavitation. The results of the fully 3D conjugate heat transfer analysis are then the values of the inlet gage pressure and the inlet coolant average speed. Together with the fixed width of the micro pin-fin array and the specified pin-fin heights, this determines the ideal pumping power requirement. That is, rather than directly minimizing the pumping power requirement, we minimized it indirectly by simultaneously minimizing pressure drop and inlet coolant speed, thus, explicitly exposing their individual contributions.

Fully 3D conjugate heat transfer analyses performed in this work utilized ANSYS Fluent® [13] software to solve Navier–Stokes equations with low Reynolds number k-ε turbulence model in the fluid domain and only energy balance equation (with velocity components explicitly set to zero) in the solid domain. Each fully 3D conjugate heat transfer analysis was converged until each of the residuals in the Navier–Stokes equations solver was reduced by six orders of magnitude.

To test the numerical results for their grid independency, three types of cooling arrays of micro pin-fins were also analyzed with a hybrid computational grid containing 12 million cells. Maximum temperature and inlet pressure calculated with this refined grid deviated less than 0.5% from the analysis results obtained on a typical grid of 4.2 million points used in this study, confirming grid independency.

It can be reported that the Reynolds number ranged from 235 to 1200 in test cases presented in this paper. Although this is in the laminar flow regime, Alfieri et al. [14] and Dennis and Dulikravich [15] demonstrated it can still lead to vortex shedding. The laminar flow model was numerically tested for a micro pin-fin array cooled electronic chip with a hot spot. However, the maximum temperature obtained with the low intensity turbulence flow model was 2°C lower then when analyzing the same configuration conjugate heat transfer with the laminar flow model. The pressure drop was significantly larger in the case of low Reynolds number turbulence model. This is why we decided to use the low Reynolds number k-ε turbulence model in this work.

Multi-objective design optimization

Having seen the effect of each pin-fin cross-section shape on the temperature and the flow-field in case of nonoptimized cooling arrays with a hot spot [11] and in case of Pareto-optimized cooling arrays without a hot spot [12], an optimization of cooling micro arrays with a hot spot is appealing. To allow for a fair comparison, each of the three array configurations was optimized. For the circular cross section pin-fins, three design variables were used to define the configuration: cooling fluid average inlet speed, pin-fin height, and the pin-fin diameter. The design variables for the pin-fins with symmetric airfoil cross sections and the pin-fins with symmetric convex cross sections were: cooling fluid average inlet speed, pin-fin height, pin-fin thickness and its chord length.

Although an ideal optimization study would involve simultaneously optimizing shapes and sizes of individual pin-fins in an array, this would significantly increase the total number of design variables thus making such a large scale optimization process too time consuming when using a relatively small parallel computer. For this reason, all pin-fins in a given micro array were identically shaped and sized during the optimization. That is, when updating the sizes of the pin-fins in a given array, all the pin-fins were equally updated. Locations of the pin-fins were kept constant and the bounds for each variable (pin-fin height, diameter, chord length, maximum thickness) were chosen so they do not produce self-intersecting geometries.

The simultaneous objectives of the multi-objective optimization were:

a) minimize the maximum temperature, and
b) minimize the inlet pressure of the cooling fluid (for a fixed exit pressure, thus minimizing the required pumping power).

The constraint was that maximum temperature must be less than 85°C as dictated by the relevant properties of silicon used for the solid parts of the cooling arrays of micro pin-fins.

Figure 3 shows various software modules used and the workflow in this design optimization effort. A computational grid was created for each of the initial candidate designs using ANSYS Meshing® [13], a 3D conjugate heat transfer analysis was carried out in ANSYS Fluent®, and the steady-state, stress-deformation analysis was carried out in ANSYS Structural®. The multi-objective constrained optimization in the entire study was performed using modeFRONTIER [16] software. No other analysis or optimization algorithms were used in this work.

When run in parallel on eight cores, each 3D conjugate heat transfer analysis took approximately seven hours.

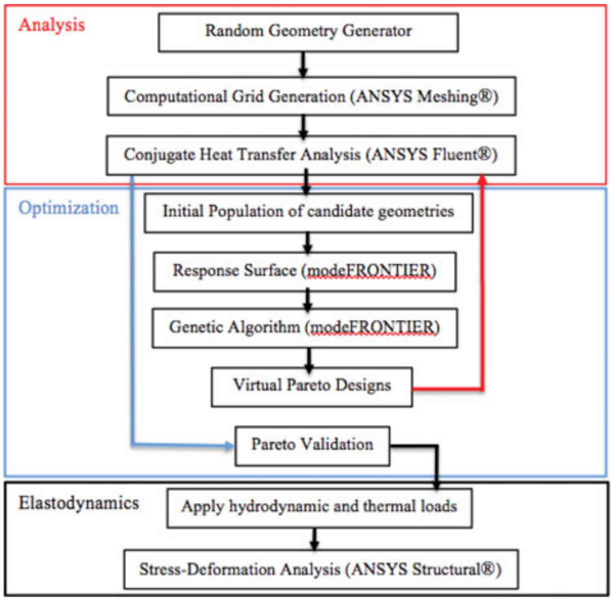

Figure 3. Workflow of different stages and software modules used.

Due to this computationally expensive analysis, a metamodel was used in this study in the form of a response surface approximation based on polynomials of Multiquadric Radial Basis Functions [17]. The response surface was created using objective function values of 30 fully 3D high fidelity conjugate heat transfer analyses for cooling arrays having micro pin-fins with circular cross sections and 50 such analyses in cases of pin-fins with symmetric airfoil and symmetric convex cross sections. It is known that the distribution of these initial designs within the design space created by the variable bounds will influence the accuracy of the response surface. For this reason, the initial population of candidate designs was created using a pseudo-random sequence generator [18] to allow for a uniform distribution of candidate designs within the multi-dimensional design space.

To carry out the optimization, the response surfaces (one for each objective function) were coupled with the Non-Dominated Sorting Genetic Algorithm (NSGA-II), developed by Deb and co-workers [19,20], that is an option in the mode Frontier optimization software [16]. The NSGA-II algorithm was used to search the objective function topology produced by the response surface to arrive at a Pareto frontier of the best trade-off solutions. The response surface "construction" in this study took less than 20 seconds, while the optimization took approximately 300 seconds.

Results of multi-objective optimization

Since results of the detailed 3D conjugate heat transfer analyses for the three types of the micro pin-fin arrays with a hot spot were already published [12], we will focus here on presenting results of multi-objective optimization of the cooling arrays with a hot spot and comparison with results for optimized cooling arrays without a hot spot [11].

A total of three optimization studies were carried out; one for each configuration of pin-fins.

Table 1 shows the range for each design variable and the step size used to define the three micro-pin fin geometries. The same range was used, since both the airfoil and convex lens geometries are defined in the same manner.

Table 1. Range for design variables defining inlet conditions and the pin-fin configuration when simultaneously minimizing the maximum temperature and inlet pressure, while keeping the maximum temperature below 85°C.

Design variable	Range	Step size
Inlet velocity (m s^{-1})	1–5	0.2
Height of pin-fins (μm)	100–250	50
Diameter of circular pin-fins (μm)	100–200	10
Chord length of pin-fins (μm)	200–300	10
Thickness of pin-fins (μm)	80–160	10

Figure 4 shows Pareto frontiers obtained using two-objective optimization by coupling the NSGA-II optimization algorithm [19,20] and the response surface in modeFRONTIER software [16] as well as the initial candidate designs used to create the response surfaces. Although the response surfaces were validated for certain designs, their global accuracy is extremely difficult to achieve and validate. To investigate any discrepancies in the Pareto frontier due to local imperfections in the response surfaces, six Pareto-optimized designs were randomly selected and analyzed in ANSYS Fluent®. The results from the response surface predictions for these selected Pareto designs were within 3% error compared to those obtained from the fully 3D conjugate heat transfer analysis, thereby demonstrating the acceptable accuracy of the metamodel used.

Green curves in Figure 4 are Pareto frontiers, which are envelopes of the entire set of configurations analyzed. In case of multi-objective optimization, there is no single optimum solution. There are only trade-off solutions [19]. Solutions that form the Pareto frontier represent the best trade-off solutions among the multiple objectives. That is, the Pareto-optimal solutions cannot be improved further in any of the individual objectives without causing deterioration of the performance in at least one of the other simultaneous objectives [19].

The Pareto frontier curves show that with an increase in the maximum temperature anywhere in a particular array of the micro pin-fins, the needed inlet pressure will have lower value. Conversely, with a more stringent constraint on lowering the maximum temperature, the inlet pressure will have to increase in order to provide higher

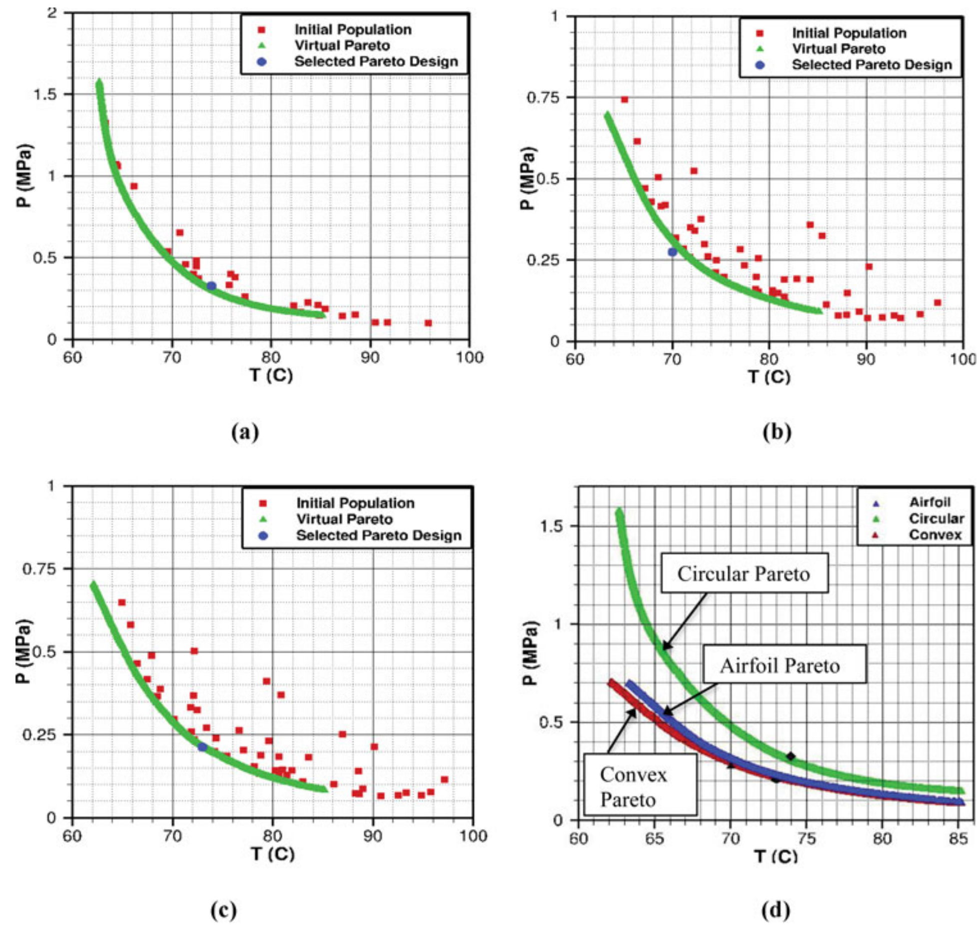

Figure 4. Maximum temperatures and corresponding inlet gage pressures for initial population and virtual Pareto designs for arrays of micro pin-fins having: (a) circular, (b) symmetric airfoil, (c) symmetric convex cross sections, and (d) superimposed Pareto frontiers for the three configurations. Figure 4d shows that micro pin-fins with symmetric convex cross sections offer superior performance in the presence of a hot spot.

average coolant speed, thus, increase in the coolant mass flow rate. The individual orange squares in Figure 4 are not results of the optimization; they are just performances of the initial, random, non-optimized individual configurations of pertinent arrays of micro pin-fins. This is why in Figures 4b and 4c some of the (not Pareto-optimal) solutions are having increased inlet pressure requirements as maximum temperature increases beyond approximately 95°C.

Since in the case of multi-objective optimization, there is no single optimum design, a lower pressure was given priority when selecting a Pareto design point from the Pareto front for comparison purposes. However, it should be mentioned that a different Pareto design can be selected from the Pareto front that best satisfies the needed performance.

Figure 5 shows the temperature distributions on the outer surfaces of the cooling arrays having Pareto-optimized micro pin-fins with circular, symmetric airfoil, and symmetric convex-shaped cross sections. It can be seen from Figure 5 that the maximum temperature for the Pareto-optimized array of micro pin-fins with no hot spot (Figure 5a and 5e and Figure 6c) is significantly lower than the maximum temperature of the arrays with Pareto-optimized pin-fins with a hot spot (Figures 5b, 5d, 5f). The higher temperatures in the lower region of the pin-fins having circular cross sections (Figures 5a and 5b) indicate that the heat is not being efficiently transferred between the pin-fins and the moving cooling fluid by convection and that it is being carried towards the bottom end of the pin-fins by conduction. This is not the case in the cooling arrays with optimized pin-fins having symmetric airfoil (Figures 5c and 5d) and symmetric convex (Figures 5e and 5f) cross sections, as can be seen from the lower temperatures toward the bottom of the pin-fins in these cases.

Figure 6 shows the spatial variation of the coolant speed and streamlines for each of the three Pareto-optimized arrays' configurations at mid height in a case with no hot spot (Figures 6a, 6c, 6e) and with a hot spot (Figures 6b, 6d, 6f). The reason for poor convection heat transfer performance of circular cross-section pin-fins in both cases is now evident since there is a large flow

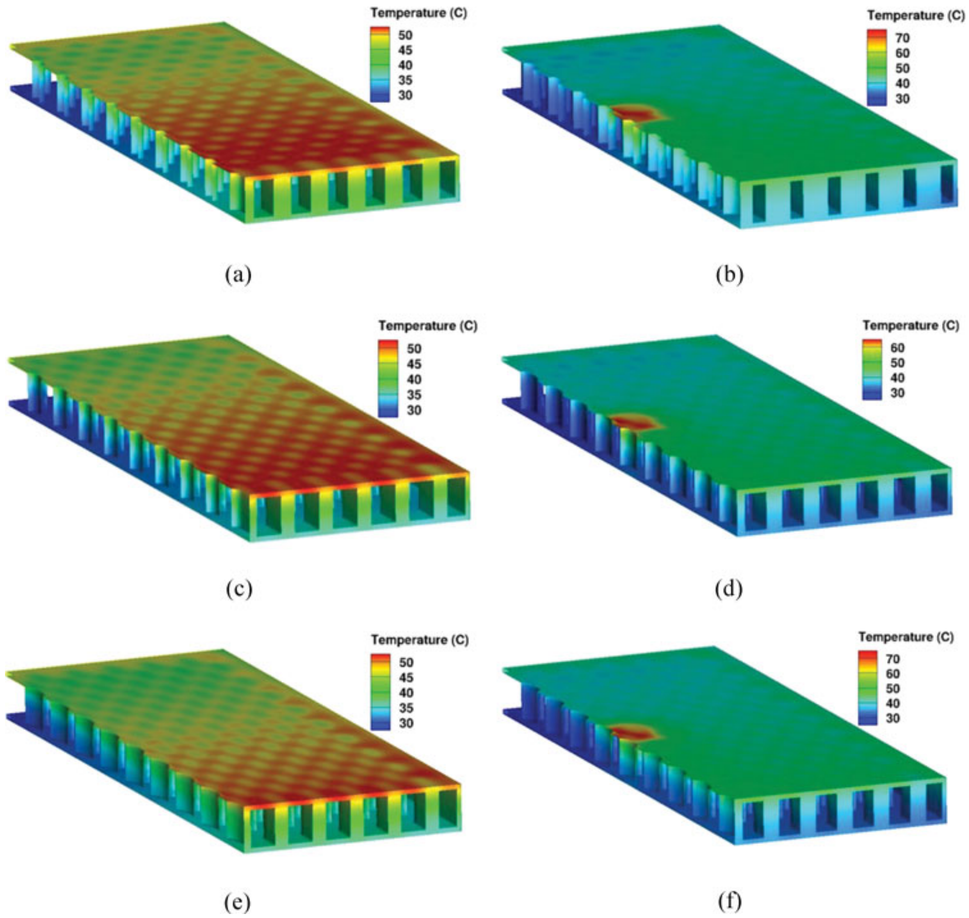

Figure 5. Comparison of surface temperature distribution in case of Pareto-optimized arrays of micro pin-fins 250 μm high with no hot spot (a, c, e) and with a hot spot (b, d, f). Cross sections of pin-fins were circular (a, b), symmetric airfoil (c, d), and symmetric convex (e, f).

separation behind each circular cross-section pin-fin. Optimization can reduce the size of this domain of recirculating warm fluid, but it cannot eliminate it in the case of micro pin-fins with circular cross sections (Figures 6a and 6b). In the case of micro pin-fins having symmetric airfoil (Figures 6c and 6d) and symmetric convex (Figures 6e and 6f) cross sections, optimization effectively eliminated flow separation by properly determining thickness and chord length of such pin-fins.

Potentially larger improvements in the heat transfer and efficiency of the cooling arrays of micro pin-fins are possible by allowing for more flexible geometries of the symmetric airfoil and symmetric convex cross sections. In addition, allowing for sizes and clustering of the pin-fins in an array, to optimally vary with a distance from the known hot spot location, could help reduce the hot spot temperature.

Table 2 provides a numerical comparison between parameters defining Pareto-optimized cooling arrays without a hot spot (left columns) and with a hot spot (right columns) for four heights of the micro pin-fins having circular cross sections. It should be pointed out that all of these results were obtained by enforcing adiabatic thermal boundary conditions on the bottom and side walls of the cooling arrays. That is, by assuring that the entire amount of the input thermal energy is removed by convection. In the case of Pareto-optimized cooling arrays without a hot spot (Table 2), the maximum temperature only weakly depends on the variation of pin-fin aspect ratio (height / diameter). On the other hand, the maximum temperature

Table 2. Pareto-optimized design variables for arrays of micro pin-fins having circular cross section: left columns—no hot spot, right columns—with a hot spot. Reynolds number is based on inlet coolant speed and pin-fin diameter.

Pin-fin height	100 μm		150 μm		200 μm		250 μm	
Diameter (μm)	140	180	120	190	110	180	120	170
Inlet fluid speed (m s^{-1})	3.0	2.6	2.6	2.2	2.8	2.2	2.2	2.8
Max. temp. (°C)	58	81	56	76	54	74	56	74
Inlet gage pressure (kPa)	278.58	212.96	146.45	391.03	131.80	287.58	101.41	327.77
Reynolds number	375	418	278	373	275	354	236	425

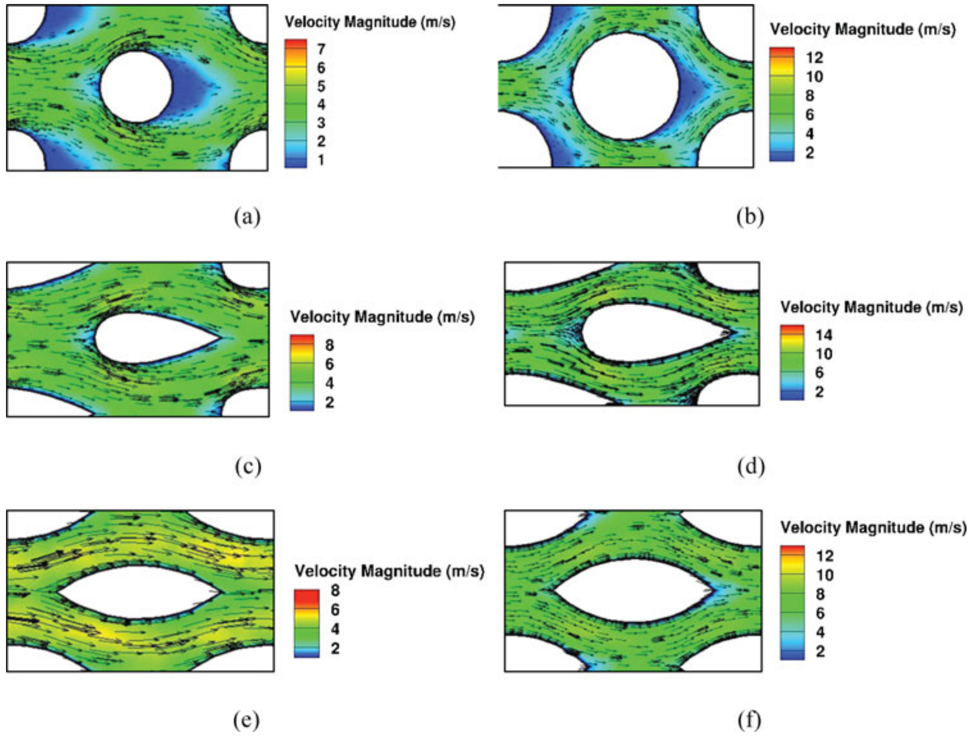

Figure 6. Velocity magnitude fields and streamlines for Pareto-optimized cross section shapes at the mid-height of the micro pin-fins 250 µm high in case of no hot spot (a, c, e) and with a hot spot (b, d, f).

noticeably decreases in the optimized cooling arrays with a hot spot as aspect ratio of the pin-fins increases.

Table 3 provides a numerical comparison between parameters defining Pareto-optimized cooling arrays without a hot spot (left columns) and with a hot spot (right columns) for four heights of the micro pin-fins having symmetric airfoil cross sections. It is noticeable that maximum temperature decreased monotonically for the optimized cooling arrays with a hot spot as the aspect ratio (height / chord length) of the pin-fins monotonically increased, while the chord length stayed almost constant. In the case of Pareto-optimized cooling arrays without a hot spot, the maximum temperature decreased more rapidly as the aspect ratio and inlet velocity increased, while chord length monotonically decreased and maximum thickness remained unchanged.

Table 4 provides a numerical comparison between parameters defining Pareto-optimized cooling arrays without a hot spot (left columns) and with a hot spot (right columns) for four heights of the micro pin-fins having symmetric convex cross sections. It is noticeable that maximum temperature decreased monotonically with the increase in the aspect ratio of the pin-fins for the optimized cooling arrays with a hot spot as other optimized quantities varied nonmonotonically. In the case of Pareto-optimized cooling arrays without a hot spot, chord length increased monotonically and maximum thickness remained unchanged with increase in the aspect ratio.

Calculated inlet pressures (Tables 2, 3 and 4) are different because exit pressure is the same for each of the three micro pin-fin arrays, while each cooling array contains differently sized and shaped pin-fins creating different levels of pressure loss.

The idealized pumping power required to fully compensate for viscous losses can be formulated [21] on the basis of total pressure loss between inlet and exit of the micro pin-fin array. Since average coolant speeds at these two stations must be the same because of the mass

Table 3. Pareto-optimized design variables for arrays of micro pin-fins having symmetric airfoil cross section: left columns—no hot spot, right columns—with a hot spot. Reynolds number is based on inlet coolant speed and pin-fin chord length.

Pin-fin height	100 µm		150 µm		200 µm		250 µm	
Chord length (µm)	280	290	270	280	220	280	210	280
Thickness (µm)	90	90	90	90	90	100	90	110
Inlet fluid speed (m s^{-1})	1.8	4.6	2.2	4.6	2.6	4.4	3.0	4.4
Max. temp. (°C)	73	75	60	74	55	73	52	70
Inlet gage pressure (kPa)	87.69	305.20	87.57	240.56	87.03	235.76	96.55	274.42
Reynolds number	450	1191	530	1150	510	1100	562	1100

Table 4. Pareto-optimized design variables for arrays of micro pin-fins having symmetric convex cross section: left columns—no hot spot, right columns—with a hot spot.

Pin-fin height	100 μm		150 μm		200 μm		250 μm	
Chord length (μm)	230	280	250	290	270	260	290	270
Thickness (μm)	90	90	90	140	90	130	90	110
Inlet fluid speed (m s^{-1})	2.6	4	2.4	4.2	2.8	3.4	2.6	4
Max. temp. (°C)	63	79	59	67	53	72	53	73
Inlet gage pressure (kPa)	118.49	224.97	87.18	370.10	98.06	244.63	89.00	212.55
Reynolds number	534	1000	536	1087	675	789	673	964

Reynolds number is based on inlet coolant speed and pin-fin chord length.

conservation, the idealized pumping power requirement can be calculated from Eq. (1) as

$$P_{ideal} = (\rho VA)_{in}(\Delta p/\rho) = A_{in}V_{in}(p_{in} - p_{exit}) \tag{1}$$

Coefficient of efficiency of the cooling scheme can be defined as depicted in Eq. (2)

$$\eta = \frac{\dot{Q}_{conv}}{\dot{Q}_{input} + P_{ideal}} = \frac{\dot{Q}_{conv}}{\dot{Q}_{input} + (VA)_{in}(p_{in} - p_{exit})} \tag{2}$$

It should be noticed that in the cooling arrays discussed in this paper, the total amount of heat that entered each cooling array equals the total amount of heat convected with the moving cooling fluid as given in Eq. (3).

$$\dot{Q}_{conv} = \dot{Q}_{input} = \dot{q}_{in/background}(A_{top} - A_{hotspot}) + \dot{q}_{in/hotspot}A_{hotspot} \tag{3}$$

Using dimensions and heat fluxes utilized in this paper it follows that

$$\dot{Q}_{conv} = \dot{Q}_{input} = 500\frac{\mathrm{W}}{\mathrm{cm}^2}(0.4 \times 0.3\,\mathrm{cm}^2 - 0.05 \times 0.05\,\mathrm{cm}^2) + 2000\frac{\mathrm{W}}{\mathrm{cm}^2}(0.05 \times 0.05\,\mathrm{cm}^2) \tag{4}$$

Thus, $\dot{Q}_{conv} = \dot{Q}_{input} = 63.75$ W and inlet area is $A_{in} = 0.003\ \mathrm{m} \times H$.

Table 5 presents the resulting values of the idealized pumping power, actual pumping power, coefficient of cooling efficiency and maximum temperature for four heights of the pin-fins having circular, symmetric airfoil and symmetric convex cross sections and a hot spot. Thus, Table 5 is helpful in making the final decision which shape of the pin-fins to use and what heights of the Pareto-optimized pin-fins to use. For example, if the essential objectives are to minimize the pumping power, while assuring that the maximum temperature is well below the constrained value of 85°C, then it becomes apparent from Table 5 that the best performing arrays should have pin-fin heights either 150 μm (in case of symmetric airfoil cross section) or 200 μm (in case of circular and symmetric convex cross sections). In other words, the general trend is that increase in height of the pin-fins reduces the maximum temperature, but it also increases the pumping power. The final recommendations for the best cooling arrays of micro pin-fins are given in Table 5. They all have low pumping power requirement, relatively low maximum temperature, and an almost identical high cooling efficiency.

For the arrays of micro pin-fins, this study reveals what is maximally possible in case when all pin-fins have the same shape and size. Further improvements in heat transfer performance could be anticipated when shapes, sizes and locations of individual pin-fins are optimized simultaneously in such cooling arrays of micro pin-fins. Moreover, the objective function that should be minimized should be the ideal pumping power, since Pareto-optimized inlet conditions do not necessarily result in a decrease in pressure drop and a decrease in the average inlet fluid speed.

A posteriori stress-deformation analysis

Because of the relatively high average speed of the cooling liquid, the very small sizes of the pin-fins, the relatively weak material (silicon) and potentially high thermal gradients causing high stresses, it could be argued that structural integrity of such arrays of the micro pin-fins is questionable. That is, stresses caused by the fluid flow in addition to thermal stresses could possibly cause significant deformations and even structural failure of the micro pin-fins. Consequently, steady stresses and deformations in the pin-fins and walls were calculated in an *a posteriori* fashion after the Pareto optimization was completed by using temperature field gradients and surface stresses due to hydrodynamics calculated during the 3D thermal-fluid conjugate analysis. The conservation of steady momentum for an isotropic, linear, elastic solid body can be expressed as in Eq. (5)

$$\nabla \cdot (2G\varepsilon + \lambda \overset{\leftrightarrow}{I}\,\mathrm{tr}(\varepsilon)) + \vec{F} = 0 \tag{5}$$

The combined strain tensor, ε, (accounting also for thermal expansion) is defined as

$$\varepsilon = \frac{1}{2}(\nabla\vec{u} + (\nabla\vec{u})^{*}) + \alpha_V(\Delta T)\overset{\leftrightarrow}{I} \tag{6}$$

Table 5. Idealized pumping power, actual pumping power, coefficient of cooling efficiency and maximum temperature for the Pareto-optimized cooling arrays of micro pin-fins with a hot spot having circular, symmetric airfoil and symmetric convex cross sections for four heights of the pin-fins.

	H = 100 μm	H = 150 μm	H = 200 μm	H = 250 μm
	Pareto-optimized array with circular cross section micro pin-fins and a hot spot			
P_{ideal} (W)	1.5051	2.6832	2.2120	4.3632
$P_{ideal}+\dot{Q}_{input}$	65.2551	66.4332	65.9620	68.1132
η	0.97693	0.95961	0.96646	0.93594
T_{max}(°C)	81	76	74	74
	Pareto-optimized array with symmetric airfoil cross section micro pin-fins and a hot spot			
P_{ideal} (W)	2.5563	2.4956	3.0555	5.0960
$P_{ideal}+\dot{Q}_{input}$	66.3063	66.2456	66.8055	68.8460
η	0.96144	0.96232	0.95426	0.92598
T_{max} (°C)	75	74	73	70
	Pareto-optimized array with symmetric convex cross section micro pin-fins and a hot spot			
P_{ideal} (W)	1.2837	4.7269	2.5425	2.7764
$P_{ideal}+\dot{Q}_{input}$	65.0337	68.4769	66.2925	66.5264
η	0.98026	0.93097	0.96165	0.95826
T_{max}(°C)	79	67	72	73

where the Lame coefficients are defined as

$$G = \frac{\varepsilon}{2(1+\nu)} \text{ and } \lambda = \frac{\varepsilon\nu}{(1+\nu)(1-2\nu)} \tag{7}$$

The hydrodynamic and thermal loads from the 3D conjugate heat transfer analysis were applied and all the external sides of the electronic chip were fixed to investigate the effects of the loads on the micro pin-fin configurations.

Figure 7 shows the calculated displacement field for the Pareto-optimized pin-fin configuration having circular cross sections of the pin-fins. The calculated displacements were on the nanometer level, with the maximum displacement occurring at the leading edge of the channel outlets. This is due to the increased heat conduction loads as a result of lower convection due to warmer fluid in this region. Very similar results were obtained with Pareto-optimized arrays of pin-fins having symmetric airfoil and symmetric convex cross sections.

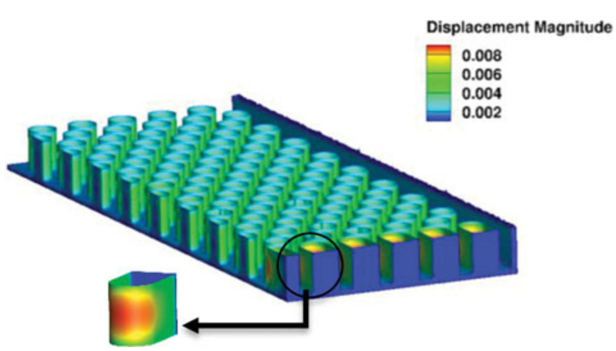

Figure 7. Displacement field (in micro-meters) due to hydrodynamic and thermal loads on Pareto-optimized array of micro pin-fins having circular cross sections.

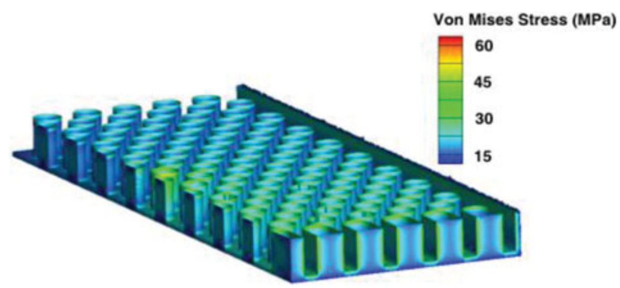

Figure 8. Von-Mises stress distribution due to hydrodynamic and thermal loads on the Pareto-optimized array of micro pin-fins having circular cross sections.

Figure 8 shows calculated Von-Mises stress for the Pareto-optimized array of pin-fins having identical circular cross sections. The higher stresses are concentrated in the pin-fins directly below the hot spot. This is due to both the increased heat conduction in this area and the fixed boundary between the ends of the pin-fins and the top and bottom walls of the chip. The magnitudes of calculated stresses for arrays having symmetric airfoil and symmetric convex Pareto-optimized cross sections were similar. The maximum calculated Von-Mises stress for the three configurations was between 67 to 97 MPa which is significantly lower than the yield strength of 7000 MPa for silicon [22]. This demonstrates that hydrodynamic loads and thermal loads can be significantly increased without compromising structural integrity of the micro pin-fins.

Conclusions

This study investigated the multi-objective design optimization of three micro pin-fin based cooling arrays in forced convection cooling of micro-electronics with a hot spot. An example studied here had a background heat flux of 500 W cm^{-2} and a hot spot heat flux of 2000 W cm^{-2}. Fully 3D conjugate heat transfer analysis was performed showing that symmetric airfoil and symmetric convex cross-section pin-fin geometries can practically eliminate the recirculation region that cannot be eliminated in case of circular cross-section pin-fins. A multi-objective optimization was carried out for the three configurations where the inlet coolant speed and the geometric parameters of the pin-fins were design variables. The two simultaneous objectives were to minimize maximum temperature and reduce inlet pressure, while keeping exit pressure fixed, the maximum temperature below 85°C, and removing the entire amount of input thermal energy via convection. The multi-objective optimization was carried out efficiently using response surfaces coupled with a genetic algorithm. The Pareto-optimized arrays having micro pin-fins with symmetric airfoil and symmetric convex cross section shapes were found to result in both lower maximum temperature and lower inlet pressure than the Pareto-optimized array of micro pin-fins with circular cross section shapes. However, in terms of the pumping power, all three cross section shapes of the optimized micro pin-fins perform comparably well. In the stress-deformation *a posteriori* analysis of the Pareto-optimized configurations, both hydrodynamic and thermal loads from the 3D conjugate heat transfer analysis were incorporated. It was found that the maximum displacement for the three Pareto-optimized array configurations was on the nanometer level. The Von-Mises stress for the three Pareto-optimized array configurations was in the range of 67 – 97 MPa, which is significantly lower than the yield strength for silicon of 7000 MPa, thus confirming the structural integrity of the micro pin-fin array design.

Nomenclature

A_{in} Inlet area, m^2
A_{top} surface area of the top surface of the top wall exposed to heating, m^2
$A_{hotspot}$ surface area of the top surface of the top wall exposed to heating, m^2
$\vec{F}$ volumetric force applied, N m^{-3}
G first Lame coefficient, N m^{-2}
H height of the micro pin-fin, m
$\overleftrightarrow{I}$ identity tensor
k thermal conductivity, W m^{-1} K^{-1}
p fluid pressure, Pa
P pumping power, W
$\dot{q}$ heat flux per unit area, W m^{-2}
Q thermal power, W
T temperature, K
ΔT temperature difference ($T - T_{ref}$), K
$\vec{u}$ deformation vector, m
V average fluid speed at inlet, m s^{-1}

Greek symbols

α_V coefficient of thermal volumetric expansion, K^{-1}
ε combined strain tensor
η coefficient of cooling efficiency
λ second Lame coefficient, N m^{-2}
ν Poisson's ratio
ρ fluid density, kg m^{-3}

Subscripts

conv convection
ref reference

Superscripts

$*$ transpose of a matrix

Acknowledgments

The authors are grateful for the partial financial support of this research provided by DARPA grant HR0011-14-1-0002 via GaTech grant RE314-G1 monitored by Dr. Muhannad Bakir in the framework of ICECool project under supervision of Prof. Avram Bar-Cohen, Dr. Joe Maurer and Dr. Kaiser Matin. Authors are also grateful for the partial financial support provided by DOE/NETL grant DE-FE0023114 and would like to express their appreciation to Prof. Carlo Poloni, president of ESTECO, for providing modeFRONTIER optimization software free of charge. FIU Instructional and Research Computing Center provided HPC resources to perform calculations for this project. The lead author gratefully acknowledges the financial support from Florida International University Presidential Fellowship.

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