

# Bayesian estimate of pre-mixed and diffusive rate of heat release phases in marine diesel engines

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**Abstract** The rate of heat released during the combustion in Diesel engines is important for many reasons, including performance evaluation, pollutant formation, and control. Combustion in Diesel engines can be generally divided into three phases: pre-mixed, diffusive or mixed-controlled, and late combustion. The objective of this paper is to estimate the rate of heat released by the fuel in a marine Diesel engine, in order to identify the pre-mixed and diffusive phases, using the Sampling Importance Resampling (SIR) Bayesian Particle Filter. Experimental pressure data obtained from a piezoelectric sensor, installed in a research marine diesel engine (MAN Innovator 4c), was used to feed the observation model in such Bayesian approach. The evolution model for the pressure was formulated in terms of a set of ordinary differential equations, coming from the First Law of Thermodynamics, together with a random walk model for the unknown state variable. The proposed

approach was able to identify the pre-mixed and diffusive combustion phases, for different engine loads. Results were compared with a simple inversion procedure, showing a good agreement. The combustion ignition delay was also calculated, showing its variation with the engine load.

**Keywords** Rate of heat release · Inverse problems · Bayesian technique

## List of symbols

$A$	Area
$A/F$	Air/fuel ratio
$B$	Piston bore
$CA$	Crankshaft angle
$f$	Linear or non-linear function of the state variables
$g$	Linear or non-linear function representing the observation model
$h$	Heat transfer coefficient
LHV	Lower heating value
$m$	Mass
$n$	Engine speed in Hz
$\mathbf{n}$	Vector of noise associated with the observation model
$P$	Pressure
$Q$	Heat
$t$	Time
$T$	Temperature
$v$	Average gas velocity within the cylinder
$\mathbf{v}$	Vector of noise associated with the evolution model
$V$	Volume
$w$	Weights of particles
$\mathbf{W}$	Covariance matrix
$x$	Mass fraction of burned fuel
$\mathbf{y}$	Vector of state variables
$\mathbf{z}$	Vector of observation variables

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**Greek letters**

$\gamma$	Polytropic coefficient
$\theta$	Crankshaft angle
$\pi$	Probability density function

**Subscripts/superscripts**

d	Displaced
f	Fuel
gas	Gas mixture
m	Mixture
meas	Measured
mot	Motored
r	Reference
w	Wall

**1 Introduction**

Diesel engines are widely used for terrestrial and marine transportation of goods and people. They are very efficient given the fact that they can operate at very high compression ratios. Since they usually work with excess of air, their emissions of total unburned hydrocarbons (THC) and Carbon monoxide (CO) are very low. However, these two characteristics (high compression ratios and temperatures, and excess of air) make them a substantial source of Nitrogen oxides ( $\text{NO}_x$ ) [1]. The combustion process in Diesel engines is very complicated, since it involves different physical processes. Once the fuel is injected in the combustion chamber, there is an ignition delay before it starts burning. During such delay, fuel vaporizes and mixes with the surrounding air, which is at a high temperature. Then, the combustion begins, initially consuming this pre-mixed mixture of air and fuel, and releasing energy at a very high rate. Longer ignition delays cause a more dramatic release of energy during this pre-mixed combustion phase, which is often an undesirable phenomenon. This problem affects the durability of the engine and also increases the emissions of  $\text{NO}_x$  [2]. Therefore, fuels used in Diesel engines must have a short ignition delay, which can be characterized by an ASTM standard test [3]. After the pre-mixed phase, the combustion continues, consuming the remaining fuel, as it mixes with the air, in a diffusive or mixed-controlled combustion phase. Recently, a paper [4] used a Bayesian technique to estimate the ignition delay in a Diesel engine. Another paper [5] also used Bayesian techniques to estimate some parameters in internal combustion engines.

In marine applications, two types of fuels are used in Diesel engines: high viscosity oils, named bunker oils or heavy fuel oils (HFO), and marine Diesel oils (MDO), which have high concentrations of Sulfur. Due to its physical characteristics HFO is less expensive than the MDO and also has a low combustion quality. In most cases, MDO

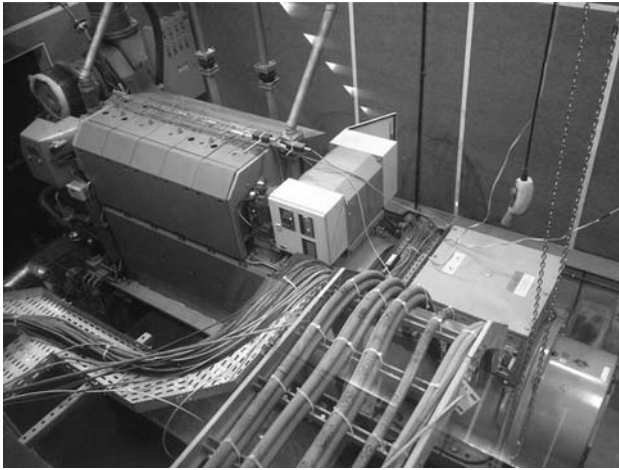
is used close to the coast and HFO for open sea transportation. The development of new heavy fuel oils for marine applications that keep their cost low while increasing their combustion quality characteristics is important for several reasons. Since large vessels make the transportation of goods between countries, non-expensive fuels are required to keep the associated costs at a low level. However, the combustion quality of these fuels has to be increased, in order to reduce the gaseous emissions and not harm the environment.

The evaluation of fuel quality in Diesel engines is, therefore, a very important task. This involves both numerical and experimental studies and it is the main objective of this paper. The numerical simulation of the combustion process in Diesel engines involves a turbulent and unsteady flow of a reacting non-homogeneous mixture with temperature-dependent properties. Different models for this problem can be found in the literature, depending on the simplifying hypothesis used [1, 6].

One of the most used models by the industry, although being very simple, considers the burned and unburned gasses as an ideal homogenous gas with uniform temperature and pressure. This model is generally called a zero-dimensional model [1, 6] and can be derived from the First Law of Thermodynamics and the equation of state for an ideal gas. Since they do not include any sub model for the chemical reactions, these models rely on some empirical or semi-empirical correlation for the rate of heat released by the fuel. The most used correlation is the Wiebe's model, which has two adjustable parameters, or the double-Wiebe's model that has four parameters [1–10]. These models also use some correlation for the heat transfer coefficient at the cylinder walls and piston head, to model the amount of heat lost [1, 6, 11]. Although these models are very simple and easy to use, they are limited by the type of function used in the Wiebe's model, or by the correlation for the heat transfer coefficient. Also, as reported by [11], correlations for the heat transfer coefficient that were obtained for a specific engine running under specific conditions, in general do not give good results for other engines and can present discrepancies in excess of 100 %. Thus, the estimation of these functions is also very important, but will not be considered in this paper.

The objective of this paper is to use experimental data available from tests made in the Laboratory of Thermal Engines (LMT) of Federal University of Rio de Janeiro (UFRJ) to validate a Bayesian technique [12] used to estimate the rate of heat released by an MDO fuel in a marine Diesel engine. The main advantage of this procedure is that the heat release rate is no longer dependent on the Wiebe's model, and therefore, different functions can be recovered.

Finally, it is worth mentioning that other recent works also proposed some procedures to estimate the mass



**Fig. 1** MAN Innovator research marine Diesel engine at LMT-UFRJ

**Table 1** Engine parameters

Parameter	Value
Engine manufacturer	MAN
Engine model	Innovator research diesel engine
Fuel	MDO or HFO
Number of cylinders	5
Valves per cylinder	2
Bore	160 mm
Stroke	240 mm
Connecting rod length	480 mm
Compression ratio	15.2
Speed	1200 rpm
Imep	20.7 bar
Maximum rated power	500 kW
Inlet closing valve	146.5° BTDC
Outlet opening valve	126.5° ATDC

fraction of burned fuel [13–22], but none of them used Bayesian techniques.

### 1.1 Physical problem

The physical problem considered in this paper involves the combustion process in a MAN Innovator research marine Diesel engine, shown in Fig. 1. This engine is capable of burning marine Diesel oil (MDO) or heavy fuel oil (HFO). It has auxiliary equipment capable of centrifuging the fuel and lubricant and also to adjust fuel’s viscosity. The engine is also equipped with a pressure transducer installed inside one of the combustion chambers and a sensor to detect the start of fuel injection. Some of the engine parameters are shown in Table 1.

In this paper, we used a zero-dimensional model to simulate the combustion occurring in one of the cylinders of

the engine shown in Fig. 1. The mixture inside the cylinder was considered an ideal gas with uniform properties. From the equation of state for an ideal gas, together with the First Law of Thermodynamics, the following equation can be obtained when both the inlet and outlet valves are closed [1]

$$\frac{dP}{d\theta} = -\gamma \frac{P}{V} \frac{dV}{d\theta} - \frac{(\gamma - 1)}{V} \frac{dQ}{d\theta} \quad (1)$$

where  $P(\theta)$  is the time-varying pressure,  $\theta$  is the crankshaft angle (which is related to time),  $V(\theta)$  is the instantaneous volume of the cylinder (which can be obtained from the engine speed and geometrical data),  $Q(\theta)$  is the heat released and  $\gamma$  is the polytropic coefficient, which was assumed constant and equal to 1.33 in this work.

Assuming a combustion process with 100 % efficiency, the rate of heat released in the combustion chamber,  $dQ/d\theta$ , can be obtained as

$$\frac{dQ}{d\theta} = \frac{dQ_f}{d\theta} - \frac{dQ_w}{d\theta} \quad (2a)$$

$$\frac{dQ_f}{d\theta} = m_f LHV \frac{dx}{d\theta} \quad (2b)$$

where  $Q_f$  is the total heat released by the fuel,  $Q_w$  is the heat lost through the combustion chamber walls and piston head,  $m_f$  is the mass of fuel injected in the combustion chamber, LHV is the lower heating value of the fuel and  $x$  is the mass fraction of burned fuel. The objective of this paper is to estimate  $dQ_f/d\theta$  using a Bayesian approach.

The wall heat transfer can be modeled considering a time-varying convection heat transfer coefficient  $h(\theta)$  [1]

$$\frac{dQ_w}{d\theta} = \frac{hA(T - T_w)}{2\pi n} \quad (3)$$

where  $T(\theta)$  is the temperature of the gas mixture inside the combustion chamber,  $A(\theta)$  is the area of the combustion chamber and piston head,  $T_w(\theta)$  is the temperature at the walls, and  $n$  is the engine speed in Hz. Although the estimation of the wall heat transfer coefficient  $h(\theta)$  in itself is a challenging task [23–25], in this paper we modeled it using the Woschni’s equation [1, 6, 11]

$$h(\text{W/m}^2\text{K}) = 3.26B(\text{m})^{-0.2}P(\text{kPa})^{0.8}T(\text{K})^{-0.55}v(\text{m/s})^{0.8} \quad (4)$$

where  $B$  is the bore (diameter) of the cylinder and  $v$  is the average cylinder gas velocity. For a four-stroke, water-cooled engine, it can be expressed as [1, 6, 11]

$$v = \left[ C_1 \bar{S}_p + C_2 \frac{V_d T_r}{P_r V_r} (P - P_{\text{mot}}) \right] \quad (5)$$

Here,  $V_d$  is the displaced volume,  $P_r$ ,  $V_r$  and  $T_r$  are taken at some reference state,  $P_{\text{mot}}$  is the motored cylinder

pressure at the same crank angle as  $P$ , and the constants  $C_1$  and  $C_2$  are given as functions of the engine stroke as

$$\begin{aligned} \text{Gas exchange: } C_1 &= 6.18; C_2 = 0 \\ \text{Compression: } C_1 &= 2.28; C_2 = 0 \\ \text{Combustion and expansion: } C_1 &= 2.28; C_2 = 3.24 \times 10^{-3} \end{aligned} \tag{6}$$

The pressure inside the combustion chamber can, therefore, be obtained by the solution of Eqs. (1)–(6), given an appropriate initial condition. In this paper, we used a four-stage Runge–Kutta algorithm to solve them. It is worth noticing, however, that the rate of heat released by the fuel,  $dQ_f/d\theta$ , is unknown in these equations, and therefore, must be determined. This will be discussed in the next section.

### 1.2 Inverse problem

The inverse problem considered in this paper deals with the estimation of the rate of heat released by the fuel,  $dQ_f/d\theta$ , given pressure measurements performed inside the combustion chamber of the engine shown in Fig. 1. For this purpose, we used a Bayesian approach, where the results are obtained in terms of a posterior probability density. Such function is the conditional probability of the unknown variables  $\mathbf{y} = \{P, dQ_f/d\theta\}$ , given some measurements  $\mathbf{z} = \{P_{\text{meas}}\}$ . In the Bayesian approach some prior information about the unknown variables used are combined with the information given by the measurements, to produce better estimates.

The posterior probability density  $\pi(\mathbf{y}|\mathbf{z})$  is related to the prior model,  $\pi(\mathbf{y})$ , which is the model for the probability density of the unknowns without the information obtained from the measurements, and the information about the measurements, which is given as the conditional probability density of the measurements given the unknowns,  $\pi(\mathbf{z}|\mathbf{y})$ . According to Bayes' theorem [26, 27]:

$$\pi(\mathbf{y}|\mathbf{z}) = \frac{\pi(\mathbf{y})\pi(\mathbf{z}|\mathbf{y})}{\pi(\mathbf{z})} \tag{7}$$

where  $\pi(\mathbf{z})$  is the marginal probability density of the measurements, which plays the role of a normalizing constant. This is the base for state estimation problems, also referred as non-stationary inverse problems [26], such as the one addressed in this paper.

If the measurements errors are Gaussian with zero mean, known covariance matrix  $\mathbf{W}$ , additive and independent of the unknown variables  $\mathbf{y}$ , it can be shown [26, 27] that the likelihood function  $\pi(\mathbf{z}|\mathbf{y})$  is given by

$$\begin{aligned} \pi(\mathbf{z}|\mathbf{y}) &= (2\pi)^{-D/2} |\mathbf{W}|^{-1/2} \\ &\times \exp \left\{ -\frac{1}{2} [\mathbf{z} - \mathbf{f}(\mathbf{y})]^T \mathbf{W}^{-1} [\mathbf{z} - \mathbf{f}(\mathbf{y})] \right\} \end{aligned} \tag{8}$$

where  $D$  is the dimension of the problem (one in the present paper), and  $\mathbf{f}$  is the solution of the direct problem, given by Eqs. (1)–(6) for the estimated variables  $\mathbf{y}$ .

According to Eqs. (7) and (8), to evaluate the posterior probability density  $\pi(\mathbf{y}|\mathbf{z})$ , a prior model  $\pi(\mathbf{y})$ , which is related to the data, and a likelihood function  $\pi(\mathbf{z}|\mathbf{y})$ , which is related to the measurements, are needed. For state evolution problems these can be translated into two models: an evolution model and an observation model. Considering that the state variables  $\mathbf{y}$  have state noise  $\mathbf{v}$ , and the measurements  $\mathbf{z}$  have noise  $\mathbf{n}$ , these two models can be written as

$$\mathbf{y}_k = \mathbf{f}_k(\mathbf{y}_{k-1}, \mathbf{v}_k) \tag{9a}$$

$$\mathbf{z}_k = \mathbf{g}_k(\mathbf{y}_k, \mathbf{n}_k) \tag{9b}$$

where  $k = 1, 2, 3, \dots$  denotes a time instant  $t_k$  in a dynamic problem.

The dynamic estimate of the state variables  $\mathbf{y}$  can be obtained initially by a prediction step, using the evolution model given by Eq. (9a). Then, an update step is performed, using the observation model, given by Eq. (9b), and the likelihood function, given by Eq. (8), in conjunction with the Bayes' theorem, given by Eq. (7). This process is called a filtering problem [26, 27].

For linear problems with Gaussian and additive noises, the optimum filter is the Kalman filter [26–30], whereas for non-linear problems, other strategies must be used. Some of those strategies involve the linearization of the Kalman filter [31, 32] or the use of the so-called particle filters. Particle filters can be applied to non-linear models with non-Gaussian errors [30, 33–40].

The main idea of the particle filter is to represent the required posterior density function by a set of random samples with associated weights and to use them to compute new estimates [30]. In this paper, we used the SIR algorithm summarized in Table 2. More details of this algorithm can be found in [30].

### 1.3 Experimental setup

As mentioned before, in this paper we used experimental data obtained from an Innovator research marine Diesel engine, manufactured by MAN, currently installed at Federal University of Rio de Janeiro and shown in Fig. 1. This is a special engine, built for lubricant oil testing and equipped with three segregated lubricant oil circuits. This engine became operational in 2010 and since then is also being used for fuel oil development in constant speed tests. Several engine parameters can be found in Table 1. The engine is connected to an electrical generator and the electricity produced is dissipated as heat in a load bank that allows for nominal load variation of 12.5 % (62.5 kW) of

**Table 2** SIR algorithm [25]

<b>Step 1</b>
For $i = 1, \dots, N$ draw new particles $\mathbf{y}_k^i$ from the prior density $\pi(\mathbf{y}_k   \mathbf{y}_{k-1}^i)$ and then use the likelihood density to calculate the correspondent weights $w_k^i = \pi(\mathbf{z}_k   \mathbf{y}_k^i)$
<b>Step 2</b>
Calculate the total weight $t = \sum_i w_k^i$ and then normalize the particle weights, that is, for $i = 1, \dots, N$ let $w_k^i = t^{-1} w_k^i$
<b>Step 3</b>
Resample the particles as follows :
Construct the cumulative sum of weights (CSW) by computing $c_i = c_{i-1} + w_k^i$ for $i = 1, \dots, N$ , with $c_0 = 0$
Let $i = 1$ and draw a starting point $u_1$ from the uniform distribution $U[0, N^{-1}]$
For $j = 1, \dots, N$
Move along the CSW by making $u_j = u_1 + N^{-1}(j-1)$
While $u_j > c_i$ make $i = i + 1$
Assign sample $x_k^j = x_k^i$
Assign sample $w_k^j = N^{-1}$

the engine rated power (500 kW). Gaseous and particulate matter emission equipments are also available in the test bench where the Innovator engine is installed.

General engine operational parameters, such as fuel, cooling water, lubricant oil and combustion air pressures and temperatures are measured and controlled by MAN-EDS, a system provided by the engine manufacturer. An AVL GU21D pressure transducer, installed in the cylinder 2 head is used to measure the instantaneous pressures inside the combustion chamber. These data are processed in an AVL indicating system, where the frequency of measurements was taken as 28.8 kHz. Instrumentation also included an AVL SL31D sensor to measure the fuel line pressure at cylinder 2, a Honeywell 3010 optical sensor to measure the engine speed, and a modified fuel injector nozzle that allows identifying the needle lift profile.

During tests data is acquired for nominal engine loads of 25, 50, 75 and 100 % of the engine maximum rated power. Engine loads are swept three times in alternating orders. For each load, after engine parameters stabilize, in-cylinder pressure data is acquired for 200 engine cycles and averaged for use in the present work.

## 2 Results and discussion

In this work we used the SIR algorithm of the particle filter to estimate the rate of heat released by the fuel,  $dQ_f/d\theta$ , in the marine Diesel engine shown in Fig. 1. The fuel used was marine Diesel oil (MDO), whose lower heating value (LHV) is 42.7 MJ/kg. This engine operates under a

**Table 3** Operational parameters

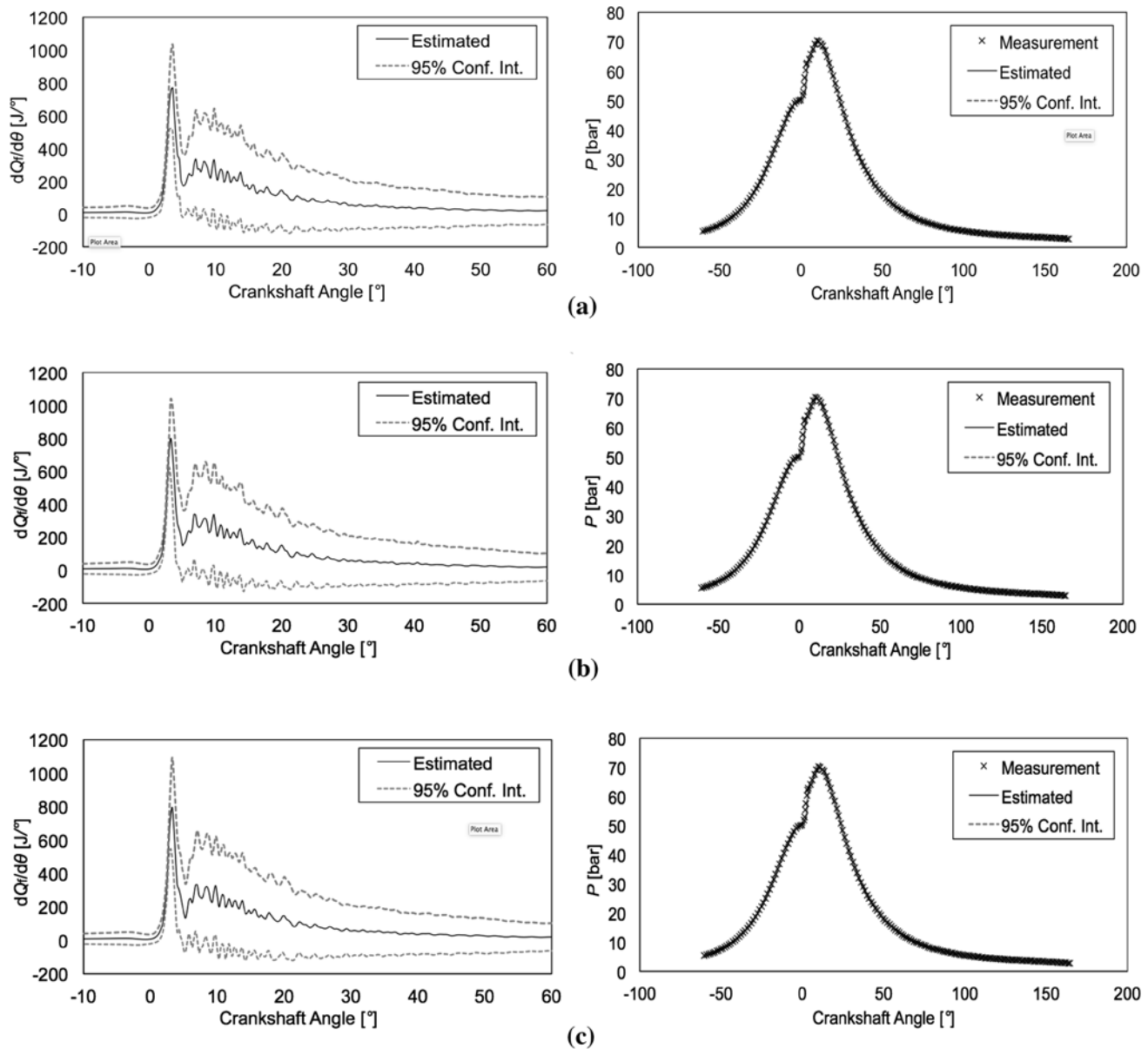
Load (%)	Power (kW)	$\dot{m}_f$ (kg/h)	A/F
25	125	32.1	34.21
50	250	53.4	31.12
75	375	75.0	31.44
100	500	99.6	31.07

constant speed of 1200 rpm and its torque can be varied by applying different loads in an electrical generator connected to it. In this paper, four loads were analyzed: 25, 50, 75 and 100 % of the engine’s full power. For these operating conditions, the power, fuel mass flow rate ( $\dot{m}_f$ ), and air/fuel ratio (A/F) are given in Table 3.

For the state estimation problem, the unknown state variables are the pressure,  $P$ , inside the combustion chamber and the rate of heat released by the fuel,  $dQ_f/d\theta$ . Therefore,  $\mathbf{y} = \{P, dQ_f/d\theta\}$ . Although there is a state evolution model for the pressure, given by the solution of Eqs. (1)–(6), we assume that no such model exists for  $dQ_f/d\theta$ . In this case the following artificial state evolution model, given as a random walk model, was considered

$$\frac{dQ_f}{d\theta}(\theta_k) = \frac{dQ_f}{d\theta}(\theta_{k-1}) + \sigma_h \varepsilon \frac{d\bar{Q}_f}{d\theta}(\theta_{k-1}) \tag{10}$$

Here,  $\sigma_h$  is the step size of the random walk (taken as 1),  $\varepsilon$  is a random variable with zero mean and uniform distribution between  $-1$  and  $1$ , and the bar indicates the mean value at the previous time step. The initial value, at time  $t_k = 0$ , was taken as zero.

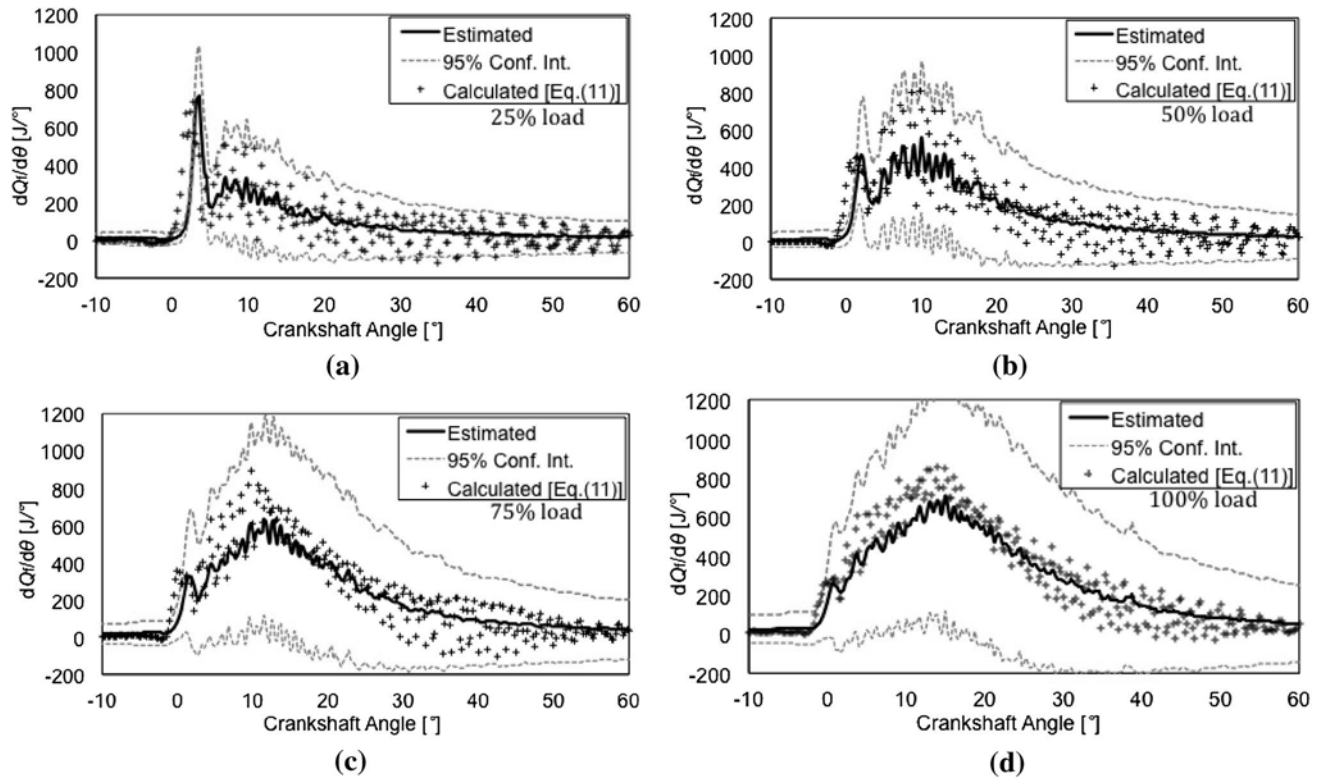


**Fig. 2** Estimate of  $dQ_p/d\theta$  and  $P$  for 25 % of load, using **a** 50, **b** 100, and **c** 200 particles

For each engine condition, 200 cycles were recorded where the mean pressure at each crankshaft angle was calculated and adopted as the measured variable  $\mathbf{z} = \{P_{\text{meas}}\}$ . Also, the standard deviation of the data at each angle was calculated considering these 200 cycles and taken into account in the experimental noise of this model. Typical values of the standard deviation varied from 0.0095 to 0.8049 bar.

In this paper, we analyzed a different number of particles to verify the convergence of the particle filter. Also, since the particle filter relies on several random numbers, we used the procedure presented in [25] to check its convergence and present the averaged computational time required for the estimate.

Figure 2 shows the estimate of  $dQ_p/d\theta$  and pressure for 25 % load using (a) 50, (b) 100, and (c) 200 particles. Results show that there is not much difference among the results indicating that the filter already converged for 50 particles. The computational time required to perform this estimate on a 1.8 GHz Intel Core i7 with 4 Gb of RAM was 3, 7, and 15 s, for 50, 100, and 200 particles, respectively. From these figures it is clear that the estimated pressure matched the measured one, with the 95 % confidence intervals coinciding with the estimated value. Also, from the estimates of  $dQ_p/d\theta$  it is possible to identify the pre-mixed and diffusive phases of combustion. This is an important result since no empirical equation was used to model this



**Fig. 3** Estimate of  $dQ_f/d\theta$  for **a** 25 %, **b** 50 %, **c** 75 %, and **d** 100 % load

function. Thus, the Bayesian particle filter was capable of dynamically identifying, at each time instant, what was the instantaneous value of  $dQ_f/d\theta$  that generated pressure data capable of matching the experimental data. Considering that the entire procedure is automatic and can be done without any user interference, its application to real time monitoring of engines could be very useful.

Once the convergence of the method was verified, and the matching between the experimental and measured pressure curves was checked, Fig. 3 shows the estimation of  $dQ_f/d\theta$  for all loads using 50 particles. This figure also shows the calculated rate of heat released by the fuel, which can be obtained from equation Eq. (11) [1], while neglecting the heat loss through the piston head and cylinder walls.

$$\frac{dQ_{f(\text{no heat loss})}}{d\theta} = \frac{1}{\gamma - 1} \left[ \gamma P \frac{dV}{d\theta} + V \frac{dP}{d\theta} \right] \quad (11)$$

Notice that the derivative of pressure,  $P$ , with respect to the crankshaft angle,  $\theta$ , can be obtained using a finite difference approximation of the experimental data. Therefore, the calculated values are prone to some oscillations, as shown in Fig. 3. It is quite interesting that the confidence interval embraces the calculated values, except at the beginning of the combustion. Also, the mean values obtained by the particle filter are much smoother than the ones obtained by Eq. (11). This comparison shows that the

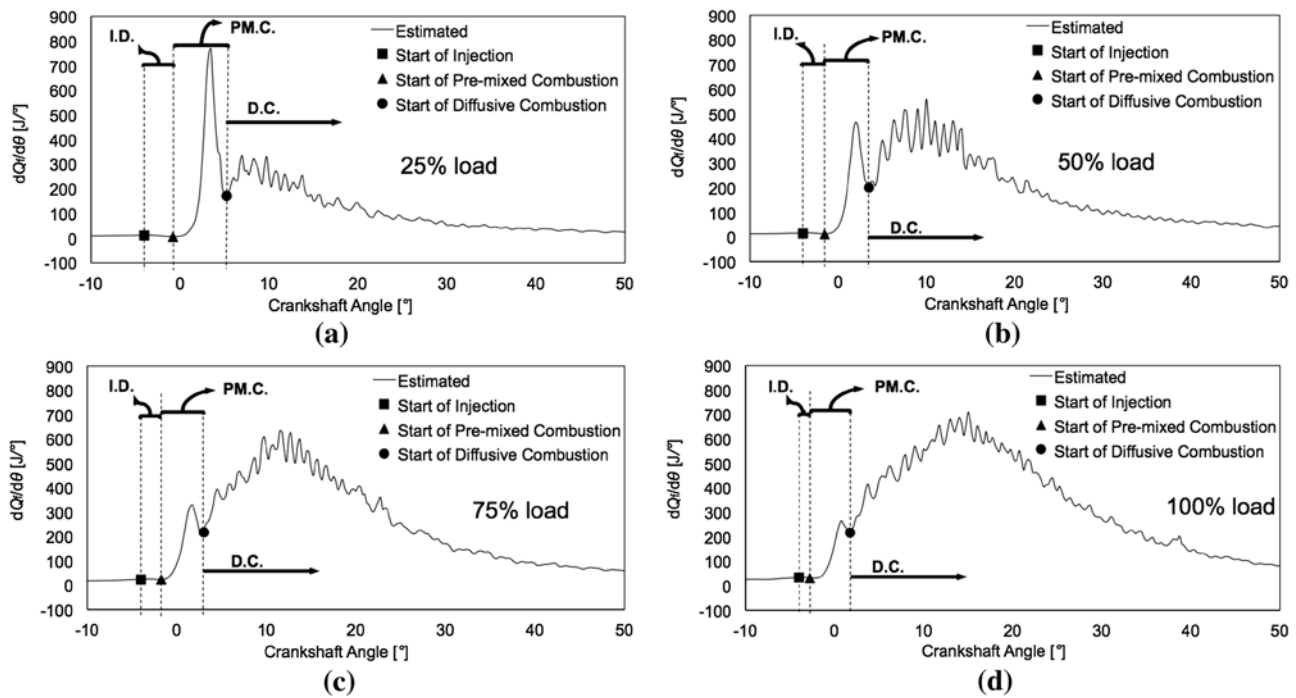
**Table 4** Ignition delay calculated from the pressure data

Load (%)	Mean of ignition delay (CA°)	Variance of ignition delay (CA°)	99 % uncertainty interval of ignition delay (CA°)
25	3.93	0.04	[3.57, 4.29]
50	2.72	0.04	[2.39, 3.05]
75	1.73	0.07	[1.09, 2.37]
100	1.31	0.04	[0.84, 1.78]

values predicted by the particle filters are in good agreement with those calculated by Eq. (11).

For these operational conditions the ignition delay was also calculated, using the method based on the maximum of the second derivative of the pressure [41–43]. For this purpose, the ignition delay was calculated for each one of the 200 pressure curves as well as its mean value and the associated variance. Table 4 shows these results. As expected, when the load is increased, the ignition delay decreases, mainly due to the high temperatures and pressures associated.

Analyzing Fig. 3 and Table 4, it is clear that for 25 % load, where the mean of the ignition delay is equal to 3.93 CA degrees, the rate of heat released by the fuel presents a very strong pre-mixed phase (Fig. 3a). When the load is



**Fig. 4** Combustion phases for **a** 25 %, **b** 50 %, **c** 75 %, and **d** 100 % load (ID ignition delay, PMC pre-mixed combustion phase, DC diffusive combustion phase)

increased to 50 %, the mean ignition delay decays to 2.72 CA degrees and the pre-mixed phase is less pronounced (Fig. 3b). Also, one can notice by comparing Fig. 3a, b that the integral under the diffusive phase of the combustion increases when a shorter ignition delay is found. Finally, for 75 % (Fig. 3c) and 100 % (Fig. 3d) loads, the ignition delays become very short and the rate of heat release plots show an almost pure diffusive combustion phase.

Figure 4 presents the mean results for the rate of heat released by the fuel, where the ignition delay (ID), pre-mixed diffusive combustion phase (PMD) and diffusive combustion phase (DC) are marked. The integral under the curves plotted in this figure were used to calculate the duration of the pre-mixed combustion phase and the percentage of total energy released during this phase. Table 5 shows these values where it is clear that as the load increases, the percentage of energy released during the pre-mixed combustion phase decreases, while the diffusive phase releases more energy. It is also clear that although the duration of the pre-mixed phase is almost constant in crankshaft angle degrees, the amount of energy released during this phase varies substantially. This is mainly due to the larger ignition delays for low loads where more fuel is evaporated and mixed with the air prior to combustion. From the analysis of Tables 4 and 5, a decrease in the ignition delay from 3.93 CA degrees (for 25 % load) to 2.72 CA degrees (for 50 % load) reduces the percentage of energy released by the pre-mixed combustion phase from

**Table 5** Mean combustion parameters obtained from the estimated rate of heat released by fuel

Load (%)	Duration of pre-mixed combustion (CA°)	Percentage of total energy released during the pre-mixed combustion phase (%)	Percentage of total energy released during the diffusive and residual combustion phases (%)
25	5.32	18.07	81.93
50	4.78	7.88	92.12
75	5.27	5.32	94.63
100	4.44	2.64	94.36

18.07 to 7.88 %. Thus, the development of fuels with shorter ignition delays is crucial for better combustion characteristics in marine Diesel engines.

As a final result, Table 6 compares the overall estimated energy released by the fuel, given as the integral of  $dQ_f/d\theta$ , with the real value taken into account the mass of fuel ( $m_f$ ) multiplied by its lower heating value (LHV). The values are very close, with differences varying from 10 % (for 25 % load) to 21 % (for 100 % load). This difference is probably due to the correlation used for the heat transfer coefficient. Therefore, although the estimate is reasonably good, for a more accurate result a simultaneous estimation of  $dQ_f/d\theta$  and  $h(\theta)$  is necessary and shall be investigated in the future.



**Table 6** Difference between the overall estimated and real energy released by the fuel

Load (%)	Integral of $dQ_p/d\theta$ given in Fig. 4 (J)	Total amount of energy contained in the fuel $m_f$ LHV (J)	Relative difference (%)
25	6907	7614	10
50	10,864	12,668	17
75	14,933	17,792	19
100	19,554	23,627	21

From the previous analysis, it is clear that Bayesian techniques provide a good way to estimate state variables in the combustion process, where the rate of heat released by the fuel could be estimated without using any empirical model, such as the Wiebe's function. This tool can be used to monitor engines in real time and also to help developing new fuels with desired combustion characteristics.

### 3 Conclusion

In this paper, we used a Bayesian technique to estimate the transient rate of heat released by the fuel in a marine Diesel engine. Pressure data was taken by a pressure transducer located inside the combustion chamber and was used as an observation model. A zero-dimensional model was employed as the evolution model. A comparison between the estimated functions and the calculated ignition delays was conducted, showing good physical agreement between them. The Bayesian particle filter was capable of identifying the pre-mixed and diffusive combustion phases without using any correlation for the sought function. Therefore, such methodology could be used for real time monitoring of Diesel engines to identify possible anomalies during their operations.

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