INVERSE DESIGN OF COOLING OF ELECTRONIC CHIPS SUBJECT TO SPECIFIED HOT SPOT TEMPERATURE AND COOLANT INLET TEMPERATURE

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ABSTRACT
Most methods for designing electronics cooling schemes do not offer the information on what levels of heat fluxes are maximally possible to achieve with the given material, boundary and operating conditions. Here, we offer an answer to this inverse problem posed by the question below. Given a micro pin-fin array cooling with these constraints:
- given maximum allowable temperature of the material,
- given inlet cooling fluid temperature,
- given total pressure loss (pumping power affordable), and
- given overall thickness of the entire electronic component,
find out the maximum possible average heat flux on the hot surface and find the maximum possible heat flux at the hot spot under the condition that the entire amount of the inputted heat is completely removed by the cooling fluid. This problem was solved using multi-objective constrained optimization and metamodeling for an array of micro pin-fins with circular, airfoil and symmetric convex cross sections that is removing all the heat inputted via uniform background heat flux and by a hot spot. The goal of this effort was to identify a cooling pin-fin shape and scheme that is able to push the maximum allowable heat flux as high as possible without the maximum temperature exceeding the specified limit for the given material. Conjugate heat transfer analysis was performed on each of the randomly created candidate configurations. Response surfaces based on Radial Basis Functions were coupled with a genetic algorithm to arrive at a Pareto frontier of best trade-off solutions. The Pareto optimized configuration indicates the maximum physically possible heat fluxes for specified material and constraints.

INTRODUCTION
High heat flux thermal loads on electronic components, having multiple hot spots with heat fluxes at least an order of magnitude larger than the average heat flux values, present a serious challenge to designers of electronics cooling schemes. The main issue is the extreme temperature that results at the hot spots. It is known that next generation electronic chips are expected to produce around 500 W cm$^{-2}$ as the background and over 1000 W cm$^{-2}$ at the hot spots [1-2]. Special application microelectronics already need adequate efficient cooling for background heat fluxes as high as several kilowatts per square centimeter with hot spots having an order of magnitude higher heat fluxes. Consequently, maximum temperature of such electronics components will be much higher than silicon can endure. Specifically, silicon carbide is an up-and-coming material which can be used for electronic components reaching temperatures as high as 250°C. Several materials research teams are already working on developing electronic materials that can operate at temperatures as high as 800°C. Thus, it would be of interest to any electronics cooling system designer to know a priori how high background heat flux and how high hot spot heat flux can possibly be handled with a particular cooling scheme if electronic material is given with its maximum allowable temperature. Various forced convection cooling schemes have been reported in the literature for their attempted efforts to handle such extreme heat fluxes, while keeping hot spot temperatures below the limit dictated by the physical properties of the material. For example, Abdoli et al. [3] investigated the cooling capability of two-floor microchannels under the background heat flux of 1000 W cm$^{-2}$ and a hot spot heat flux of 2000 W cm$^{-2}$. Siu-Ho et al. [4] investigated the performance of micro pin-fin heat sinks under single-phase and two-phase regimes. The current methodology for obtaining an efficient
cooling scheme employs a multi-objective optimization to find a configuration capable of cooling a specified heat flux. This approach has been applied by Abdoli and Dulikravich [5] to optimize two-floor and three-floor, single-phase, through-flow and counter-flow micro-heat exchangers to cool a uniform heat flux of 1000 W cm$^{-2}$. Cooling arrays of micro pin-fins with a 4 x 3 mm footprint and a 0.5 x 0.5 mm hot spot having circular, airfoil and convex cross-sections were analyzed by Abdoli et al. [6] and optimized by Reddy et al. [7]. The two thermal loading schemes considered in their work were a uniform heat flux of 500 W cm$^{-2}$ and a hot spot heat flux of 2000 W cm$^{-2}$. The size of the array of micro pin-fins in this work was 4 mm x 3 mm. Only one half was actually computationally analyzed, because of geometric symmetry. Fabbri and Dhir [8] optimized arrays of micro-jets to cool microelectronic devices. Prasher and Chang [9] numerically analyzed the use of microchannels and micro-pin fin heat exchangers to cool a heat flux of 1250 W cm$^{-2}$.

It should be noted that all of these cooling configurations were designed to cool a specified heat flux, and that very little is known of their performance under different thermal maps.

The current design approach only allows for a scheme to cool electronics up to a specified heat flux. Due to the rapid growth in the performance electronic chips and thermal energy produced, higher performing and robust cooling configurations must be designed.

Thus, the question that should be asked is: What is the maximum possible average heat flux and what is the maximum possible heat flux at the hot spots that will still not exceed the maximum specified temperature? That is, what is the maximum possible achievable performance of a given cooling scheme with given inlet coolant temperature and given properties of the coolant and the material of the chip?

This is an inverse design approach that can be solved as a constrained multi-objective optimization problem. The design variables include coolant inlet velocity and the geometric design parameters for the array of micro pin-fins. The three objectives in this study were to minimize maximum temperature and pumping power, while maximizing the background and hotspot heat fluxes. A constraint was imposed on the inlet temperature and maximum allowable temperature. Figure 1 shows the typical configurations used in this study. Conjugate heat transfer was carried out on each micro-pin fin configuration to obtain the objective function values which were then used to create response surfaces. They were coupled with a genetic algorithm to create a Pareto frontier of the best trade-off solutions.

**CONJUGATE HEAT TRANSFER ANALYSIS**

The three proposed geometries (Figure 1) for the pin-fins (having circular, symmetric airfoil, and symmetric convex cross sections) have been optimized by Reddy et al. [7] for a specified heat flux. The electronic chip in the current study has the same dimensions as those used in [6-7]. The proposed configurations include channeled outlets to suppress backflow at the outlet. The thickness of the channeled outlets was the same as the thickness of the pin-fins. Water was used as the cooling fluid. A computational grid was created for each of the initial candidate designs using ANSYS Meshing® [11] and 3-D conjugate heat transfer analysis was performed using ANSYS Fluent®.

A 3-D computational grid of approximately 7 million cells was created and used for the analysis of each configuration in this study. The grid consisted of four structured layers smoothly clustered at each solid boundary, with tetrahedral cells filling the rest of the space. Reynolds Averaged Navier-Stokes (RANS) equations with standard k-$\varepsilon$ turbulence model was used to simulate the turbulent flow and convective heat transfer in all cases. It can be reported that the Reynolds number ranged from 80 to 180. Although this is in the laminar regime, Alfieri et al. [12] demonstrated it can still lead to vortex shedding. The steady-state heat conduction equation was solved in the 3-D solid domain. The fluid and solid domains were coupled through their shared boundary and the iteration process was stopped when the residuals for both domains and on the shared boundary converged to six orders of magnitude below their initial values.

A grid convergence study was performed utilizing a grid with 7 million and a grid with 12 million cells. The results obtained using these two grids deviated by less than 0.5% confirming grid independence beyond 7 million grid cells.
GEOMETRY DEFINITION

The performances of all three proposed micro pin-fin shapes were optimized, allowing for a fair comparison. The geometric parameters for circular cross section pin-fins were the diameter and the height of the pin-fin. The geometric parameters for the airfoil and convex cross section shapes were defined using the pin-fin height, chord length and thickness. The airfoil configuration was defined using symmetric, four series NACA 00XX airfoils. The thickness of all walls of the chip (top, bottom, and side walls) was kept constant at 30 µm. The bottom surface and the external sides of the chip were thermally insulated thus assuring that 100% of the heat is removed by the fluid. The outlet gage pressure was kept constant at 20 kPa. Water was used as the heat removing fluid where the inlet temperature was considered as a constraint. Table 1 shows the range for each of the geometric variables, inlet fluid conditions, and thermal loads that were used to construct the response surfaces.

Table 1. Range of design parameters and increment sizes.

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Range</th>
<th>Step size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pin-fin diameter (µm)</td>
<td>100 - 200</td>
<td>10</td>
</tr>
<tr>
<td>Pin-fin chord length (µm)</td>
<td>200 - 300</td>
<td>10</td>
</tr>
<tr>
<td>Pin-fin thickness (µm)</td>
<td>80 - 160</td>
<td>5</td>
</tr>
<tr>
<td>Pin-fin height (µm)</td>
<td>100 - 250</td>
<td>50</td>
</tr>
<tr>
<td>Coolant inlet speed (m s⁻¹)</td>
<td>1 - 8</td>
<td>0.1</td>
</tr>
<tr>
<td>Background heat flux (W cm⁻²)</td>
<td>650 - 2000</td>
<td>5</td>
</tr>
<tr>
<td>Hot spot heat flux (W cm⁻²)</td>
<td>2300 - 5000</td>
<td>10</td>
</tr>
</tbody>
</table>

INVERSE DESIGN APPROACH

The multi-objective constrained optimization was performed using a commercial software package modeFRONTIER [13]. The optimization methodology utilizes a response surface coupled with a genetic algorithm to arrive at a Pareto frontier of the best trade-off solutions. The optimization process of this scale requires analysis of typically several thousand candidate solutions. Because the computational time for each 3-D conjugate heat transfer analysis was over eight hours on the parallel computer that we used, directly coupling the optimizer with the 3-D conjugate heat transfer analysis code would be too costly and time consuming. For this reason, a metamodel in the form of a multi-dimensional interpolation (response surface) that can calculate the interpolated objective function values for any set of design variables in less than one second is very appealing. Several response surface methods, including Multiquadrics, Gaussian and Inverse Quadric Radial Basis Functions, Kriging and Gaussian Process, were tested with Multiquadric Radial Basis Functions [14] outperforming the rest. Values of the objective functions obtained from the Multiquadric Radial Basis Functions interpolation differed by less than 2% from those obtained by the full 3-D conjugate heat transfer analysis.

It is known that the accuracy of the multi-dimensional response surface is greatly dependent on both the number and the distribution of its high-fidelity support points. For this reason, sets of design variables defining initial population of 120 candidate micro pin-fin arrays were created using Sobol’s [15] algorithm capable of uniformly distributing points throughout the multi-dimensional design variables space.

It should be noted that Table 1 not only lists the range for geometric parameters, but also inlet conditions and thermal loads. This allows various constraints to be placed on inlet temperature while utilizing the same response surface rather than constructing an entirely different response surface using a separate population of candidate designs. It also allows a single response surface to be used to predict the effects of various magnitudes of the applied heat fluxes on maximum temperature. This allows a constraint to be effectively placed on the maximum allowable temperature without having to construct a separate response surface for it. A constraint can also be enforced on the heat flux to mimic the traditional approach of optimizing the configuration for specified externally applied heat flux. Finally, constraint can also be placed on total pressure drop (pumping power) thus offering an effective comparison of maximum temperature and maximum heat flux possible among micro pin-fin arrays having different pin-fin cross section shapes.

The three response surfaces (one for each objective function) were coupled with the NSGA-II evolutionary optimization algorithm [16] which then searched the response surfaces in a sequence to arrive at a Pareto frontier of non-dominated (the best trade-off) solutions.

In summary, the simultaneous objectives in this paper are:
1. Maximize background heat flux
2. Maximize hot spot heat flux
3. Minimize inlet pressure
subject to specified maximum temperature equality constraint $T_{\text{max}} = 85 \degree C$, specified inlet coolant temperature of 30.85 °C and specified exit absolute static pressure of 20 kPa.

Figure 2 and Figure 3 show the workflow of design variables, objectives and constraints implemented for the multi-objective optimization process utilizing the response surfaces.

![Figure 2. Workflow of different stages and software used.](image-url)
Figure 3. Formulation of the optimization problem.

OPTIMIZATION RESULTS

This optimization approach incorporates constraints on the inlet temperature and the electronic chip overall thickness (bottom plate thickness, height of the pin-fins connecting the bottom and the top plates, and the top plate thickness), allowing for the optimization of the configuration for different restrictions placed. The multi-objective constrained optimization was run for 100 generations and again for 200 generations. The Pareto-optimal solutions did not differ noticeably. The results presented in Fig. 4 and Table 3 are after 200 optimization generations.

Figure 4a shows the Pareto optimized designs for arrays of pin-fins having circular cross sections. Figure 4b shows the Pareto-optimized designs for arrays of pin-fins having symmetric NACA00XX airfoil cross section shapes. Pareto front in this case is not well defined, indicating that more shape defining parameters should be used as design variables in this case to define more flexible symmetric airfoil shapes.

Table 2 shows the Pareto-optimized design parameters of the cooling schemes for each of the three pin-fin cross section shapes. As one of the possible relevant bases for comparison of the performance of the arrays of micro pin-fins with three pin-fin shapes, examples in Table 2 were chosen to have similar optimized inlet coolant pressures (with exit pressure constrained at 20 kPa).

On this basis, micro pin-fins having symmetric convex cross sections appear to have superior performance allowing for highest background heat flux and the lowest mass flow rate.
Table 2. An example of Pareto-optimized design parameters and objectives for three cross section shapes of micro pin-fins subject to specified maximum temperature of 85°C, inlet coolant temperature of 30.85°C and exit static pressure of 20 kPa. The three designs were chosen with similar inlet static pressures.

<table>
<thead>
<tr>
<th>Pin-fin cross section</th>
<th>Circular</th>
<th>Airfoil</th>
<th>Convex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pin-fin diameter (µm)</td>
<td>120</td>
<td>280</td>
<td>220</td>
</tr>
<tr>
<td>Pin-fin chord length (µm)</td>
<td>280</td>
<td>120</td>
<td>130</td>
</tr>
<tr>
<td>Pin-fin thickness (µm)</td>
<td>250</td>
<td>200</td>
<td>250</td>
</tr>
<tr>
<td>Pin-fin height (µm)</td>
<td>250</td>
<td>200</td>
<td>250</td>
</tr>
<tr>
<td>Coolant inlet speed (m s⁻¹)</td>
<td>6.1</td>
<td>4.7</td>
<td>4.6</td>
</tr>
<tr>
<td>Background heat flux (W cm⁻²)</td>
<td>805</td>
<td>760</td>
<td>995</td>
</tr>
<tr>
<td>Hot spot heat flux (W cm⁻²)</td>
<td>2200</td>
<td>2540</td>
<td>2240</td>
</tr>
<tr>
<td>Coolant inlet pressure (kPa)</td>
<td>395.171</td>
<td>409.746</td>
<td>393.390</td>
</tr>
</tbody>
</table>

Figure 5 shows the temperature distribution on the top surfaces of the three Pareto-optimized micro pin-fins array cooling configurations each having pin-fins with different cross section shapes. It can be seen that the airfoil and convex configurations resulted in a lower hotspot temperature even when a large heat flux was applied to both the background and the hot spot area. The airfoil and convex designs are known [7] to eliminate the flow recirculation at the trailing edge of the pin-fins, leading to improved heat transfer. It should be noted that each of the three configurations presented is a randomly selected Pareto design from the corresponding Pareto frontiers. Another designs can be selected from the Pareto frontiers that best achieve the required performance. For example, the three design could have been chosen with similar background heat fluxes.

Figure 6 shows the temperature distribution on the bottom, insulated surface of the three Pareto optimized configurations.

The higher temperatures on the bottom surface of the Pareto-optimized array of micro pin-fins having airfoil cross sections are due to more heat been conducted through such pin-fins, because of their larger cross sectional areas resulting from the optimization process (Table 2).
It was observed that a large height of the pin-fins resulted in lower temperatures on the bottom surface. Implementing a two-floor cooling configuration [5] could possibly further reduce the maximum temperatures on both the top and bottom surfaces.

The basic conclusion is that much higher average heat fluxes and hot spot heat fluxes can be handled by the optimized arrays of micro pin-fins and the optimized inlet coolant conditions.

Figure 7 shows the pressure distribution at mid height of the three Pareto-optimized configurations. A large stagnation region can be seen at the leading edge of the circular configuration. Previous research [6-7] has shown that, under similar conditions, the convex shape requires less pumping power followed by the airfoil and circular configurations. The conditions under which these Pareto designs were analyzed are not the same, making a fair comparison more difficult. Although the airfoil and convex configurations here require more pumping power (Table 3), they have been shown to handle a higher applied heat flux while resulting in lower maximum temperature. Since any other Pareto design can be selected from the Pareto frontier, an optimized design can be easily found that best satisfies the needed performance with a pressure drop similar to the value offered by the pin-fins with circular cross section.

It can be reported that nowhere in the fluid domain did the pressure drop to vaporization pressure of water.

![Figure 7. Pressure distribution at mid-height of the Pareto-optimized arrays of pin-fins having: a) circular, b) symmetric airfoil, and c) symmetric convex cross sections.](image1)

**PERFORMANCE MAPS FOR DIFFERENT PIN-FINS**

A visual perception of the interaction among inlet fluid temperature, inlet pressure and resulting maximum temperature is also a valuable tool when predicting performance of a given array of micro pin-fins with given values for applied heat fluxes.

Figure 8 gives an example of such a performance surface when the maximum allowable temperature is 85°C and pin-fins have circular cross sections. For example, if inlet water temperature increases from approximately 30°C to 40°C, inlet water speed will have to increase from approximately 4 m s$^{-1}$ to 6 m s$^{-1}$ to keep the maximum temperature below 85°C.

![Figure 8. Performance map for a larger array of micro pin-fins having circular cross sections.](image2)

Figure 9 provides an example of such performance surface when the maximum allowable temperature is 85°C and pin-fins have symmetric convex cross sections. It is apparent that micro pin-fins with these cross sections have much tighter range of inlet
fluid temperature and pressure when trying to maintain the maximum temperature below a specified value.

Results in Fig. 8 and Fig. 9 were created for a 10 mm x 10 mm electronic chip and assuming symmetry. The entire external surface of the top plate was exposed to the uniform heat flux and hot spot had a size of 0.25 mm x 0.25 mm.

CONCLUSIONS

A novel inverse approach to design of cooling schemes for high heat flux cooling micro pin-fin array based devices is proposed. It combines geometric parameters and operating conditions to formulate a multi-objective optimization problem. This approach not only allows for the optimization of cooling scheme for a specified heat flux, but also allows for the design of a cooling configuration that can maximize the allowable applied heat flux for a given material temperature limit and the available inlet coolant temperature. A multi-objective optimization was carried out for arrays of micro pin-fins having circular, symmetric airfoil and symmetric convex cross sections to attain maximum allowable heat flux, while simultaneously reducing the pumping power and maximum temperature. The input parameters used to define the cooling scheme include the size and height of the pin-fins and inlet coolant temperature and speed. A constraint was placed on the maximum allowable temperature and can also be applied to the inlet temperature. The optimization was efficiently carried out using response surfaces coupled with a genetic algorithm to arrive at a Pareto frontier of the best trade-off solutions. All configurations obtained using the inverse approach in this study, show that efficient thermal management is possible for heat fluxes higher than those reported in the literature in the case of silicon based arrays of micro pin-fins.

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