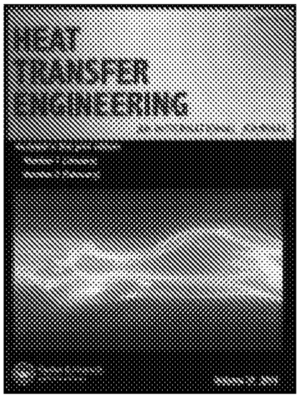

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Pipeline Heating Method Based on Optimal Control and State Estimation

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Pipeline Heating Method Based on Optimal Control and State Estimation

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In the production of oil and gas in deep waters, the flow of the produced hydrocarbon through pipelines is a challenging problem. High hydrostatic pressures and low seabed temperatures may result in the formation of solid deposits, which in critical operating conditions like unplanned shutdowns can cause pipeline blockages. One of the possible methods for flow assurance, which can be jointly used with other approaches, is to heat the pipeline. This design concept aims at heating the produced fluid, if needed, to above a safe reference temperature in order to avoid the formation of solid deposits. The objective of this article is to utilize the particle filter method for the solution of a state estimation problem, in which the state variables are considered as the transient temperatures within a pipeline cross section. In addition, the minimum temperature in the region, predicted with the particle filter method, is used in the optimal control theory as a design tool for a typical heating system, during simulated shutdown conditions. An application example is presented to illustrate the control of the minimum temperature in the region, from an observer based on the particle filter method, where temperature measurements are assumed to be available on the external surface of the pipeline.

INTRODUCTION

One of the key factors in the success of oil production in deep waters is subsea thermal management, which determines the requirements to maintain the fluid temperature, inside pipelines and inside other production equipment, above a minimum reference value. Indeed, the flow of the produced fluid through subsea pipelines in deep waters is a challenging problem. This environment presents high hydrostatic pressures and low seabed temperatures, which can favor the formation of solid deposits. Under critical operating conditions, such as unplanned shutdowns, these deposits may result in a pipeline blockage and, consequently, incur in large financial losses [1–3]. Thermal management includes both steady-state and transient heat transfer analyses involving the different stages of the oil field during its prospective lifetime. Hence, it must serve as a design tool for the selection of methods to avoid the formation of solid deposits. In steady-state operations, the temperature of the production fluid decreases as it flows through the pipeline, due to heat transfer to the seawater. This steady-state temperature profile is used to identify the minimum insulation requirements that are needed to keep the system above a critical temperature during production. If steady-state conditions are interrupted, such as during the critical periods of production shutdowns, a transient heat transfer analysis for the subsea system is necessary to ensure that the temperature of the fluid be maintained above that of formation of solid deposits. The main solid deposits formed inside subsea pipelines are wax and hydrates. For a given fluid, these solid deposits are formed at certain combinations of pressure and temperature. Wax deposits typically appear in temperatures ranging from 30 to 50°C. Hydrate formation temperatures, on the other hand, are typically around 20°C at 100 bar [3].

There are different techniques to avoid and/or minimize the formation of these solid deposits. The basic current strategies are the appropriate design of the thermal insulation and the

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injection of chemical inhibitors, but an alternative method is to heat the pipeline. This concept, generally known as active heating, aims at heating the produced fluid to above a safe reference temperature in order to avoid the formation of the solid deposits.

The pipeline can be heated by several methods, but typical concepts are based on the so-called direct electrical heating system (DEH) [4] and indirect electrical heating system (IEH) [5]. In the direct electrical heating system, electric current flows axially through the pipe wall, causing Joule heating. On the other hand, in the indirect electrical heating system, the electric current flows through heating elements (e.g., electrical cables) on the pipe surface.

The objective of this article is to use a Bayesian approach for the state estimation problem, in which the state variables are considered as the transient temperatures within a pipeline cross section, and to apply optimal control for a typical heating system. The particle filter method [6–22] is used to reconstruct the temperature field from transient temperature measurements available at one single point on the external surface of the pipeline. The minimum temperature predicted with the particle filter is then utilized in a control approach for the heating system [23–25], with the objective of maintaining the temperature within the pipeline above the critical temperature of formation of solid deposits. The physical problem consists of a pipeline cross section represented by a circular domain, with four heating cables on its surface. The fluid is considered to be stagnant, homogeneous, isotropic, and with constant thermophysical properties. The optimal control was based on a linear quadratic controller and the associated quadratic cost functional was minimized through the solution of Riccati's equation [23, 24].

STATE ESTIMATION PROBLEMS

In state estimation problems, observations obtained during the evolution of the system are used together with prior knowledge about the physical phenomena and the measuring devices, in order to sequentially produce estimates of the desired dynamic variables. State estimation problems can be solved with the so-called Bayesian filters [6–22].

In order to define the state estimation problem, consider a model for the evolution of the state variables $\boldsymbol{x}$ in the form [10]:

$$\boldsymbol{x}_k = \boldsymbol{f}_{k-1}(\boldsymbol{x}_{k-1},\ \boldsymbol{u}_{k-1},\ \boldsymbol{v}_{k-1}) \tag{1}$$

where $\boldsymbol{f}$ is, in the general case, a nonlinear function of $\boldsymbol{x}$, of the control input to the system $\boldsymbol{u}$ and of the state noise or uncertainty vector given by $\boldsymbol{v} \in \boldsymbol{R}^{n_v}$.

The vector $\boldsymbol{x}_k \in \boldsymbol{R}^{n_x}$ is called the state vector and contains the variables to be dynamically estimated. This vector advances in time in accordance with the state evolution model (1). The subscript $k = 1, 2, 3, \ldots,$ denotes time t_k in a dynamic problem.

The observation model describes the dependence between the state variable $\boldsymbol{x}$ to be estimated and the measurements $\boldsymbol{z}$ through a general, possibly nonlinear, function $\boldsymbol{h}$. This can be represented by

$$\boldsymbol{z}_k = \boldsymbol{h}_k(\boldsymbol{x}_k, \boldsymbol{n}_k) \tag{2}$$

where $\boldsymbol{z}_k \in \boldsymbol{R}^{n_z}$ are available at times t_k, $k = 1, 2, 3, \ldots$. Eq. (2) is referred to as the observation/measurement model. The vector $\boldsymbol{n}_k \in \boldsymbol{R}^{n_n}$ represents the measurement noise or uncertainty.

The evolution and observation models, given by Eqs. (1) and (2), respectively, are based on the following assumptions [6]:

(a) The sequence $\boldsymbol{x}_k$ for $k = 1, 2, 3, \ldots,$ is a Markovian process, that is,

$$\pi(\boldsymbol{x}_k | \boldsymbol{x}_0, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_{k-1}) = \pi(\boldsymbol{x}_k | \boldsymbol{x}_{k-1}) \tag{3a}$$

(b) The sequence $\boldsymbol{z}_k$ for $k = 1, 2, 3, \ldots,$ is a Markovian process with respect to the history of $\boldsymbol{x}_k$, that is,

$$\pi(\boldsymbol{z}_k | \boldsymbol{x}_0, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_k) = \pi(\boldsymbol{z}_k | \boldsymbol{x}_k) \tag{3b}$$

(c) The sequence $\boldsymbol{x}_k$ depends on the past observations only through its own history, that is,

$$\pi(\boldsymbol{x}_k | \boldsymbol{x}_{k-1}, \boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_{k-1}) = \pi(\boldsymbol{x}_k | \boldsymbol{x}_{k-1}) \tag{3c}$$

where $\pi(\boldsymbol{a} | \boldsymbol{b})$ denotes the conditional probability of $\boldsymbol{a}$ when $\boldsymbol{b}$ is given.

For the state and observation noises, the following assumptions are made [6–22]:

(a) For $i \neq j$, the noise vectors $\boldsymbol{v}_i$ and $\boldsymbol{v}_j$, as well as $\boldsymbol{n}_i$ and $\boldsymbol{n}_j$, are mutually independent and also mutually independent of the initial state $\boldsymbol{x}_0$.
(b) The noise vectors $\boldsymbol{v}_i$ and $\boldsymbol{n}_j$ are mutually independent for all i and j.

Different problems can be considered for the evolution-observation model just described, such as [6, 10]:

(i) The prediction problem, when the objective is to obtain $\pi(\boldsymbol{x}_k | \boldsymbol{z}_{1:k-1})$.
(ii) The filtering problem, when the objective is to obtain $\pi(\boldsymbol{x}_k | \boldsymbol{z}_{1:k})$.
(iii) The fixed-lag smoothing problem, when the objective is to obtain $\pi(\boldsymbol{x}_k | \boldsymbol{z}_{1:k+p})$, where $p \geq 1$ is the fixed lag.
(iv) The whole-domain smoothing problem, when the objective is to obtain $\pi(\boldsymbol{x}_k | \boldsymbol{z}_{1:K})$, where $\boldsymbol{z}_{1:K} = \{\boldsymbol{z}_i,\ i = 1, \ldots, K\}$ is the complete set of measurements.

The filtering problem is dealt with in this article, as described next.

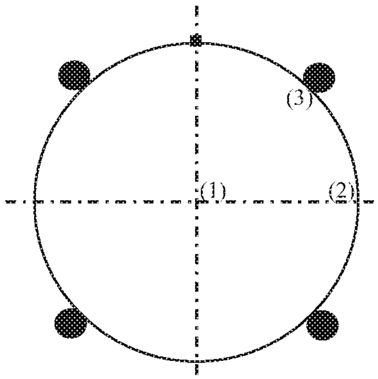

Figure 1 Hypothetical heating system on a pipeline cross section.

PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The idealized problem addressed here considers a critical operational condition involving the cooling of a pipeline,during a production shutdown. This work involves a two-dimensional heat transfer analysis in a cross section of the pipeline, where temperature measurements are available. Temperature measurements can be obtained through specially designed optical fiber sensors [5]. The physical problem under consideration consists of a pipeline cross section represented by a circular domain filled with a stagnant fluid and bounded by a constant-thickness pipe wall, with four electrical cables evenly located over its external surface [5], as illustrated in Figure 1. The fluid is considered as homogeneous, isotropic, and with constant thermal properties. The idealized pipeline will be treated here with a transient heat conduction problem in a single medium, thus not taking into account the pipe wall. The heat flow rate resulting from the Joule effect in the electrical cables is considered in the form of a transient heat flux imposed as a boundary condition for the problem.

The dimensionless mathematical formulation for this problem in cylindrical coordinates is given by

$$\frac{\partial\theta\left(R,\varnothing,\tau\right)}{\partial\tau}=\frac{\partial^2\theta}{\partial R^2}+\frac{1}{R}\frac{\partial\theta}{\partial R}+\frac{1}{R^2}\frac{\partial^2\theta}{\partial\varnothing^2}\quad 0\le R<1,$$

$$0\le\varnothing<2\pi,\quad \tau>0 \tag{4a}$$

where $\theta\left(R,\varnothing,\tau\right)$ is the dimensionless temperature distribution into the pipeline. This equation was solved subjected to the following boundary and initial conditions:

$$\frac{\partial\theta}{\partial R}+Bi\theta=Q(\varnothing,\tau)\quad R=1,\ 0\le\varnothing<2\pi,\ \tau>0 \tag{4b}$$

$$\theta=1\quad 0\le R<1,\ 0\le\varnothing<2\pi,\ \tau=0 \tag{4c}$$

where the following dimensionless groups were defined:

$$\theta\left(R,\varnothing,\tau\right)=\frac{T\left(r,\varnothing,t\right)-T_\infty}{T_0-T_\infty} \tag{5a}$$

$$\tau=\frac{\alpha t}{r^{*2}} \tag{5b}$$

$$R=\frac{r}{r^*} \tag{5c}$$

$$Bi=\frac{hr^*}{k} \tag{5d}$$

$$Q\left(\varnothing,\tau\right)=\frac{q\left(\varnothing,t\right)r^*}{k\left(T_0-T_\infty\right)} \tag{5e}$$

Here, T_∞ is the surrounding environment temperature, T_0 is the uniform initial temperature of the fluid, h is the convective heat transfer coefficient between the pipe and the surrounding environment, k and α are the fluid thermal conductivity and diffusivity, respectively, r^* is the external radius, Bi is the *Biot* number, and $q(\varnothing,t)$ is the heat flux imposed on the external surface by the heating cable. Uncertainties inherent to the parameters used in the model are taken care of as described in the next section.

The mathematical formulation governing the heat conduction problem, given by Eqs. (4a)–(4c), was solved with the finite-volume method. The computer code developed for this purpose was verified by using an analytical solution obtained with the classical integral transform technique.

OPTIMAL CONTROL BASED ON PARTICLE FILTER OBSERVER

The state space representation of a dynamical system consists of the specification of the evolution model for the state variables and observation model that links the measurements to the state variables. Thus, for the classical linear time-invariant discrete state estimation problem, the evolution model may be written in the form

$$\boldsymbol{x}_k=\boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1}+\boldsymbol{G}_{k-1}\boldsymbol{u}_{k-1}+\boldsymbol{v}_{k-1} \tag{6}$$

where $\boldsymbol{F}$ is the linear evolution matrix of the state variables $\boldsymbol{x}_{k-1}$ and $\boldsymbol{G}$ is the input matrix. The state uncertainty or noise $\boldsymbol{v}_{k-1}$ is assumed to be a Gaussian random variable with zero mean and known covariance Ω_v.

The linear observation equation is given in the form

$$z_k=\boldsymbol{H}_k\boldsymbol{x}_k+\boldsymbol{n}_k \tag{7}$$

where z_k is the measurement vector and $\boldsymbol{H}$ is the linear observation matrix, which relates the state variables to the measurements. The observation noise $\boldsymbol{n}_k$ is assumed to be a Gaussian random variable with zero mean and known covariance Ω_n.

In the application under study, the evolution model is given by the finite volume representation of Eqs. (4a)–(4c). The state vector $\boldsymbol{x}_k$ contains the values of the temperatures at each of the volumes and the control variable $\boldsymbol{u}$ is given by the heat flux imposed on the external boundary. Uncertainties in the evolution model come from the fact that different quantities in the formulation are not exactly known, such as the Biot number.

The main objective of the pipeline heating system is to keep the fluid temperature above the critical one, before solid deposits can be formed. Such a critical temperature might be reached by the fluid during cooling periods. Thus, for the application of the control strategy in accordance with the optimal control theory for linear problems, an objective function is established in order to find the control input $\boldsymbol{u}$ (the boundary heat flux) that minimizes the difference between the fluid temperature field and a desired reference temperature, where the formation of solid deposits can be safely avoided.

For the implementation of the control strategy we consider [24]:

$$\bar{\boldsymbol{u}}_k = \boldsymbol{u}_k^* - \boldsymbol{u}_d \tag{8a}$$

$$\bar{\boldsymbol{x}}_k = \boldsymbol{x}_k^* - \boldsymbol{x}_d \tag{8b}$$

where $\boldsymbol{u}_d$ and $\boldsymbol{x}_d$ refer to the steady values of the control input and of the state variables, respectively. Hence, $\bar{\boldsymbol{u}}_k$ and $\bar{\boldsymbol{x}}_k$ are considered as deviations from their steady-state values.

In terms of the linear quadratic regulator problem, the optimal values of the control input $\bar{\boldsymbol{u}}_k$ are obtained by minimizing the following quadratic cost function [24]:

$$J = \lim_{t_k \to \infty} \frac{1}{t_k} \sum_{t=0}^{t_k} \left[(\bar{\boldsymbol{x}}_k)^T \boldsymbol{Q} (\bar{\boldsymbol{x}}_k) + \bar{\boldsymbol{u}}_k^T \boldsymbol{R} \bar{\boldsymbol{u}}_k \right] \tag{9}$$

where the weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ are symmetric positive definite.

The solution to the optimal control problem is the state feedback control law [24] given by

$$\bar{\boldsymbol{u}}_k = -\boldsymbol{K} \bar{\boldsymbol{x}}_k \tag{10}$$

where the discrete-time state feedback gain $\boldsymbol{K}$ is of the form

$$\boldsymbol{K} = \left(\boldsymbol{R} + \boldsymbol{G}^T \boldsymbol{S} \boldsymbol{G} \right)^{-1} \boldsymbol{G}^T \boldsymbol{S} \boldsymbol{F} \tag{11}$$

The matrix $\boldsymbol{S}$ is the steady-state solution to the time-discrete Riccati's equation [24]:

$$\boldsymbol{F}^T \boldsymbol{S} \boldsymbol{F} - \boldsymbol{S} + \boldsymbol{Q} - \boldsymbol{F}^T \boldsymbol{S} \boldsymbol{G} \left(\boldsymbol{R} + \boldsymbol{G}^T \boldsymbol{S} \boldsymbol{G} \right)^{-1} \boldsymbol{G}^T \boldsymbol{S} \boldsymbol{F} = 0 \tag{12}$$

Thus, the control input $\boldsymbol{u}_k^*$ can be calculated from the control law (10) as:

$$\boldsymbol{u}_k^* = \boldsymbol{u}_d - \boldsymbol{K} \left(\boldsymbol{x}_k^* - \boldsymbol{x}_d \right) \tag{13}$$

However, when the state variables are not directly available for control, an observer must be built to estimate the state variables from the input and output variables of the system. For the solution of the state estimation problem considered here, which involves the estimation of the transient temperature field in the medium from temperature measurements taken at the surface of the pipe (Figure 1), the particle filter method is used [6–22]. The particle filter predicts the transient temperature field in the cross section of the domain, from which the minimum value is extracted for control purposes at each time instant.

The particle filter is a Monte Carlo technique used for the solution of state estimation problems, where the main idea is to represent the required posterior density function by a set of random samples with associated weights and to compute the estimates based on these samples and weights. Let $\{\boldsymbol{x}_{0:k}^i, i = 0, \ldots, I\}$ be the particles with associated weights $\{\boldsymbol{w}_k^i, i = 0, \ldots, I\}$ and $\boldsymbol{x}_{0:k} = \{x_j, j = 0, \ldots, k\}$ be the set of all states up to t_k, where I is the number of particles. The weights are normalized, so that $\sum_{i=1}^{I} w_k^i = 1$. Then the posterior density at t_k can be discretely approximated by:

$$\pi \left(\boldsymbol{x}_{0:k} | z_{1:k} \right) \approx \sum_{i=1}^{I} w_k^i \delta \left(\boldsymbol{x}_{0:k} - \boldsymbol{x}_{0:k}^i \right) \tag{14}$$

where $\delta(.)$ is the Dirac delta function. Similarly, its marginal distribution, which is of interest for the filtering problem, can be approximated by:

$$\pi(\boldsymbol{x}_k | z_{1:k}) \approx \sum_{i=1}^{N} w_k^i \, \delta \left(\boldsymbol{x}_k - \boldsymbol{x}_k^i \right) \tag{15}$$

A common problem with the particle filter method is the degeneracy phenomenon, where after a few states all but one particle may have negligible weight. The degeneracy implies that a large computational effort is devoted to updating particles whose contribution to the approximation of the posterior density function is almost zero. This problem can be overcome by increasing the number of particles. In addition, the use of the resampling technique is recommended to avoid the degeneracy of the particles [9, 10]. Resampling involves a mapping of the random measure $\{\boldsymbol{x}_k^i, w_k^i\}$ into a random measure $\{\boldsymbol{x}_k^i, I^{-1}\}$ with uniform weights; it deals with the elimination of particles originally with low weights and the replication of particles with high weights. It can be performed if the number of effective particles with large weights falls below a certain threshold number. Alternatively, resampling can also be applied indistinctively at every instant t_k, as in the sampling importance resampling (SIR) algorithm used here [9, 10]. This algorithm can be summarized in the steps presented in Table 1, as applied to the system evolution from t_{k-1} to t_k.

Although the resampling step reduces the effects of the degeneracy problem, it may lead to a loss of diversity and the resultant sample can contain many repeated particles. This problem, known as sample impoverishment, can be severe in the case of small evolution model noise. In this case, all particles collapse to a single particle within a few instants. Another drawback of the particle filter is related to the large computational costs due to the Monte Carlo method, which may not allow its application to complicated physical problems. On the other hand, algorithms more involved than the one just presented have been developed [10, 18] and can reduce the number of particles required for an appropriate representation of the posterior density, thus resulting in the reduction of associated computational times,

Table 1 Sampling importance resampling algorithm

Step 1
For $i = 1, \ldots, I$ draw new particles x_k^i from the prior density $\pi(x_k \mid x_{k-1}^i)$ and then use the likelihood density to calculate the correspondent weights $w_k^i = \pi(z_k \mid x_k^i)$.
Step 2
Calculate the total weight $T_w = \sum_{i=1}^{I} w_k^i$ and then normalize the particle weights, that is, for $i = 1, \ldots, I$ let $w_k^i = T_w^{-1} w_k^i$.
Step 3
Resample the particles as follows: Construct the cumulative sum of weights (CSW) by computing $c_i = c_{i-1} + w_k^i$ for $i = 1, \ldots, I$, with $c_0 = 0$. Let $i = 1$ and draw a starting point d_1 from the uniform distribution $U[0, I^{-1}]$. For $j = 1, \ldots, I$ Move along the CSW by making $d_j = d_1 + I^{-1}(j-1)$. While $d_j > c_i$ make $i = i + 1$. Assign sample $x_k^j = x_k^i$. Assign sample $w_k^j = I^{-1}$.

especially when associated with parallel computing techniques. In addition, the use of reduced models or the use of response surfaces for the solution of the direct problem appear as promising approaches for the reduction of the computational time, thus enabling the use of sampling methods for more involved cases.

RESULTS AND DISCUSSIONS

In order to examine a test case involving typical conditions resulting from a shutdown of the flow through the pipeline, a hypothetical situation was simulated where the stagnant fluid was assumed to be initially at the uniform temperature of $T_0 = 80°C$ in a circular domain with external diameter of 0.1682 m (6 inches). The surrounding temperature was considered of $T_\infty = 4°C$. The thermophysical properties were assumed constant and given by $k = 12.54$ W m^{-1} °C^{-1}, $\rho = 933.59$ kg m^{-3}, and $c_p = 1826.80$ J kg^{-1} °C^{-1}. The objective of the heating system is to drive the minimum temperature of the stagnant fluid to a reference value of 30°C. The heating system was turned on when the lowest predicted temperature in the domain reached the critical value of formation of solid deposits, which was assumed to be 20°C. For the results presented in the following, the Biot number was taken as 1.

For the prediction of the state variables, one single sensor was considered available, located at the surface of the circular domain (Figure 1). The simulated measurements contain additive, uncorrelated, Gaussian errors, with zero mean and a constant standard deviation of 3°C. This corresponds to 3.75% of the maximum temperature in the region, that is, the initial temperature of the stagnant fluid (80°C). Errors in the evolution model are also supposed to be additive, Gaussian, uncorrelated, with zero mean and constant standard deviation. The effects of the errors in the evolution model, on the prediction of the temperature field in the region, are examined next by considering two different standard deviations for such errors: 0.1°C and 3°C, corresponding, namely, to test case 1 and test case 2, respectively. For the results presented next, 5000 particles were used in the particle filter method. Numerical experiments revealed that such number of particles would be sufficient to represent the posterior distribution of the predicted states.

Figures 2a and b present the simulated measured temperatures, during both the cooling and heating periods, for standard deviations in the evolution model of 0.1°C and 3°C, respectively. The critical temperature for the formation of solid deposits, as well as the aimed reference temperature, is also presented in these figures. Figures 2a and b also show the temperatures predicted with the particle filter at the measurement position, for test cases 1 and 2, respectively. An analysis of Figures 2a and b reveals the capabilities of the particle filter in accurately predicting the temperatures, even under a large standard deviation of errors of the evolution model, such as for test case 2 (Figure 2b). As the evolution model becomes more uncertain, the predicted

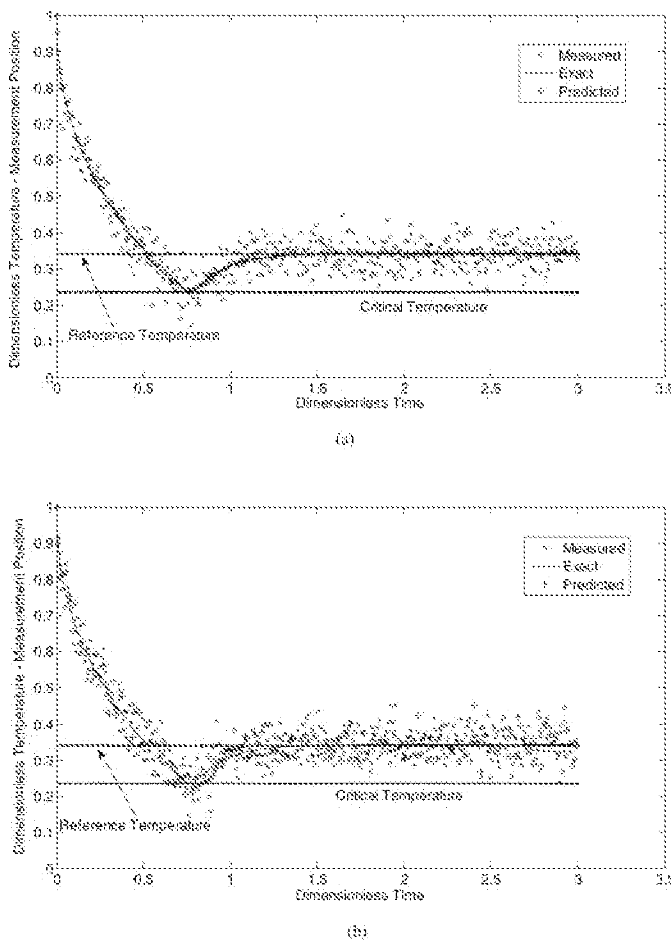

Figure 2 Exact temperatures, simulated measurements, and predicted temperatures for a standard deviation in the evolution model errors of (a) 0.1°C and (b) 3°C.

temperatures at the measurement location tend to follow the measurements (Figure 2b). On the other hand, for evolution models with low uncertainty, the predicted temperatures follow the exact ones very closely (Figure 2a).

We now present the results obtained for the state estimation problem and optimal control, by using simulated experiments. The minimum temperature predicted by the particle filter (implemented in accordance with the SIR algorithm; Table 1) in the whole domain was used in the control strategy described earlier, with the weighting matrices $\boldsymbol{Q} = \boldsymbol{R} = \boldsymbol{I}$ (identity matrix). For both cases, we compare the exact temperature (obtained with the numerical solution with finite volumes) and predicted temperatures at three positions: $R = 0$ (centerline), $R = 1$ under the heater, and $R = 1$ between heaters (Figure 1). Figures 3a and 3b show the time evolution of the predicted temperatures at these three positions, for test cases 1 and 2, respectively. One can clearly see that the heating is turned on when the lowest temperature in the domain (at $R = 1$) reaches the critical value. During the cooling period, the temperature variation of the medium is purely radial. On the other hand, after the heating starts the temperature behavior is two-dimensional as a result of the nonuniform heating over the boundary, because the controlled heat flux is imposed on the regions where the electrical cables are located (Figure 1). Anyhow, the minimum temperature inside the domain (at point 2) is accurately predicted by the particle filter at each time instant, and such a minimum temperature is driven to the reference value to avoid the formation of solid deposits. Conspicuously, the temperatures at other points in the domain are above this minimum temperature due to the external heating. In fact, the largest temperatures take place in the region below the applied heat flux (point 3).

The optimal heat flux obtained through the control strategy already described is presented in Figures 4a and b, for test cases 1 and 2, respectively. This figure shows that the heat flux attains large values when the heating is turned on, but gradually tends to a constant value that makes the minimum temperature in the medium approach the desired reference value. It is important to note that a completely erratic heat flux would be obtained if the measurements shown in Figure 1 were directly used in the control approach, without the use of the particle filter to predict the minimum temperature in the region at each time instant.

Figures 5a and b were prepared in order to illustrate the effects of giving different weights to the terms containing the

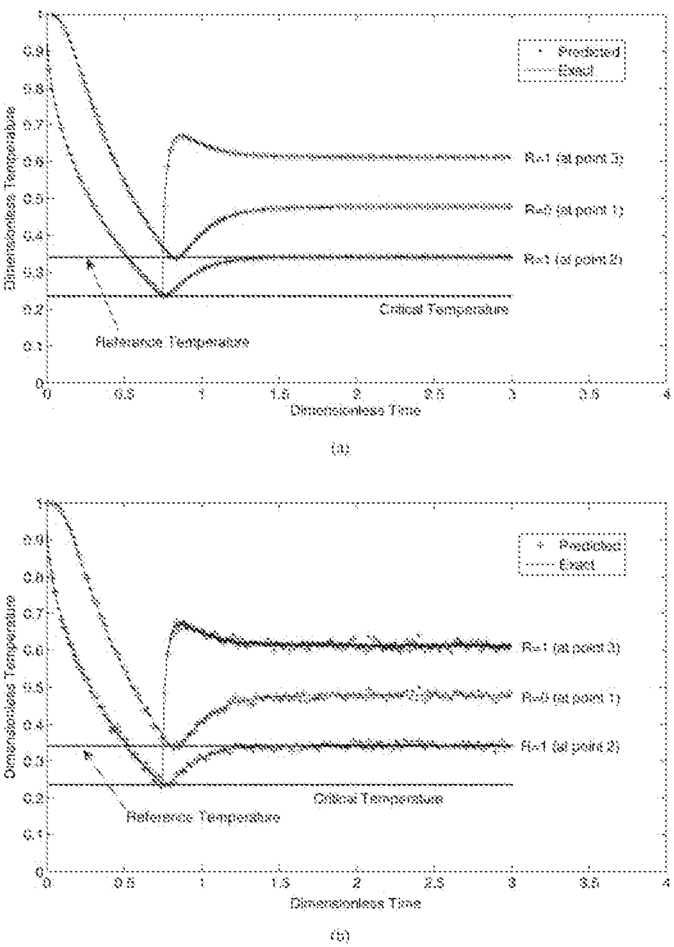

Figure 3 Predicted temperatures and comparison with exact ones with a standard deviation in the evolution model errors of (a) 0.1°C and (b) 3°C.

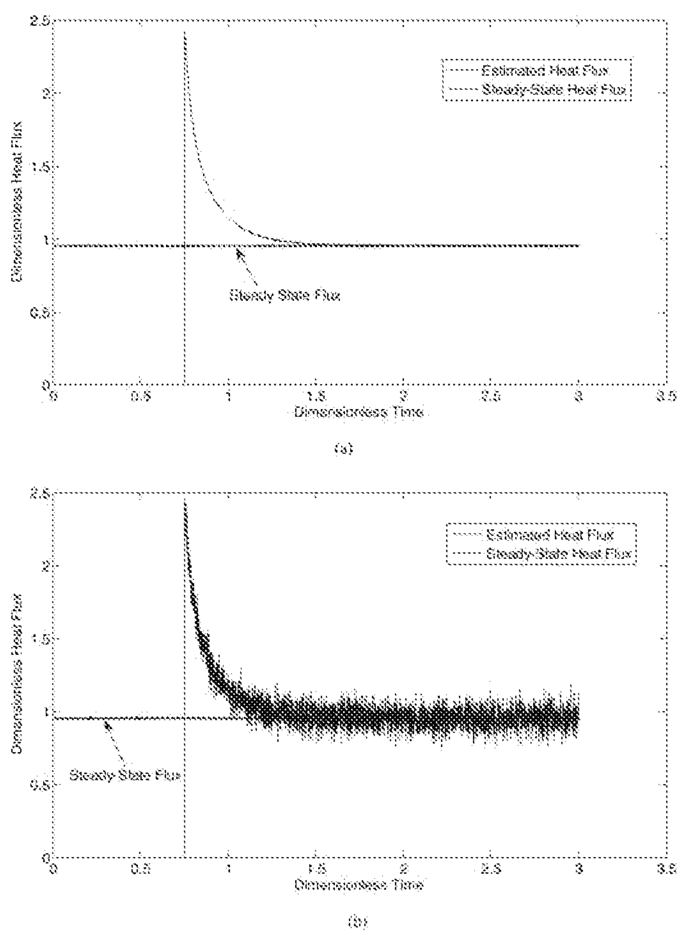

Figure 4 Optimal heat flux on the boundary surface with a standard deviation in the evolution model errors of (a) 0.1°C and (b) 3°C.

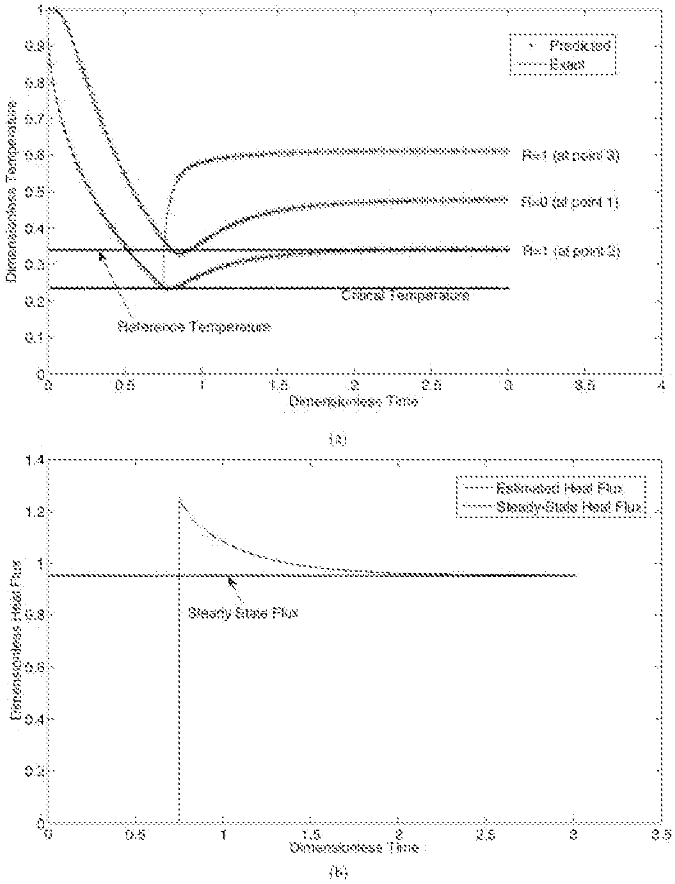

Figure 5 (a) Predicted temperatures and comparison with exact ones. (b) Optimal heat flux on the boundary surface, for test case 1, with $\boldsymbol{Q} = \boldsymbol{I}$ and $\boldsymbol{R} = 10\,\boldsymbol{I}$.

state variables and the control variables in the objective function (9), which was used for the optimal control strategy; the results presented correspond to test case 1, but they were obtained with $\boldsymbol{Q} = \boldsymbol{I}$ and $\boldsymbol{R} = 10\,\boldsymbol{I}$. As expected, increasing the weighting matrix related to the control variable results in a smaller imposed heat flux (Figures 4a and 5b), but in a larger time required for the minimum temperature to reach its desired reference value (Figures 3a and 5a). The opposite effect (larger heat flux and smaller time) can be observed if the weighting matrix $\boldsymbol{Q}$ is made larger than $\boldsymbol{R}$.

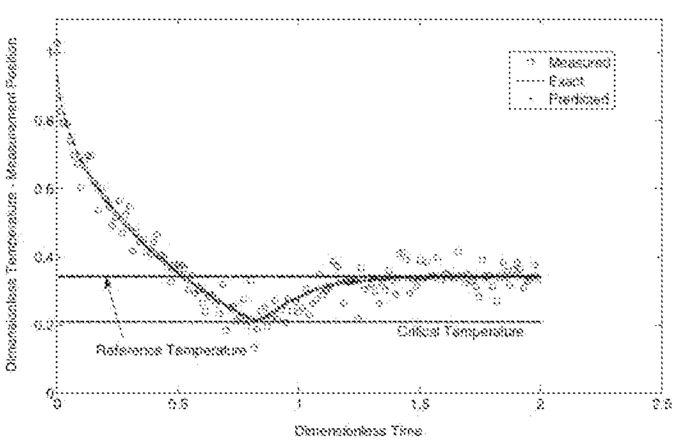

Figure 6 Exact temperatures, simulated measurements, and predicted temperatures obtained with the Kalman filter for test case 1.

Finally, we compare the results obtained here with the particle filter to those obtained with the classical Kalman filter [6–18]. Figure 6 shows the exact, simulated, and predicted temperatures obtained with the Kalman filter for test case 1, with $\boldsymbol{Q} = \boldsymbol{R} = \boldsymbol{I}$. An analysis of Figures 2a and 6 reveals that the particle filter is capable of predicting the exact temperatures with accuracy comparable to that of the Kalman filter, which is the optimal solution for problems with linear and Gaussian evolution and measurement models. On the other hand, for nonlinear and/or non-Gaussian models the basic hypotheses required for the application of the Kalman filter are not valid. Therefore, the particle filter can be used for extensions of the present work (e.g., fluids with temperature-dependent properties), where the classical Kalman filter cannot be applied.

CONCLUSIONS

The objective of this article was to apply an optimal control strategy to a heating system, in order to avoid the formation of solid deposits in pipelines. The optimal control input was determined with a linear quadratic regulator, where a quadratic cost functional was minimized through the solution of Riccati's equation. The minimum temperatures in the domain, predicted with the particle filter, were used in the control strategy instead of the direct measurements. The particle filter was capable of providing accurate estimates for the temperature field in the region, even for large errors in the observation and measurement models. With the present approach, the control strategy could be effectively applied and the minimum temperature in the region maintained above the critical one during the time range of interest.

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NOMENCLATURE

$\boldsymbol{F}$	linear evolution matrix
$\boldsymbol{G}$	input matrix
h	global heat transfer coefficient
$\boldsymbol{H}$	linear observation matrix
$\boldsymbol{I}$	identity matrix
I	number of particles
J	quadratic cost functional
k	fluid thermal conductivity
$\boldsymbol{K}$	discrete-time state feedback gain matrix
$\boldsymbol{n}$	measurement noise or uncertainty
q	heat flux

r^* external radius
T_∞ surrounding environment temperature
$\boldsymbol{v}$ state noise or uncertainty vector
$\boldsymbol{u}$ control input
$\boldsymbol{x}$ state variables
z measurements

Greeks Symbols

α diffusivity
Ω covariance matrix
θ dimensionless temperature

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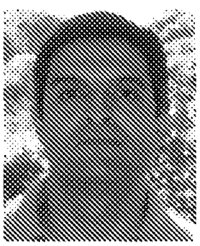

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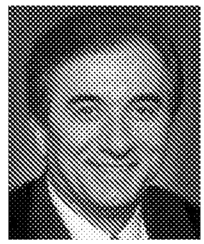

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