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Inverse Approaches to Drying of Thin Bodies With Significant Shrinkage Effects

This paper deals with the application of inverse concepts to the drying of bodies that undergo changes in their dimensions. Simultaneous estimation is performed of moisture diffusivity, together with the thermal conductivity, heat capacity, density, and phase conversion factor of a drying body, as well as the heat and mass transfer coefficients and the relative humidity of drying air. This was accomplished by using only temperature measurements. A mathematical model of the drying process of shrinking bodies has been developed where the moisture content and temperature fields in the drying body are expressed by a system of two coupled partial differential equations. The shrinkage effect was incorporated through the experimentally obtained changes of the specific volume of the drying body in an experimental convective dryer. The proposed method was applied to the process of drying potatoes. For the estimation of the unknown parameters, the transient readings of a single temperature sensor located in the midplane of the potato slice, exposed to convective drying, have been used. The Levenberg–Marquardt method and a hybrid optimization method of minimization of the least-squares norm are used to solve the present parameter estimation problem. Analyses of the sensitivity coefficients and of the determinant of the information matrix are presented as well.

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Introduction

There are several methods for describing the direct problem of complex simultaneous heat and moisture transport processes within a drying material. In the approach proposed by Luikov [1] the moisture and temperature fields in the drying body are expressed by a system of two coupled partial differential equations. The system of equations incorporates coefficients that must be determined experimentally.

All the coefficients, except for the moisture diffusivity, can be relatively easily determined by experiments [2,3]. A number of methods for the experimental determination of the moisture diffusivity exist such as: sorption kinetics methods, permeation methods, concentration-distance methods, drying methods, radiotracer methods, and methods based on the techniques of electron spin resonance and nuclear magnetic resonance, but there is no standard method. The adoption of a generalized method for moisture diffusivity estimation would be of great importance.

We have recently analyzed a method for moisture diffusivity estimation by the temperature response of a drying body [4–11]. The main idea of this method is to make use of the interrelation between the heat and mass transport processes within the drying body and from its surface to the surroundings. Then, the moisture diffusivity can be estimated on the basis of an accurate and easy to perform single thermocouple temperature measurement by using an inverse approach.

The objective of this paper is an analysis of the possibility of the simultaneous estimation of the moisture diffusivity, together with other thermophysical properties of vegetables, as well as the heat and mass transfer coefficients. The method requires a single drying experiment and a single temperature measurement probe. As a representative drying vegetable product, thin slices of potato have been chosen. An analysis of the influence of the drying air velocity, temperature and relative humidity, drying body dimensions, and drying time on the moisture diffusivity estimation, enables the design of appropriate experiments to be conducted as well. In order to realize this analysis, the sensitivity coefficients and the determinant of the information matrix were calculated for the characteristic drying regimes and drying body dimensions.

Physical Problem and Mathematical Formulation

The physical problem involves a single slice of a potato of thickness $2L$ initially at uniform temperature and uniform moisture content (Fig. 1). The surfaces of the drying body are in contact with the drying air, thus resulting in a convective boundary condition for both the temperature and the moisture content. The

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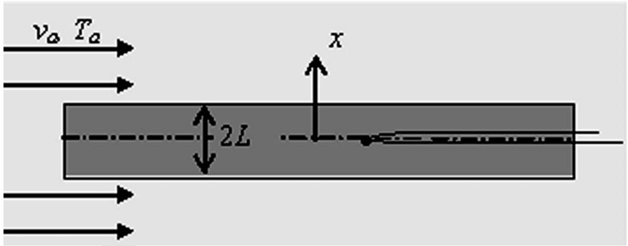

Fig. 1 Scheme of the drying experiment

problem is symmetrical relative to the mid-plane of the slice. The thickness of the body changes during the drying from $2L_0$ to $2L_f$.

In the case of an infinite flat plate the unsteady temperature, $T(x,t)$, and moisture content, $X(x,t)$, fields in the drying body are expressed by the following system of coupled nonlinear partial differential equations for energy and moisture transport

$$c\rho_s\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\varepsilon\Delta H\frac{\partial(\rho_s X)}{\partial t} \tag{1}$$

$$\frac{\partial(\rho_s X)}{\partial t}=\frac{\partial}{\partial x}\left(D\rho_s\frac{\partial X}{\partial x}+D\rho_s\delta\frac{\partial T}{\partial x}\right) \tag{2}$$

Here, t, x, c, k, ΔH, ε, δ, D, and ρ_s are time, normal distance from the midplane of the plate, specific heat, thermal conductivity, latent heat of vaporization, ratio of water evaporation rate to the reduction rate of the moisture content, thermogradient coefficient, moisture diffusivity, and density of dry solid, respectively.

From the experimental and numerical examinations of the transient moisture and temperature profiles [12] it was concluded that for practical calculations, the influence of any thermodiffusion is small and can be ignored. Consequently, $\delta=0$ was utilized in this paper.

The shrinkage effect of the drying body was incorporated through the changes of the specific volume of the drying body. There are several models for describing the changes of the specific volume of the body during drying. In this paper, linear relationship between the specific volume, ν_s, and the moisture content, X, has been used

$$\nu_s=\frac{1}{\rho_s}=\frac{V}{m_s}=\frac{1+\beta' X}{\rho_{b0}} \tag{3}$$

Here, m_s is the mass of the dry material of the drying body, V is the volume of the drying body, ρ_{b0} is the density of a fully dried body, and β' is the shrinkage coefficient.

Substituting the above expression for $\rho_s(=1/\nu_s)$ into Eqs. (1) and (2) and rearranging with $\delta=0$, results in

$$\frac{\partial T}{\partial t}=\frac{k}{\rho_s c}\frac{\partial^2 T}{\partial x^2}+\frac{\varepsilon\Delta H}{c}\frac{\rho_s}{\rho_{b0}}\frac{\partial X}{\partial t} \tag{4}$$

$$\frac{\partial X}{\partial t}=D\frac{\rho_{b0}}{\rho_s}\frac{\partial^2 X}{\partial x^2}+\frac{\rho_{b0}}{\rho_s^2}\frac{\partial(D\rho_s)}{\partial x}\frac{\partial X}{\partial x} \tag{5}$$

The problem of the moving boundaries due to the changes of the dimensions of the body during the drying was resolved by introducing the dimensionless coordinate

$$\psi=\frac{x}{L(t)} \tag{6}$$

Consequently, the resulting system of equations for the temperature and moisture content prediction becomes

$$\frac{\partial T}{\partial t}=\frac{k}{\rho_s c}\frac{1}{L^2}\frac{\partial^2 T}{\partial\psi^2}+\frac{\psi}{L}\frac{dL}{dt}\frac{\partial T}{\partial\psi}+\frac{\varepsilon\Delta H}{c}\frac{\rho_s}{\rho_{b0}}\left(\frac{\partial X}{\partial t}-\frac{\psi}{L}\frac{dL}{dt}\frac{\partial X}{\partial\psi}\right) \tag{7}$$

$$\frac{\partial X}{\partial t}=D\frac{\rho_{b0}}{\rho_s}\frac{1}{L^2}\frac{\partial^2 X}{\partial\psi^2}+\left[\frac{\rho_{b0}}{\rho_s^2}\frac{1}{L^2}\frac{\partial(D\rho_s)}{\partial\psi}+\frac{\psi}{L}\frac{dL}{dt}\right]\frac{\partial X}{\partial\psi} \tag{8}$$

The initial conditions are

$$t=0:\quad T(\psi,0)=T_0,\quad X(\psi,0)=X_0 \tag{9}$$

The temperature and the moisture content boundary conditions on the surfaces of the drying body in contact with the drying air are

$$-k\frac{1}{L}\left(\frac{\partial T}{\partial\psi}\right)_{\psi=1}+j_q-\Delta H(1-\varepsilon)j_m=0$$

$$D\rho_s\frac{1}{L}\left(\frac{\partial X}{\partial\psi}\right)_{\psi=1}+j_m=0 \tag{10}$$

The convective heat flux, $j_q(t)$, and mass flux, $j_m(t)$, on these surfaces are

$$j_q=h(T_a-T_{x=L})$$

$$j_m=h_D(C_{x=L}-C_a) \tag{11}$$

where h is the heat transfer coefficient, and h_D is the mass transfer coefficient, T_a is the temperature of the drying air, and $T_{x=L}$ is the temperature on the surfaces of the drying body. The water vapor concentration in the drying air, C_a, is calculated from

$$C_a=\frac{\varphi p_s(T_a)}{R_w T_{k,a}} \tag{12}$$

where φ is the relative humidity of the drying air and p_s is the saturation pressure. The water vapor concentration of the air in equilibrium with the surface of the body exposed to convection is calculated from

$$C_{x=L}=\frac{a(T_{x=L},X_{x=L})p_s(T_{x=L})}{R_w T_{k,x=L}} \tag{13}$$

The water activity, a, or the equilibrium relative humidity of the air in contact with the convection surface at temperature $T_{x=L}$ and moisture content $X_{x=L}$ are calculated from experimental water sorption isotherms.

The boundary conditions on the midplane of the drying slice are

$$\left(\frac{\partial T}{\partial\psi}\right)_{\psi=0}=0,\quad\left(\frac{\partial X}{\partial\psi}\right)_{\psi=0}=0 \tag{14}$$

Problem defined by Eqs. (7)–(14) is referred to as a direct problem when initial and boundary conditions as well as all the parameters appearing in the formulation are known. The objective of the direct problem is to determine the temperature and moisture content fields in the drying body.

In order to approximate the solution of Eqs. (7) and (8), an explicit numerical procedure has been used.

The Drying Body Properties

In this paper, application of the proposed method for the estimation of the thermophysical properties of vegetables has been analyzed. As a representative vegetable product, a potato was chosen.

Heat capacity of food materials can be taken as equal to the sum of the heat capacity of solid matter and water absorbed by that solid

$$c=c_s+c_w X \tag{15}$$

Although the heat capacity of solid matter, c_s, and water, c_w, are functions of the temperature, constant values have been most widely used.

From Ref. [12] it was also concluded that for practical calculations the system of the two simultaneous partial differential equations could be used by treating the thermal conductivity, k, and the phase conversion factor, ε, as constants.

Moisture diffusivity of foods is a function of the temperature and the moisture content as well. The moisture diffusivity dependence of the moisture content for a potato is not clearly expressed [[13], p. 216], and it is very often considered as an Arrhenius-type temperature function [14,15]

$$D = D_0 \exp[-E_0/(RT_k)] \tag{16}$$

with constant values of the Arrhenius factor, D_0, and the activation energy for moisture diffusion, E_0.

The variation in water activity with change in moisture content of samples at a specified temperature is defined by sorption isotherms. There are many different models for describing the sorption isotherms of foods [3]. In recent years, the most widely accepted and efficient model for sorption isotherms of foods has been the Guggenheim–Anderson–de Boer (GAB) model

$$X = \frac{X_m C K a}{(1 - Ka)(1 - Ka + CKa)} \tag{17}$$

The monolayer moisture, X_m, and the adsorption constants C and K are related as Arrhenius type equations with the Arrhenius factors X_{m0}, C_0, and K_0, and the energy terms ΔH_X, ΔH_C, and ΔH_K, respectively.

Inverse Approach

The inverse problem in this paper is solved as a parameters estimation approach. The estimation methodology used is based on the minimization of the ordinary least square norm

$$E(\mathbf{P}) = [\mathbf{Y} - \mathbf{T}(\mathbf{P})]^{\mathrm{T}}[\mathbf{Y} - \mathbf{T}(\mathbf{P})] \tag{18}$$

Here, $\mathbf{Y}^{\mathrm{T}} = [Y_1, Y_2, \ldots, Y_{\mathrm{imax}}]$ is the vector of measured temperatures, $\mathbf{T}^{\mathrm{T}} = [T_1(\mathbf{P}), T_2(\mathbf{P}), \ldots, T_{\mathrm{imax}}(\mathbf{P})]$ is the vector of estimated temperatures at time t_i $(i = 1, 2, \ldots, \mathrm{imax})$, $\mathbf{P}^{\mathrm{T}} = [P_1, P_2, \ldots P_{\mathrm{N}}]$ is the vector of unknown parameters, imax is the total number of measurements, and N is the total number of unknown parameters ($\mathrm{imax} \geqslant \mathrm{N}$).

A hybrid optimization algorithm OPTRAN [16] and the Levenberg–Marquardt method [17–19] have been utilized for the minimization of $E(\mathbf{P})$ representing the solution of the present parameter estimation problem.

The Levenberg–Marquardt method is a quite stable, powerful, and straightforward gradient search minimization algorithm that has been applied to a variety of inverse problems. It belongs to a general class of damped least square methods. The solution for vector $\mathbf{P}$ is achieved using the following iterative procedure

$$\mathbf{P}^{r+1} = \mathbf{P}^r + [(\mathbf{J}^r)^T \mathbf{J}^r + \mu^r \mathbf{I}]^{-1} (\mathbf{J}^r)^T [\mathbf{Y} - \mathbf{T}(\mathbf{P}^r)] \tag{19}$$

where r is the number of iterations, $\mathbf{I}$ is identity matrix, μ is the damping parameter, and $\mathbf{J}$ is the sensitivity matrix defined as

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial T_1}{\partial P_1} & \cdots & \dfrac{\partial T_1}{\partial P_N} \\ \vdots & & \\ \dfrac{\partial T_{\mathrm{imax}}}{\partial P_1} & \cdots & \dfrac{\partial T_{\mathrm{imax}}}{\partial P_N} \end{bmatrix} \tag{20}$$

Near the initial guess, the problem is generally ill conditioned so that large damping parameters are chosen thus making the term $\mu\mathbf{I}$ large as compared to term $\mathbf{J}^T\mathbf{J}$. The term $\mu\mathbf{I}$ damps instabilities due to the ill-conditioned character of the problem. So, the matrix $\mathbf{J}^T\mathbf{J}$ is not required to be nonsingular at the beginning of the iterations and the procedure tends towards a slow-convergent steepest descent method. As the iteration process approaches the converged solution, the damping parameter decreases, and the Levenberg–Marquardt method tends towards a Gauss method. In fact, this method is a compromise between the steepest descent and Gauss method by choosing μ so as to follow the Gauss method to as large an extent as possible, while retaining a bias towards the steepest descent direction to prevent instabilities. The presented iterative procedure terminates if the norm of gradient of $E(\mathbf{P})$ is sufficiently small, if the ratio of the norm of the gradient of $E(\mathbf{P})$ to $E(\mathbf{P})$ is small enough, or if the changes in the vector of parameters are very small.

An alternative to the Levenberg–Marquardt algorithm, especially when searching for a global optimum of a function with possible multiple minima, is the hybrid optimization program OPTRAN [16]. OPTRAN incorporates six of the most popular optimization algorithms: the Davidon–Fletcher–Powell gradient search [20], sequential quadratic programming algorithm [21], Pshenichny–Danilin quasi-Newtonian algorithm [22], a modified Nelder–Mead simplex algorithm [23], a genetic algorithm [24], and a differential evolution algorithm [25]. Each algorithm provides a unique approach to optimization with varying degrees of convergence, reliability, and robustness at different stages during the iterative optimization procedure. The hybrid optimizer OPTRAN includes a set of rules and switching criteria to automatically switch back and forth among the different algorithms as the iterative process proceeds in order to avoid local minima and accelerate convergence towards a global minimum.

The population matrix was updated every iteration with new designs and ranked according to the value of the objective function, in this case the ordinary least square norm. As the optimization process proceeded, the population evolved towards its global minimum. The optimization problem was completed when one of several stopping criteria was achieved: the maximum number of iterations or objective function evaluations was exceeded, the best design in the population was equivalent to a target design, or the optimization program tried all six algorithms, but failed to produce a nonnegligible decrease in the objective function. The last criterion usually indicated that a global minimum had been found.

Parameters Estimation Analysis

For the inverse problem of interest here, the moisture diffusivity parameters, together with other thermophysical properties of the potato as well as the heat and mass transfer coefficients and the relative humidity of the drying air, are treated as unknown parameters.

Thus, in the inverse problem the analyzed vector of the unknown parameters was

$$\mathbf{P}^T = [D_0, E_0, \rho_s, c_s, k, \varepsilon, h, h_D, \varphi] \tag{21}$$

For the simultaneous estimation of these unknown parameters, the transient reading of a single temperature sensor located at the position $x = 0$, has been considered.

The possibility of the simultaneous estimation of the temperature-dependent moisture diffusivity together with the other thermophysical properties of the potato as well as the heat and mass transfer coefficients and the relative humidity of the drying air depends on the boundary conditions and dimensions of the drying sample. An analysis of the influence of the drying air parameters and dimensions of the drying sample needed for the design of the appropriate experiment have been conducted in this paper. In order to perform this analysis, the sensitivity coefficients have been calculated.

The sensitivity coefficients analysis has been carried out for an infinite flat plate model of a slice of potato with initial moisture content of $X(x,0) = 5.00$ kg/kg and initial temperature $T(x,0) = 20.0$°C. The drying air bulk temperature, T_a, was varied between 40 and 80°C, the convection heat transfer coefficient between 27 and 33 W m^{-2} K^{-1}, and the initial thickness, $2L_0$, of the potato slice between 2 and 6 mm.

From the sensitivity coefficients analysis the following experimental parameters were chosen: $T_a = 60$°C, $2L_0 = 3$ mm, $h = 30$ W m^{-2} K^{-1}, $h_D = 3.36 \times 10^{-2}$ m s^{-1}, and $\varphi = 0.09$. Figure 2

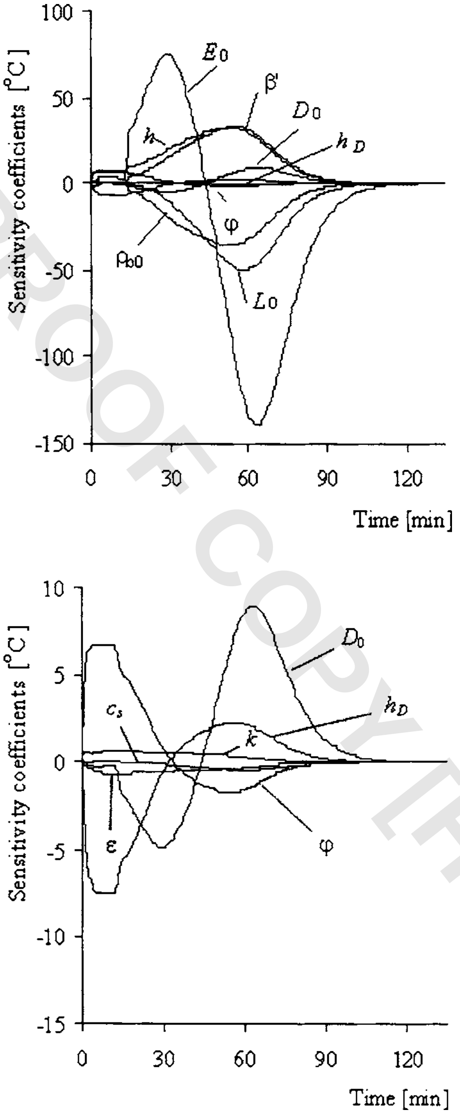

Fig. 2 Relative sensitivity coefficients for temperature

shows the relative sensitivity coefficients $P_j \partial T_i / \partial P_j$, $i = 1, 2, \ldots, \text{imax}$, for temperature with respect to the unknown parameters, for this case.

It can be seen that the relative sensitivity coefficients with respect to the phase conversion factor, ε, and the thermal conductivity, k, are very small. This indicates that ε and k cannot be estimated in this case. This also indicates that the influence of the phase conversion factor and the thermal conductivity on the transient moisture content and temperature profiles is very small in this case. This can be explained by the very small heat transfer Biot number ($\text{Bi}=hL/k \leq hL_0/k=0.11$) and consequently very small temperature gradients inside the body during the drying process. For these reasons, the phase conversion factor and the thermal conductivity were treated as known quantities for the examination described below.

The heat capacity of wet potato was taken as equal to the sum of the heat capacity of solid matter and absorbed water, Eq. (15). Since the heat capacity of the solid matter, c_s, presents only a few percent of the overall heat capacity of the potato, the relative sensitivity coefficients with respect to the heat capacity of solid matter is also very small. Consequently, the value of the heat capacity of the solid matter was also taken as known.

The relative sensitivity coefficients with respect to the density of the fully dried body, ρ_{b0}, and the shrinkage coefficient, β', are relatively high. Despite this, because the shrinkage effect of the drying body was incorporated through the changes of the specific volume of the drying body, these parameters were determined by separate experiments.

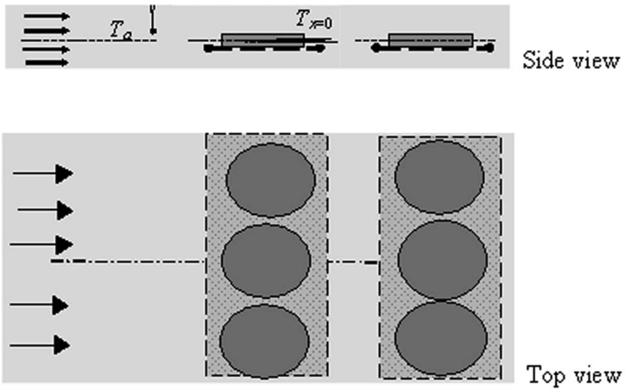

Fig. 3 Scheme of the experimental arrangement

The relative sensitivity coefficients with respect to the initial potato slice thickness are high as well, but the initial slice thickness was measured with sufficient accuracy, so it is also taken as a known parameter.

It can be seen that the temperature sensitivity coefficient with respect to the convection mass transfer coefficient h_D is very small relative to the temperature sensitivity coefficient with respect to the convection heat transfer coefficient, h. The very high mass transfer Biot number and the very small heat transfer Biot number can explain this. To overcome this problem, in this paper the mass transfer coefficient was related to the heat transfer coefficient through the analogy between the heat and mass transfer processes in the boundary layer over a drying body [10]

$$h_D = 0.95 \frac{D_a}{k_a} h \qquad (22)$$

where D_a and k_a are the moisture diffusivity and thermal conductivity in the air, respectively. The obtained relation is very close to the well-known Lewis relation. By using the above relation between the heat and mass transfer coefficients, they can be estimated simultaneously, so that only the heat transfer coefficient is regarded as an unknown parameter.

Experiment

Real experiments have been conducted to investigate the applicability of the method to food processing, when involving drying of thin flat samples. The experiments have been conducted on the experimental setup that is designed to imitate an industrial convective dryer.

Drying of approximately three millimetres thick potato slices have been examined. The slices have been in contact with the drying air from the top and the bottom surfaces. Two shelves, (Fig. 3), each holding three moist potato slices have been introduced into the rectangular experimental channel of dimensions 25 × 200 mm. A microthermocouple was inserted in the midplane, ($x=0$), of each of the three slices on the first shelf. An arithmetical mean of the readings from the three thermocouples was used as a transient temperature reading, ($T_{x=0}$), for the estimation of the unknown parameters. The potato slices on the second shelf were weighed every ten minutes in order to obtain the volume-averaged moisture content change during drying. The temperature of the drying air, T_a, has been recorded as well. The initial moisture content, X_0, and the initial potato slices thickness, $2L_0$, were measured for each of the experiments.

The change of the specific volume of the drying body was determined by a separate experiment. The cylindrical potato slices

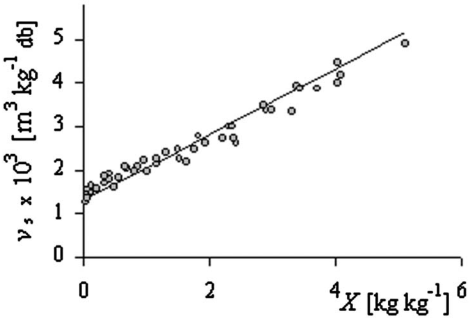

Fig. 4 Change of the specific volume during the drying of potato slices

with diameter of approximately 40 mm and thickness of approximately 3 mm have been placed on the second shelf and dried until the equilibrium moisture content has been reached. The dimensions and the mass of the slices were measured every 10 min. The initial moisture content and the initial potato slices thicknesses were measured as well. The experiment was repeated for different temperatures and speed of the drying air. The drying air temperature was varied between 50 and 70°C, and the drying air speed between 1.0 and 3.0 m s^{-1}.

The relative errors of the measurements were estimated between 0.1% and 1.0% for the mass and 0.3–2.5% for the dimensions of the slices. Microthermocouples were calibrated, relative to each other, within 0.2°C in the range of 20–80°C.

Results and Discussion

From the parameters estimation analysis it was concluded that the moisture diffusivity parameters, D_0 and E_0, the convection heat and mass transfer coefficients, h and h_D, and the relative humidity of the drying air, φ, will be treated as unknown parameters in this paper. All other quantities appearing in the direct problem formulation were taken as known.

Our experimental results for the changes of the specific volume of drying potato slices, (Fig. 4), confirm the expression (3) with $\rho_{b0}=755$ kg m^{-3} and the shrinkage coefficient $\beta'=0.57$. The heat capacity was calculated from Eq. (15). The following values, proposed in Ref. [26] for potatoes, were used: $c_s=1381$ J kg^{-1} K^{-1} and $c_w=4187$ J kg^{-1} K^{-1}. A mean value of $k=0.40$ W m^{-1} K^{-1} from the results obtained in Ref. [27] for the thermal conductivity of potato was utilized in this paper. The influence of the phase conversion factor ($0 \leq \varepsilon \leq 1$) on the transient moisture content and temperature profiles is very small. A mean value of $\varepsilon=0.5$ was used in the paper. For the GAB isotherm equation parameters the Gane experimental results for potatoes [[3], p. 45] were used in this paper ($C_0=6.609\times 10^{-1}$; $\Delta H_C=528.4$ kJ kg^{-1}; $K_0=0.606$; $\Delta H_K=53.33$ kJ kg^{-1}; $\Delta H_X=123.6$ kJ kg^{-1}), except for X_{m0}. The value of $X_{m0}=3.8\times 10^{-2}$ was obtained from our experimental results.

A number of drying experiments with similar experimental conditions, ($T_a=56.6-59.5$°C, $2L_0=2.36-3.14$ mm, $X_0=3.70-4.83$ kg/kg, and $T_0=14.9-17.7$°C), have been carried out.

The experimental drying time was estimated from the determinant of the information matrix. Figure 5 presents the transient variation of the determinant of the information matrix if D_0, E_0, h, h_D, and φ are simultaneously considered as unknown. Elements of this determinant of the information matrix were defined [19] for a large, but fixed number of transient temperature measurements (451 in this case).

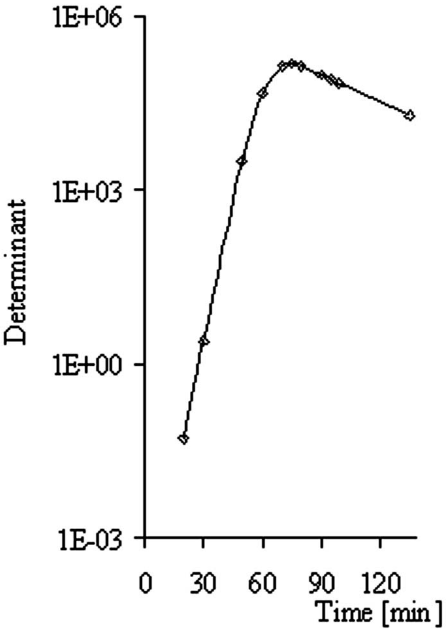

Fig. 5 Determinant of the information matrix

The maximum determinant value corresponds to the drying time when near equilibrium moisture content and temperature profiles have been reached.

The relative temperature sensitivity coefficients with respect to the moisture diffusivity parameters, D_0 and E_0, are almost linearly dependent, (Fig. 2). Despite this, we were able to obtain results using the OPTRAN [16] and the Levenberg–Marquardt algorithm [18]. Table 1 shows the computationally obtained parameters and rms error for experiment $E1$: $T_a=58.13$°C, $2L_0=3.14$ mm, $X_0=4.80$ kg/kg, and $T_0=17.53$°C. The rms changes and the convergence of the estimated values of the unknown parameters to the final values during the iterative process of the Levenberg–Marquardt method, for experiment $E1$ are shown in Fig. 6.

In Fig. 7 the estimated moisture diffusivities are compared with the results published by other authors that used different methods.

In Fig. 8 the experimental transient temperature reading, $T_{x=0}$, and the experimental volume-averaged moisture content change during drying are compared with numerical solutions for the estimated parameters. Very good agreements were obtained. The temperature changes during the weighing of the slices on the second shelf (every 10 min the second shelf together with the slices was taken outside the channel for 15 s to be weighed) can be seen in Fig. 8.

In Fig. 9, the experimental transient temperature reading, $T_{x=0}$, and the experimental volume-averaged moisture content changes during drying are compared with the numerical solutions with the estimated parameters in the cases when the shrinkage effect was

Table 1 Estimated parameters and rms error

	$D_0\cdot 10^3$ (m s^{-1})	E_0 (kJ mol^{-1})	h (W m^{-2} K^{-1})	$h_D\cdot 10^2$ (m s^{-1})	φ (–)	rms (°C)
Initial guess	0.25	35	25	2.8	0.125	4.93
Estimated values	7.985	43.3	31.08	3.48	0.0899	0.55

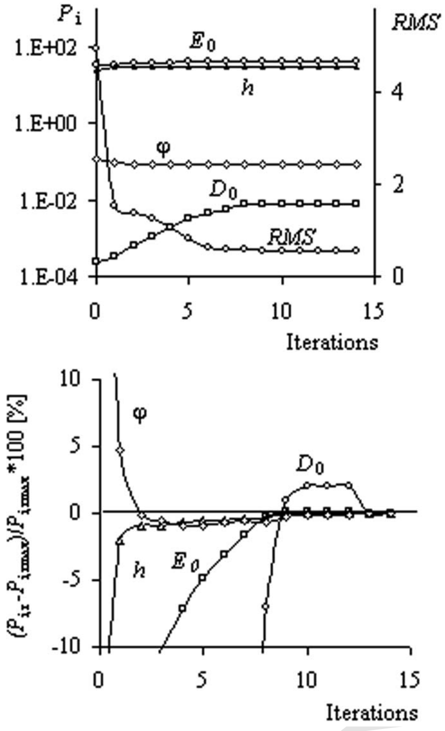

Fig. 6 Convergence history of rms errors and estimated parameters

incorporated and when it was not incorporated. It is very clear that the shrinkage effect cannot be ignored in the calculations of the drying processes of potato slices.

Conclusions

The inverse problem of simultaneous estimation of thermophysical properties and the boundary condition parameters of drying thin slices of vegetables by using only temperature measurements has been analysed. For this, a mathematical model of drying of shrinking bodies has been developed. As a representative vegetable product, a slice of a potato has been chosen.

It can be concluded that in the convective drying experiment it is possible, based on a single thermocouple temperature response, to estimate simultaneously the two moisture diffusivity parameters, the convection heat and mass transfer coefficients, and the relative humidity of the drying air.

Estimated moisture diffusivities compare well with the values obtained by other authors who utilized different methods.

Very good agreement between the experimental and numerical temperature and volume-averaged moisture content changes during drying has been obtained.

Since the relative temperature sensitivity coefficients with respect to the moisture diffusivity parameters in the Arrhenius-type

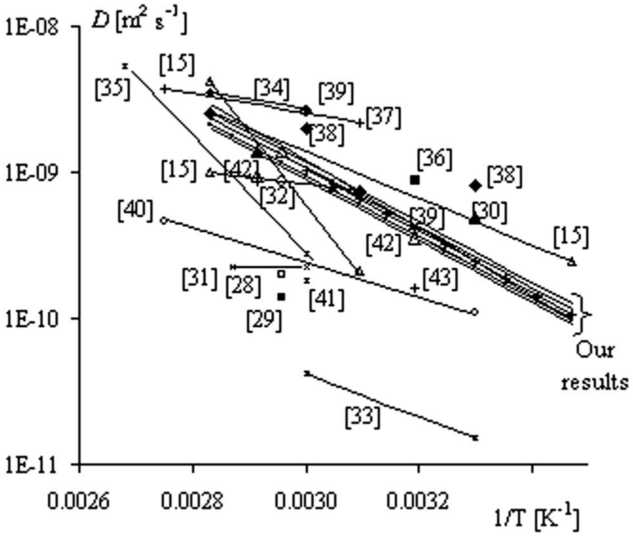

Fig. 7 Moisture diffusivity of potatoes

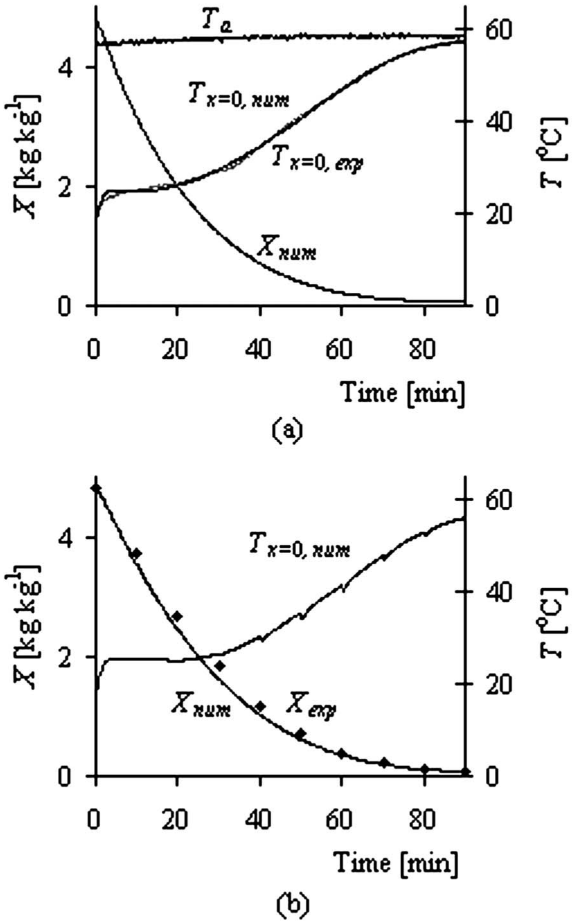

Fig. 8 Time-variations during drying: The midplane temperature, $T_{x=0}$, the temperature of the drying air, T_a, and the volume-averaged moisture content, X: (*a*) the first shelf, and (*b*) the second shelf

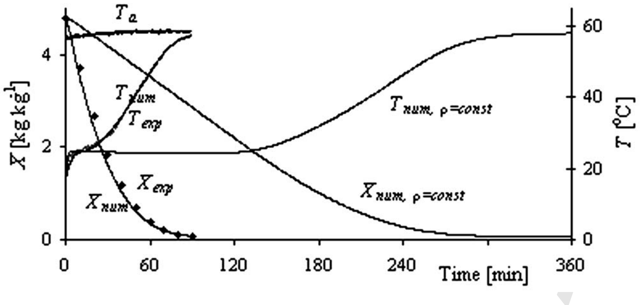

Fig. 9 Changes during drying with shrinkage effect and without shrinkage effect: The midplane temperature, $T_{x=0}$ and the volume-averaged moisture content, X

function are linearly dependent, other models for describing the moisture content and temperature-dependent moisture diffusivity could be analyzed in the future [28–43].

Nomenclature

a = water activity
c = heat capacity (dry basis), J K^{-1} kg^{-1} db
C = concentration of water vapor, kg m^{-3}
D = moisture diffusivity, m^{2} s^{-1}
D_0 = Arrhenius factor, m^{2} s^{-1}
E_0 = activation energy J kg^{-1}
E = ordinary least square norm (°C)^{2}
h = heat transfer coefficient, W m^{-2} K^{-1}
h_D = mass transfer coefficient, m s^{-1}
ΔH = latent heat of vaporization, J kg^{-1}
$\mathbf{I}$ = identity matrix
j_m = mass flux, kg m^{-2} s^{-1}
j_q = heat flux, W m^{-2}
$\mathbf{J}$ = sensitivity matrix
k = thermal conductivity, W m^{-1} K^{-1}
L = flat plate thickness, m
m = mass, kg
p_s = saturation pressure, Pa
$\mathbf{P}$ = vector of unknown parameter
R = absolute gas constant, J K^{-1} mol^{-1}
R_w = specific gas constant, J K^{-1} kg^{-1}
t = time, s
T = temperature, °C
T_k = temperature, K
$\mathbf{T}$ = vector of estimated temperature, °C
v = specific volume, m^{3} kg^{-1}
V = volume, m^{3}
x = distance from the midplane, m
X = moisture content (dry basis), kg kg^{-1} db
$\mathbf{Y}$ = vector of measured temperature, °C

Greek symbols

β' = shrinkage coefficient, -
δ = thermo-gradient coefficient, K^{-1}
ε = phase conversion factor
μ = damping parameter
ρ = density, kg m^{-3}
φ = relative humidity
ψ = dimensionless coordinate

Subscripts

a = drying air
$b0$ = fully dried body
f = final
m = monolayer
w = water
s = dry solid

Superscripts

r = number of iterations
T = transposed

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