8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

MAGNETO-HYDRODYNAMIC SIMULATIONS USING RADIAL BASIS FUNCTIONS

***Marcelo J. Colaço[1] and George S. Dulikravich[2]**

[1] Military Institute of Engineering
Dept. of Mech. and Materials Eng.
Rio de Janeiro, RJ, 22290-270, Brazil
colaco@ime.eb.br

[2] Florida International University
Dept. of Mech. and Materials Eng.
Miami, Florida 33174, U.S.A.
dulikrav@fiu.edu

Key Words: *Radial Basis Functions, Magnetohydrodynamic.*

ABSTRACT

This work deals with the application of the Radial Basis Functions (RBF) to the solution of the classical Magneto-Hydro-Dynamics (MHD) problem [1,2].

The use of RBFs followed by collocation, a technique first proposed by Kansa [3], after the work of Hardy [4] on multivariate approximation, is now becoming an established approach. Kansa's method (or asymmetric collocation) starts by building an approximation to the field of interest (normally displacement components) from the superposition of RBFs (globally or compactly supported) conveniently placed at points in the domain and/or at the boundary. The unknowns (which are the coefficients of each RBF) are obtained from the approximate enforcement of the boundary conditions as well as the governing equations by means of collocation. Usually, this approximation only considers regular RBFs, such as the globally supported multiquadrics or the compactly supported Wendland functions [5].

Use of meshless methods such as the RBF promises to significantly reduce the computing time, especially in arbitrary shaped domains, for the complex classes of problems such as EHD and MHD, which involves the solution of coupled mass, momentum and energy conservation equations and the Maxwell's equations in a moving media.

Table 1 shows the very small CPU time required to solve a MHD fluid flow for $Ra=10^4$ and various Hartmann (Ha) numbers. As the Hartmann number increases, more pronounced is the magnetic field in the *x* direction, thus reducing the thermal buoyancy forces. All test cases presented in this paper were run in an Intel Centrino Duo (T2300 @ 1.66Ghz) with 1Gb of RAM memory. The code was written in Fortran90 and the "cpu_time" intrinsic function was used to measure the computing time. The numbers between parentheses correspond to the number of centers used in each solution.

Table 1. CPU time for solving the MHD problem using RBF with different number of centers.

$Gr=10^4$	CPU time (s) - RBF (6x6)	CPU time (s) - RBF (8x8)	CPU time (s) - RBF (15x15)
Ha = 0	0.7187500	2.046875	50.35938
Ha = 10	0.6562500	1.703125	34.03125
Ha = 15	0.6250000	1.828125	40.14062
Ha = 25	0.5625000	1.687500	42.59375
Ha = 50	0.4218750	1.171875	25.53125

Figure 1 shows the velocity and temperature profiles at mid-location of the cavity, both for the RBF-MHD formulation with 15x15 centers, and for the results presented in [6]. From this figure, one can see that even for this extremely low number of collocation centers, the velocity and the temperature profiles are very well captured. This becomes even more impressive when one looks at Table 1 and sees that such results were obtained in less then one minute.

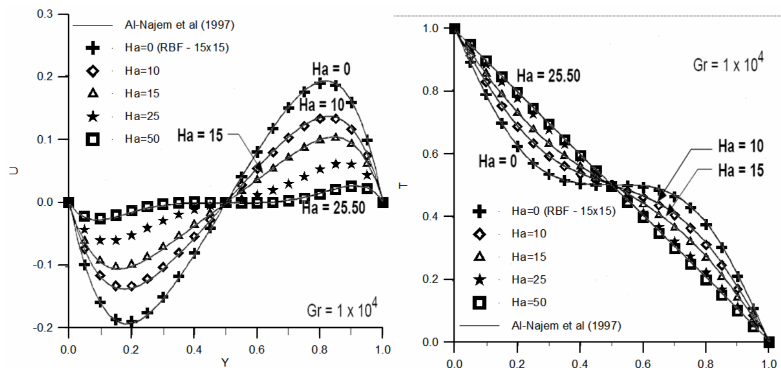

Figure 1. Velocity and temperature profiles for 15x15 RBF centers.

REFERENCES

[1] G.S. Dulikravich and S.R. Lynn, "Unified electro-magneto-fluid dynamics (EMFD): A survey of mathematical models", *Int. J. of Non-Linear Mechanics*, Vol. **32** (5) 923-932 (1997).

[2] M.J. Colaço, and G.S. Dulikravich, "A Multilevel Hybrid Optimization of Magnetohydrodynamic Problems in Double-Diffusive Fluid Flow", *Journal of Physics and Chemistry of Solids*, Vol. **67**, pp. 1965-1972 (2006).

[3] E.J. Kansa, "Multiquadrics – A Scattered Data Approximation Scheme with Applications to Computational Fluid Dynamics – II: Solutions to Parabolic, Hyperbolic and Elliptic Partial Differential Equations", *Comput. Math. Applic.*, Vol. **19**, pp. 149-161 (1990).

[4] R.L. Hardy, "Multiquadric Equations of Topography and Other Irregular Surfaces", *Journal of Geophysics Res.*, Vol. **176**, pp. 1905-1915 (1971).

[5] H. Wendland, "Error Estimates for Interpolation by Compactly Supported Radial Basis Functions of Minimal Degree", *Journal of Approximation Theory*, Vol. **93**, pp. 258-272 (1998).

[6] N.M. Al-Najem, K.M. Khanafer, and N.M. El-Refaee, "Numerical Study of Laminar Natural Convection in Tilted Enclosure with Transverse Magnetic Field", *International Journal of Numerical Methods for Heat & Fluid Flow*, Vol. **8** (6), pp. 651–672 (1998).