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DRAFT

ESTIMATION OF THERMOPHYSICAL PROPERTIES OF A DRYING BODY AT HIGH MASS TRANSFER BIOT NUMBER

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ABSTRACT

The inverse problem of simultaneously estimating the moisture content and temperature-dependent moisture diffusivity together with the heat and mass transfer coefficients by using only temperature measurements is analysed in this paper. In the convective drying practice, usually the mass transfer Biot number is very high and the heat transfer Biot number is very small. This leads to a very small temperature sensitivity coefficient with respect to the mass transfer coefficient relative to the temperature sensitivity coefficient with respect to the heat transfer coefficient. Under these conditions the relative error of the estimated mass transfer coefficient is high. To overcome this problem, in this paper the mass transfer coefficient is related to the heat transfer coefficient through the analogy between the heat and mass transfer processes in the boundary layer.

KEYWORDS

Inverse approach, drying, thermophysical properties, heat and mass transfer coefficients

INTRODUCTION

Inverse approach to parameter estimation in last few decades has become widely used in various scientific disciplines. This paper deals with the application of the inverse approaches in drying.

Drying is a complex process of simultaneous heat and moisture transport within material and from its surface to the surroundings caused by a number of mechanisms. There are several different methods of describing the drying process. In the approach proposed by Luikov [1] from the concepts of irreversible thermodynamics the moisture and temperature fields in the dried body are described by a system of two coupled partial differential equations. The system of equations incorporates coefficients, which are functions of temperature and moisture content, and must be determined experimentally. For practical calculations the influence of the temperature and moisture content on all the transport coefficients except for the moisture diffusivity is small and can be neglected. The moisture diffusivity dependence on moisture and temperature exerts a strong influence on the drying process calculation. This effect cannot be ignored for the most of practical cases. All the coefficients except for the moisture diffusivity can be relatively easily determined by experiments. The main problem in the moisture diffusivity determination by classical or inverse methods is the difficulty of local moisture content measurements within the drying body.

Kanevce, Kanevce and Dulikravich [2, 3, 4, 5] and Dantas, Orlande and Cotta [6, 7] recently analysed application of inverse approaches to estimation of a drying body parameters. The main idea of the applied method is to take advantage of the interrelation between the heat and mass (moisture) transport processes within the drying body and from its surface to the surrounding media. Then, the drying body parameters' estimation can be performed on the basis of accurate and easy-to-perform thermocouple temperature measurements by using an inverse approach. We analysed this idea of the drying body parameters' estimation by using temperature response of a body exposed to convective drying. An analysis of the influence of the drying air parameters and the drying body dimensions was conducted. In order to perform this analysis, the sensitivity coefficients and the sensitivity matrix determinant were calculated.

In the convective drying practice, usually the mass transfer Biot number is very high and the heat transfer Biot number is very small due to the low moisture diffusivity value relative to the thermal conductivity for most of the moist materials. This leads to a very small temperature sensitivity coefficient with respect to the mass transfer coefficient relative to the temperature sensitivity coefficient with respect to the heat transfer coefficient. This indicates that in these cases the mass transfer coefficient cannot be estimated simultaneously with the heat transfer coefficient with sufficient accuracy. To overcome this problem, in this paper the mass transfer coefficient is related to the heat transfer coefficient through the analogy between the heat and mass transfer processes in the boundary layer.

The objective of this paper is an analysis of the possibility of simultaneous estimation of the thermophysical properties of a drying body and the heat and mass transfer coefficients at high mass transfer Biot number by using only temperature measurements.

MATHEMATICAL MODEL OF DRYING

In the case of an infinite flat plate of thickness $2L$, if the shrinkage of the material can be neglected (ρ_s = const), the unsteady temperature field, $T(x,\ t)$, and moisture content field, $X(x,\ t)$, in the drying body are expressed by the following system of coupled nonlinear partial differential equations

$$c\rho_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) + \varepsilon \rho_s \Delta H \frac{\partial X}{\partial t} \tag{1}$$

$$\frac{\partial X}{\partial t} = \frac{\partial}{\partial x}\left(D \frac{\partial X}{\partial x} + D\delta \frac{\partial T}{\partial x} \right) \tag{2}$$

Here, t, x, c, k, ΔH, ε, δ, D, ρ_s are time, distance from the mid-plane of the plate, heat capacity, thermal conductivity, latent heat of vaporization, ratio of water evaporation rate to the reduction rate of the moisture content, thermo-gradient coefficient, moisture diffusivity, and density of the dry plate material, respectively.

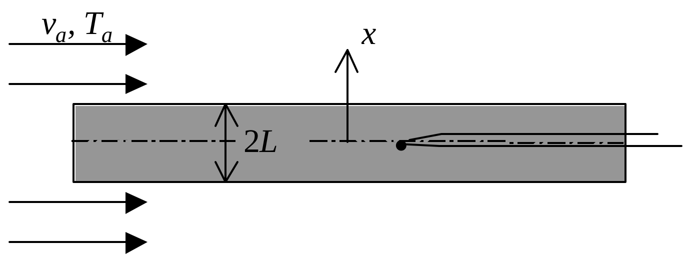

Fig.1. Scheme of the drying experiment

As initial conditions, uniform temperature and moisture content profiles are assumed

$$t = 0 \quad T(x,0) = T_0\,, \quad X(x,0) = X_0 \tag{3}$$

In the convective drying experiment (Fig. 1) the surfaces of the drying body are in contact with the drying air thus resulting in a convective boundary conditions for both the temperature and the moisture content

$$\begin{aligned} -k\left(\frac{\partial T}{\partial x}\right)_{x=L} + j_q - \Delta H\,(1-\varepsilon)\,j_m &= 0 \\ D\rho_s\left(\frac{\partial X}{\partial x}\right)_{x=L} + D\delta\rho_s\left(\frac{\partial T}{\partial x}\right)_{x=L} + j_m &= 0 \end{aligned} \tag{4}$$

The convective heat flux, $j_q(t)$, and mass flux, $j_m(t)$, on these surfaces are

$$\begin{aligned} j_q &= h\left(T_a - T_{x=L}\right) \\ j_m &= h_D\left(C_{x=L} - C_a\right) \end{aligned} \tag{5}$$

where h is the convection heat transfer coefficient and h_D is the mass transfer coefficient, while T_a is the drying air bulk temperature.

The convection heat and mass transfer coefficients can be expressed by the Nesterenko's relations [1] for the heat and mass Nusselt numbers in drying conditions

$$Nu = 0.0270\ Pr^{0.33}\ Re^{0.9}\ Gu^{0.175} \tag{6}$$

$$Nu_D = 0.0248\ Sc^{0.33}\ Re^{0.9}\ Gu^{0.135} \tag{7}$$

where Pr, Sc, Re, Gu are Prandtl, Schmidt, Reynolds, and Guhman number, respectively.

The water vapor concentration in the drying air, C_a, is calculated by

$$C_a = \frac{\varphi\, p_s(T_a)}{461.9\,(T_a + 273)} \tag{8}$$

where p_s is the saturation pressure. The water vapor concentration of the air in equilibrium with the surface of the body exposed to convection is calculated by

$$C_{x=L} = \frac{a(T_{x=L}, X_{x=L})\ p_s(T_{x=L})}{461.9\ (T_{x=L} + 273)} \tag{9}$$

The water activity, a, or the equilibrium relative humidity of the air in contact with the convection surface at temperature $T_{x=L}$ and moisture content $X_{x=L}$ is calculated from experimental water sorption isotherms.

The problem is symmetrical, and boundary conditions on the mid-plane of the plate ($x = 0$) are

$$\left(\frac{\partial T}{\partial x}\right)_{x=0} = 0, \quad \left(\frac{\partial X}{\partial x}\right)_{x=0} = 0 \tag{10}$$

In order to approximate the solution of Eqs. (1, 2), an explicit procedure has been used [8].

ESTIMATION OF PARAMETERS

The estimation methodology used is based on minimization of the ordinary least square norm

$$\mathbf{E}(\mathbf{P}) = [\mathbf{Y} - \mathbf{T}(\mathbf{P})]^{\mathrm{T}} [\mathbf{Y} - \mathbf{T}(\mathbf{P})] \tag{11}$$

Here, $\mathbf{Y}^{\mathrm{T}} = [Y_1, Y_2, \ldots, Y_{\mathrm{imax}}]$ is the vector of measured temperatures, $\mathbf{T}^{\mathrm{T}} = [T_1(\mathbf{P}), T_2(\mathbf{P}), \ldots\ T_{\mathrm{imax}}(\mathbf{P})]$ is the vector of estimated temperatures at time t_{i} (i = 1, 2, …, imax), $\mathbf{P}^{\mathrm{T}} = [P_1, P_2, \ldots\ P_{\mathrm{N}}]$ is the vector of unknown parameters, imax is the total number of measurements, and N is the total number of unknown parameters (imax $\geq$ N).

A version of Levenberg-Marquardt method was applied for the solution of the presented parameter estimation problem [9, 10]. This method is quite stable, powerful, and straightforward and has been applied to a variety of inverse problems. It belongs to a general class of damped least square methods [11].

The solution for vector P is achieved using the following iterative procedure

$$\mathbf{P}^{r+1} = \mathbf{P}^{r} + [(\mathbf{J}^{r})^{\mathrm{T}} \mathbf{J}^{r} + \mu^{r} \mathbf{I}]^{-1} (\mathbf{J}^{r})^{\mathrm{T}} [\mathbf{Y} - \mathbf{T}(\mathbf{P}^{r})], \tag{12}$$

where r is the number of iterations, $\mathbf{I}$ is identity matrix, μ is the damping parameter, and $\mathbf{J}$ is the sensitivity matrix defined as

$$\mathbf{J} = \begin{bmatrix} \frac{\partial T_1}{\partial P_1} & \cdots & \frac{\partial T_1}{\partial P_N} \\ . & & \\ \frac{\partial T_{i\max}}{\partial P_1} & \cdots & \frac{\partial T_{i\max}}{\partial P_N} \end{bmatrix}, \tag{13}$$

The presented iterative procedure stops if the norm of gradient of $\mathbf{E}(\mathbf{P})$ is sufficiently small, or if the ratio of the norm of gradient of $\mathbf{E}(\mathbf{P})$ to the $\mathbf{E}(\mathbf{P})$ is small enough, or if the changes in the vector of parameters are very small [12].

RESULTS AND DISCUSSION

The proposed method of the moisture diffusivity estimation by temperature response of a drying body was tested for a model material which was a mixture of bentonite and quartz sand with known thermophysical properties [8]. From the experimental and numerical examinations of the transient moisture and temperature profiles [8] it was concluded that for the calculations in this study, the influence of the thermal diffusion is small and can be ignored. It was also concluded that the Luikov's system of two simultaneous partial differential equations could be used. In this case, the transport coefficients can be treated as constants except for the moisture diffusivity. The appropriate mean values for the model material are:

- density of the dry solid , ρ_s = 1738 kg/m^3,
- heat capacity, c = 1550 J/K/kg db,
- thermal conductivity, k = 2.06 W/m/K,
- latent heat of vaporization, $\Delta H = 2.31 \cdot 10^6$ J/kg,
- phase conversion factor, ε = 0.5, and
- thermo-gradient coefficient, δ = 0.

The following expression can describe the experimentally obtained relationship for the moisture diffusivity.

$$D = 9.0 \cdot 10^{-12} X^{-2} \left(\frac{T + 273}{303} \right)^{10} \tag{14}$$

The experimentally obtained desorption isotherms of the model material is presented by the empirical equation

$$a = 1 - \exp(-1.5 \cdot 10^{6} (T + 273)^{-0.91} X^{(-0.005(T+273)+3.91)}) \tag{15}$$

where the water activity, a, represent the relative humidity of the air in equilibrium with the drying object at temperature T and moisture content X.

For the direct problem solution, the system of equations Eq. (1) and Eq. (2) with the initial conditions Eq. (3) and the boundary conditions Eq. (4) and Eq. (10) was solved numerically with the experimentally determined thermophysical properties.

For the inverse problem investigated in this paper, values of the moisture diffusivity, D, and, heat and mass transfer coefficients, h and h_D, are regarded as unknown. All other quantities appearing in the direct problem formulation were assumed to be known.
The moisture diffusivity of the model material has been represented by the following function of temperature and moisture content

$$D = D_X X^{-2} \left(\frac{T + 273}{303} \right)^{D_T} \tag{16}$$

where D_X and D_T are constants. Thus, the vector of unknown parameters is

$$\mathbf{P}^T = [D_X, D_T, h, h_D] \tag{17}$$

For the estimation of these unknown parameters, the transient readings of a single temperature sensor located in the mid-plane of the sample was considered (Fig. 1). The simulated experimental data were obtained from the numerical solution of the direct problem presented above, by treating the values and expressions for the material properties as known. In order to simulate real measurements, a normally distributed error with zero mean and standard deviation, σ of 1.5 ^{O}C was added to the numerical temperature response.

The sensitivity coefficients analysis has been carried out for a plate of thickness $2L = 4$ mm, with initial moisture content of $X(x, 0) = 0.20$ kg/kg and initial temperature $T(x, 0) = 20^{O}$C. Following the conclusions of the previous works [6…..] the drying air bulk temperature of $T_a = 80$ ^{O}C, and drying air speed of V_a, = 10 m/s, have been chosen. The relative humidity of the drying air was $\varphi = 0.12$.

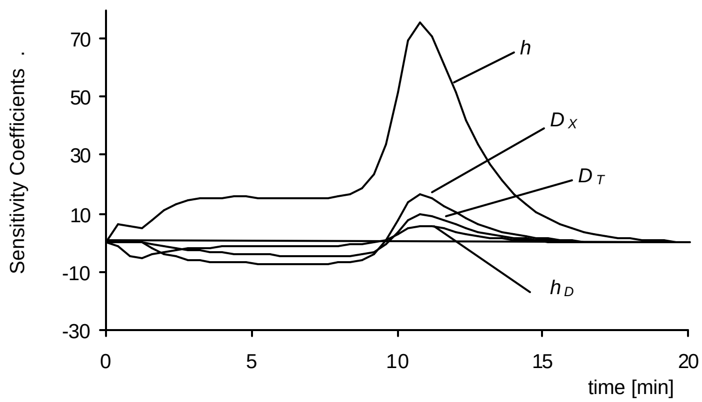

Fig. 2. Relative sensitivity coefficients for the convective drying experiment

Figure 2 shows the relative sensitivity coefficients $P_m \partial T_i / \partial P_m$, i = 1, 2,…, 101, for temperature with respect to all unknown parameters, D_X, D_T, h, h_D (m = 1, 2, 3, 4). It can be seen that the temperature sensitivity coefficient with respect to the convection mass transfer coefficient h_D is very small relatively to the temperature sensitivity coefficient with respect to the convection heat transfer coefficient h. The very high mass transfer Biot number and the very small heat transfer Biot number can explain this. The heat transfer Biot number is 0.08. The mass transfer Biot number ranged from 200 to $1{\cdot}10^6$ and changes during the drying with local moisture content and temperature change.

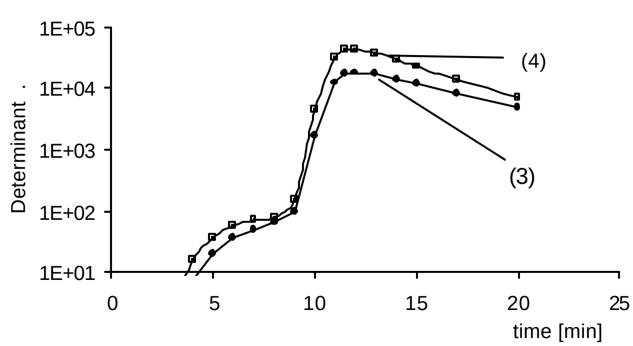

Fig. 3. Determinant of the information matrix

Figure 3 presents transient variation of the determinant of the information matrix if four, (D_X, D_T, h, h_D) and three (D_X, D_T, h) parameters are simultaneously considered as unknown. Elements of this sensitivity determinant were defined [10] for a large, but fixed number of transient temperature measurements (101 in these cases).

Table 1. Estimated parameters (σ = 1.5 ^{O}C)

Case		$D_X \cdot 10^{12}$ [m^2/s]	D_T [-]	h [W/m^2/K]	$h_D \cdot 10^2$ [m/s]
P3	Estimated	8.93	9.95	83.03	
	|e| [%]	0.83	0.54	0.09	
P4a	Estimated	9.87	8.79	83.17	8.28
	|e| [%]	9.61	12.08	0.08	10.87
P4b	Estimated	8.89	10.02	83.02	9.28
	|e| [%]	1.27	0.24	0.09	0.09
Exact values		9.00	10.0	83.1	9.29

The drying time corresponding to the maximum determinant value was used for the computation of the unknown parameters. Table 1 shows the computationally obtained results. For comparison, the exact values of parameters are shown in the bottom row. The relative errors of the estimated parameters, e, are also shown in the table.

From the obtained results in the case P3, it appears to be possible to estimate simultaneously the moisture diffusivity parameters, D_X and D_T, and the convection heat transfer coefficient, h, by a single thermocouple temperature response with the relatively high noise of 1.5 ^{O}C.

But the accuracy of computing parameters in the case of simultaneous estimation of the moisture diffusivity parameters, D_X and D_T, and the convection heat and mass transfer coefficients, h and h_D (case P4a) is small. The very small values of the relative sensitivity coefficient with respect to the mass transfer coefficient (Fig. 2) can explain this.

To overcome this problem, in this paper the mass transfer coefficient is related to the heat transfer coefficient through the Eqs. 6 and 7, obtained from the analogy between the heat and mass transfer processes in the boundary layer over the drying body. From the Eqs. 6 and 7, with accuracy within 1%, following relationship can be obtained

$$h_D = 0.95 \frac{D_a}{k_a} h \tag{18}$$

where D_a and k_a are moisture diffusivity and thermal conductivity in the air, respectively. The obtained relation is very close to the well-known Lewis relation.

By using the above relation between the heat and mass transfer coefficients, they can be simultaneously estimated with the moisture diffusivity parameters with high accuracy (case P4b in table 1).

But, it has to be stresses that in the all analysed cases local minimums have been obtained depending on the initial guesses. To overcome this problem application of the hybrid optimisation algorithm OPTRAN [13] will be analysed.

CONCLUSIONS

An analysis of the possibility of simultaneous estimation of the thermophysical properties of a drying body and the heat and mass transfer coefficients at high mass transfer Biot number by using only temperature measurements was presented. By using an interrelation between the heat and mass transfer coefficients, they were simultaneously estimated with the two moisture diffusivity parameters with high accuracy. Depending on the initial guesses local minimums have been often obtained during the analysis. To overcome this problem application of the hybrid optimisation algorithm OPTRAN will be analysed.

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