INVERSE DETERMINATION OF SMELTER WALL EROSION SHAPES USING A FOURIER SERIES METHOD

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ABSTRACT

An algorithm has been developed for the non-destructive determination of local thickness of the refractory material walls in smelter furnaces. This method uses measurements of temperature and heat flux at a number of points on the outer (fixed) surface of the furnace and assumes that the inner (guessed) surface of the furnace wall is isothermal. The temperature field is then predicted in the furnace wall subject to the measured temperature values on the external surface and the isothermal solidus temperature on the inner surface of the wall. The shape determination method then uses the difference between the measured and the computed heat fluxes on the outer surface of the furnace as a forcing function in an elastic membrane concept for the determination of the inner (melt refractory) surface shape. The initial guess of the wall inner shape can be significantly different from the final (unknown) wall shape. The entire wall shape determination procedure requires typically 5-15 temperature field analysis in the furnace wall material.

KEYWORDS

Inverse shape determination, Fourier series, elastic membrane, smelter wall erosion, BEM

INTRODUCTION

Walls of the furnaces that contain molten material (metal, glass, etc.) are made of layers of bricks of high-temperature resistive refractory material. High thermal gradients inside the melt create very strong circulation of the melt that causes erosion of the inner wall surface of the furnace. At such points, the molten material could easily soften the outer steel casing of the furnace and break it, thus causing a major industrial disaster. Obviously, it would be highly desirable to continuously monitor the actual local thickness of the entire furnace wall so that the furnace can be shut down and the wall material repaired before the conditions for a burn-through accident can develop. The use of any sensors imbedded in the inner surface of the wall is unacceptable, because the strong melt velocity field would wash such sensors away very quickly. Therefore, determination of the thickness distribution of the refractory wall should utilize non-destructive measurement techniques and inverse shape determination concepts.

Some of the pioneering work was performed by Yoshikawa et al. [1] who considered axisymmetric configurations of blast furnaces. They attempted to incorporate the effects of the solidified melt layer in their inverse formulation based on the use of the boundary elements method for heat conduction analysis and a shape optimization algorithm that could handle only a relatively small number of design variables. Another significant effort in the development of inverse methods for determination of inner wall surface shape was performed in the ex-USSR (Ukraine) by a research team of Matsevity [2]. They are concerned with the bottom of the flash smelting furnace which is the multi-layer structure consisting of refractory and heat-insulating materials. Two upper layers are built from chromo-magnesite brick and act as working and insulating lining. Underneath are the layers of refractory brick and light refractory brick, which are the heat insulators. The lower refractory brick layer lies on the concrete raft, which is cast on the horizontal steel plate that leans against the columnar concrete supports of the furnace foundation. The width of the wall domain in this problem is much larger than the wall thickness, it is practically symmetric relative to the transverse axis, and it has low heat conductivity of the component materials. These facts were used to justify the assumption of two-dimensionality of the temperature field in the furnace bottom wall. Apparently this team has not considered a simultaneous prediction of the inner surface of the furnace bottom and sidewalls.

Sorli and Skaar [3] reported on a very exact and mathematically sound inverse methodology that converges quite rapidly because it utilizes an adjoint operator formulation. However, the method was demonstrated for very simple smooth shapes of the inner surface of the wall that were not significantly different from the initially guessed wall surface configurations.

Tanaka's team [4,5] utilized a sophisticated Kalman filtering technique and boundary element method to deal with axisymmetric configurations of the blast furnaces. Katamine, Azegami and Kojima [6] have developed a method based on the distributed sensitivity function that uses adjoint variables. Their approach is able to predict quite realistic shapes of the inner contour of the furnace wall in two dimensions. However, the method does not seem to offer a consistently high accuracy in the prediction of the wall wear configuration.

It appears that despite the separate efforts of several independent research teams, reliable and affordable methodology for continuous sensing and monitoring of realistic three-dimensional variation of refractory wall thickness in smelters is still unavailable. What is available are several methods for the prediction of the furnace wall thickness variation in a two-dimensional horizontal or vertical plane assuming a perfect symmetry of the furnace inner and outer walls with respect to the vertical axis. Furthermore, the existing methods do not offer simultaneously high accuracy, reliability and speed of the prediction of the wall thickness distribution. The objective of this paper is to elaborate on an alternative method for predicting reliably and accurately realistic two-dimensional furnace wall wear configurations. The method is based on the authors' concept for inverse design of aerodynamics shapes [8.9,10].

THERMAL BOUNDARY CONDITIONS

In the problem of inverse determination of the inner surface of the refractory wall in smelters, this means that both temperature, T_o, and normal temperature derivatives, $(dT/dn)_o$ should be provided on the external surface of the refractory wall. A continuous reading of temperature T_o on this surface can be accomplished by placing inexpensive and reliable temperature measuring probes on the outer surface of the furnace refractory wall. The normal derivatives of temperature on the outer surface of the refractory wall of a blast furnace could be measured inexpensively by placing another temperature probe a few centimeters radially inward from each of the outer surface temperature probes. The difference between the temperatures read by each probe in such a pair of temperature probes can be divided by the known distance between the two probes in a pair to provide the needed outer surface normal temperature gradient. The differences in the temperature readings within any such pair of probes should be relatively small. Any pair of temperature probes that starts showing a temperature difference that is significantly larger than the majority of thermocouple pairs will be easily observed. Hence, an additional benefit of this arrangement of temperature probes is the quick detection of possible malfunctioning of any of the temperature probes that could then be promptly replaced.

The inner surface of the refractory wall of the furnace is of an unknown shape, but the temperature of this surface, T_i, is assumed to be known and equal to the solidification temperature of the melt which is recirculating in the furnace. The assumption of isothermal solidus temperature on the unknown inner surface of the refractory wall is reasonable, although not exact because there could be layers of solidified melt and slug on some parts of this surface.

ELASTIC MEMBRANE CONCEPT FOR SHAPE EVOLUTION

Inverse determination of the inner surface of the refractory wall of a blast furnace is based on the use of measured T_o and $(dT/dn)_o$ and on the postulated isothermal value of T_i. These boundary conditions are used in the following manner. Garabedian and McFadden [10] first proposed the elastic membrane approach for inverse design of aerodynamic shapes where the body surface is treated as an elastic membrane that deforms under certain surface loads until it achieves a desired (target) distribution of surface loads. An equivalent problem for the inverse determination of the inner surface of the refractory wall of a furnace could be formulated as follows. The original non-physical model for the evolution of, for example, a two-dimensional shape was given by [10]

$$\beta_0 \Delta n + \beta_1 \frac{d\Delta n}{ds} + \beta_2 \frac{d^2 \Delta n}{ds^2} = \Delta q_o \tag{1}$$

In our application, the forcing function is defined as

$$\Delta q_o = \left(\frac{dT}{dn}\right)_o^{\text{measured}} - \left(\frac{dT}{dn}\right)_o^{\text{computed}} \tag{2}$$

Here, Δn's are defined as shape corrections along outward normal vectors to the outer surface of the furnace wall, while the outer wall contour-following coordinate is s. Coefficients β_0, β_1, and β_2 are the user supplied constants that control the rate of convergence of the iterative shape determination process. The difference between the measured and the computed normal temperature derivatives on the outer surface of the refractory wall represents the forcing function, Δq_o. The ordinary differential equation with constant coefficients (Eq. 1) is analogous to a simple linear forced mass-damper-spring system type where the monotonically increasing time coordinate has been traded for a monotonically increasing surface-contour-following coordinate s. There is also an analogy between the forcing function in the mass-damper-spring system, which varies arbitrarily with time, and the outer wall surface heat flux difference Δq_o, which varies arbitrarily with the contour following coordinate s, in Eq. (1). Notice also a global periodicity of the mass-damper-spring forcing function and the outer surface heat flux difference, Δq_o, that repeats its value at the starting and the ending contour-following s-coordinate. Equation (1) is traditionally solved by evaluating its derivatives using finite differencing. The major problem with this approach is its slow convergence in conjunction with the field analysis codes of increasing non-linearity. In an attempt to alleviate these problems, we have developed a new formulation of the elastic membrane design concept, which allows a Fourier series analytical solution to the shape evolution equation [7,8,9].

FOURIER SERIES SOLUTION OF SHAPE EVOLUTION EQUATION

On the inner surface of the furnace wall configuration this leads to

$$\beta_0 \Delta n + \beta_1 \frac{d\Delta n}{d\theta} + \beta_2 \frac{d^2 \Delta n}{d\theta^2} = \Delta q_o \tag{3}$$

which has a homogeneous solution of the general form

$$\Delta n_h = F e^{\lambda_1 \theta} + G e^{\lambda_2 \theta} \tag{4}$$

where F and G are (as yet) undetermined coefficients and eigenvalues are determined from

$$\lambda_{1,2} = \frac{-\beta_1 \pm \sqrt{\beta_1^2 + 4\beta_0 \beta_2}}{-2\beta_2} \tag{5}$$

A particular solution of the elastic membrane model equation (3) can be represented in terms of a Fourier series as

$$\Delta n_p = A_0 + \sum_{N=1}^{N_{max}} [A_N \cos N\theta + B_N \sin N\theta] \tag{6}$$

The forcing function, Δq_o, can also be represented in terms of another Fourier series as

$$\Delta q_o = a_0 + \sum_{N=1}^{N_{max}} [a_N \cos N\theta + b_N \sin N\theta] \tag{7}$$

Substitution of equations (6) and its derivatives into the general evolution equation (3) and collection of like terms yields analytical links among the coefficients of the two Fourier series.

$$A_N = \frac{a_N(N^2\beta_2 - \beta_0) - b_N(\beta_1 N)}{(N^2\beta_2 - \beta_0)^2 + (\beta_1 N)^2}, \quad N = 0,1,2,\cdots,N_{max} \tag{8}$$

$$B_N = \frac{b_N(N^2\beta_2 - \beta_0) + a_N(\beta_1 N)}{(N^2\beta_2 - \beta_0)^2 + (\beta_1 N)^2}, \quad N = 0,1,2,\cdots,N_{max} \tag{9}$$

Thus, the complete solution for geometry corrections, Δn, in the locally normal direction to the outside surface of the furnace wall can be represented analytically as

$$\Delta n = Fe^{\lambda_1\theta} + Ge^{\lambda_2\theta} + A_0 + \sum_{N=1}^{N_{max}}\left[A_N \cos N\theta + B_N \sin N\theta\right] \tag{10}$$

The unknown constants, F and G, are determined to be zero from the closure conditions $\Delta n(0) = \Delta n(2\pi)$. This form of the solution of the elastic membrane model equation has significant advantages over the standard finite difference approach since any errors due to finite differencing are removed, because the formulation is exact. Consequently, the Fourier series formulation for the elastic membrane concept in inverse shape determination converges faster than the finite difference formulation [7-10].

NUMERICAL RESULTS

The Fourier series formulation of the elastic membrane inverse shape determination concept was tested for accuracy and speed of convergence on horizontal cross sections of an idealized furnace, using two simple geometries with outer surface radius Ro = 2.0 m. The first test geometry had an oval doubly symmetric inner boundary shape given as $R_i = 1.0 + 0.5\sin^2\theta$ (Fig. 1). The second test geometry had only one axis of symmetry with the inner surface represented by a fourth order polynomial where slope was discontinuous at the point $R_i(0) = R_i(2\pi)$ (Fig. 2).

$$R_i = 1.0 + 0.5\left\{\left[\frac{(2\pi - \theta)}{2\pi}\right]^4 + \frac{\theta}{2\pi}\right\} \tag{11}$$

Thermal boundary conditions were Ti = 2000.0 K and To = 350.0 K. For simplicity, the furnace wall was assumed to be made of an isotropic homogeneous material. In principle, the analysis of the steady heat conduction could account for a wall made of a finite number of sub-domains each having a different coefficient of thermal conductivity. We used our highly accurate boundary element heat conduction analysis code to solve Laplace’s equation for steady thermal field in the annular region. The target (“measured”) outer surface heat flux corresponding to the inner surface target shape was determined by solving for the temperature field subject to Ti = 2000 K and To = 350 K and computing dT/dr on the outer boundary.

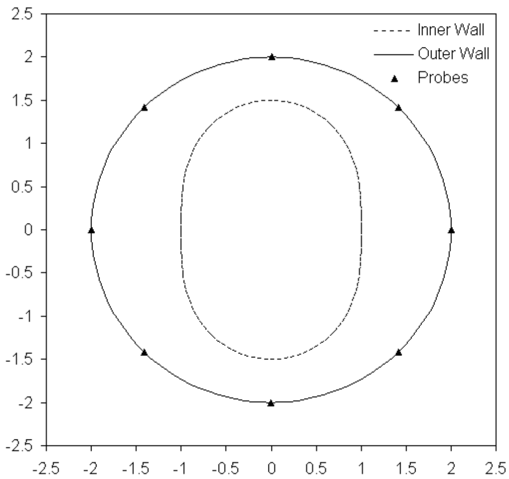

Fig. 1: Symmetric test geometry: target shape of the inner surface (vertical oval) and the outer surface (circle of radius 2.0 m) of the furnace wall with indication of the locations of eight temperature and heat flux probes.

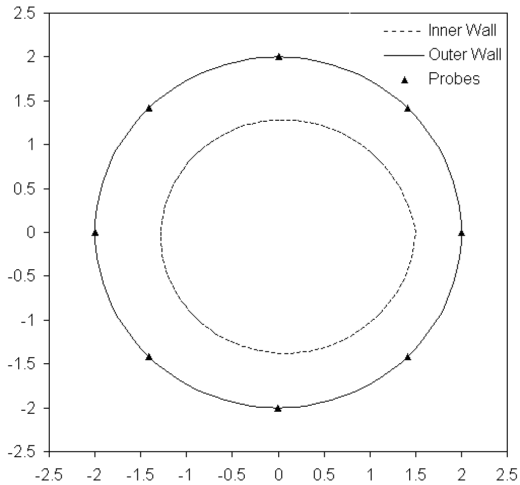

Fig. 2: Asymmetric test geometry: target shape of the inner surface and the outer surface (circle of radius 2.0 m) of the furnace wall with indication of the locations of eight temperature and heat flux probes.

Then, the inner surface was changed to a guessed shape, which was a unit circle in both test cases. Using the analysis code for heat conduction, steady thermal field was solved in this perfectly circular concentric annular region subject to Ti = 2000 K and To = 350 K. The computed values of dT/dr on the outer boundary were then treated as initial dT/dr computed values. The elastic membrane forcing function was then created by the difference between the target and the initial values of dT/dr on the outer boundary. Elastic membrane coefficients were β_o=5000.0, β_1=0.0 and β_2 = 0.0. The inverse design code solved for corrections in the wall thickness and updated the shape of the inner surface of the furnace wall as $R_i^{new} = R_i^{old} + \Delta n$. This shape was then treated as the new initial shape and the entire procedure was automatically repeated. The difference between the computed and the target heat flux on the outer surface was used as a convergence indicator, and the shape update process was stopped when the heat flux difference reached an acceptably low value (Fig. 3).

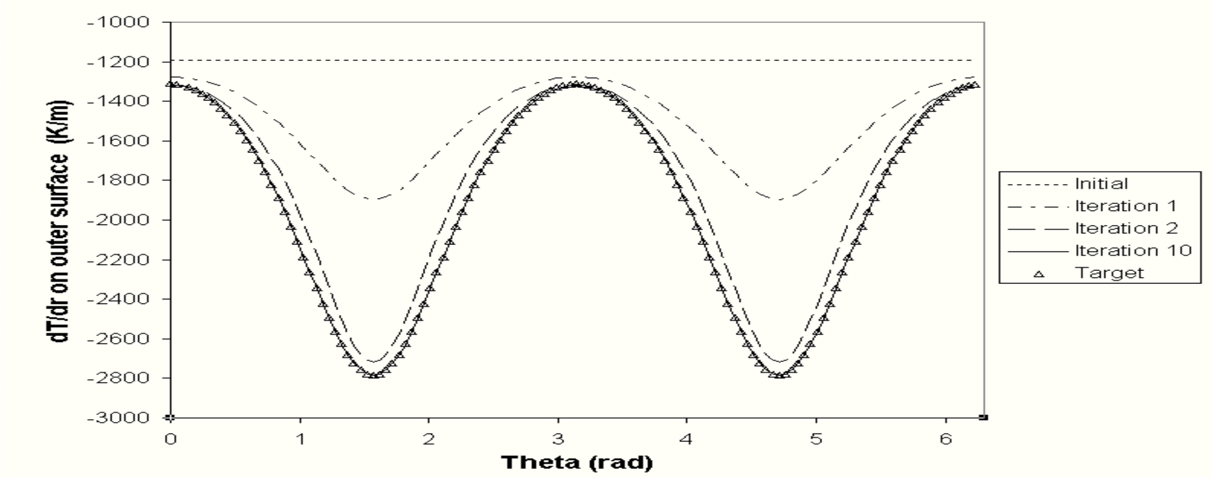

Fig. 3. Symmetric case: convergence history of the outer surface heat flux.

The shape of the inner surface of the furnace wall was also used as an indicator of convergence (Fig. 4). After ten iterations in the symmetric test case, the RMS error of dT/dr on the outer surface of the furnace wall decreased to 0.2% of its initial value (Fig. 5), while the RMS error of the radial location of the inner surface decreased to 1.0% of its initial value (Fig. 6).

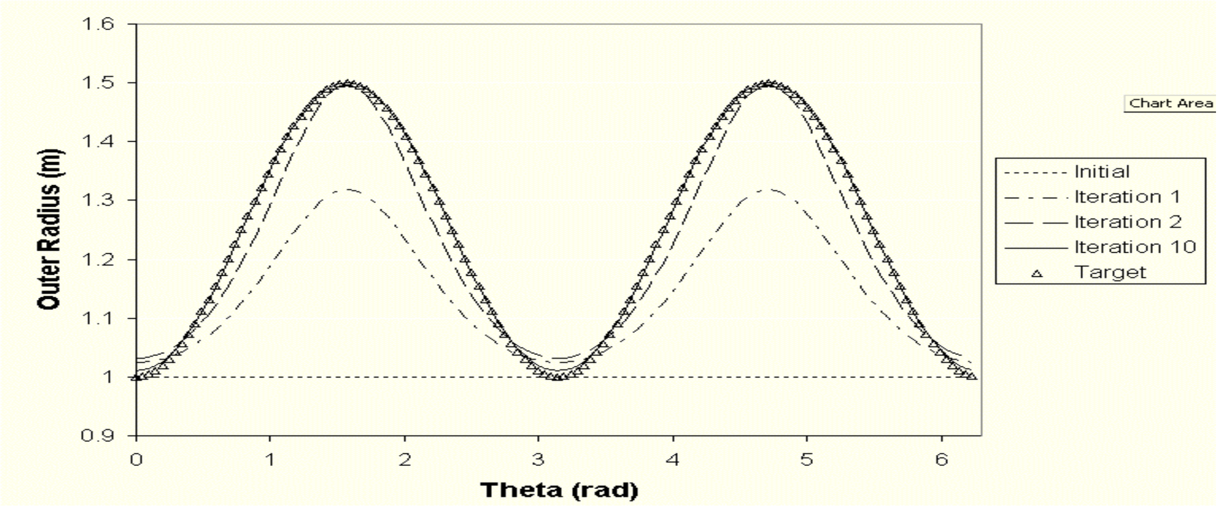

Fig. 4. Symmetric case: convergence history of the inner surface geometry.

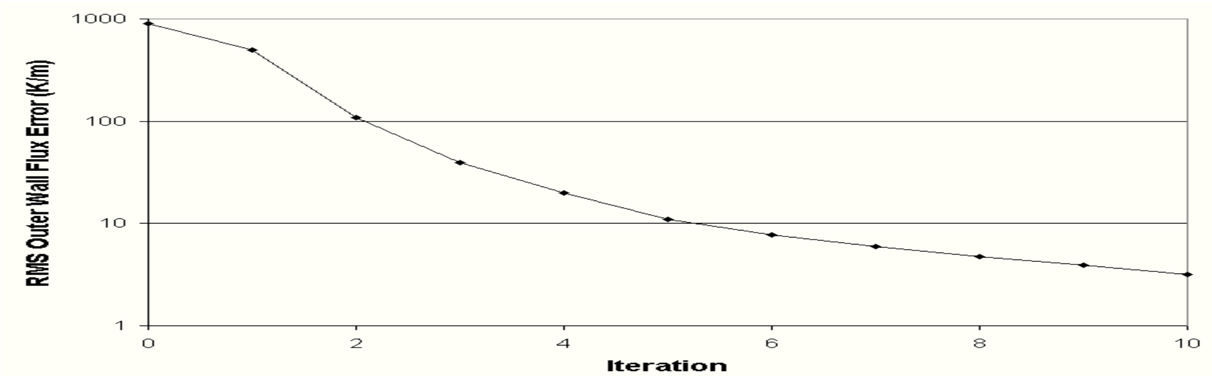

Fig. 5. Symmetric case: convergence history of the RMS error of the outer surface heat flux.

In the asymmetric geometry test case, the elastic membrane coefficients were β_o =5000.0, β_1 = 0.0 and β_2 = 0.0. After 10 iterations, the external surface heat flux difference practically disappeared (Fig. 7) and the inner surface of the furnace wall converged to the target shape (Fig. 8). The RMS error of dT/dr on the outer surface of the furnace wall decreased to 0.1% of its initial value (Fig. 9), while the RMS error on the inner surface of the furnace wall decreased to 0.8% of its initial value (Fig. 10).

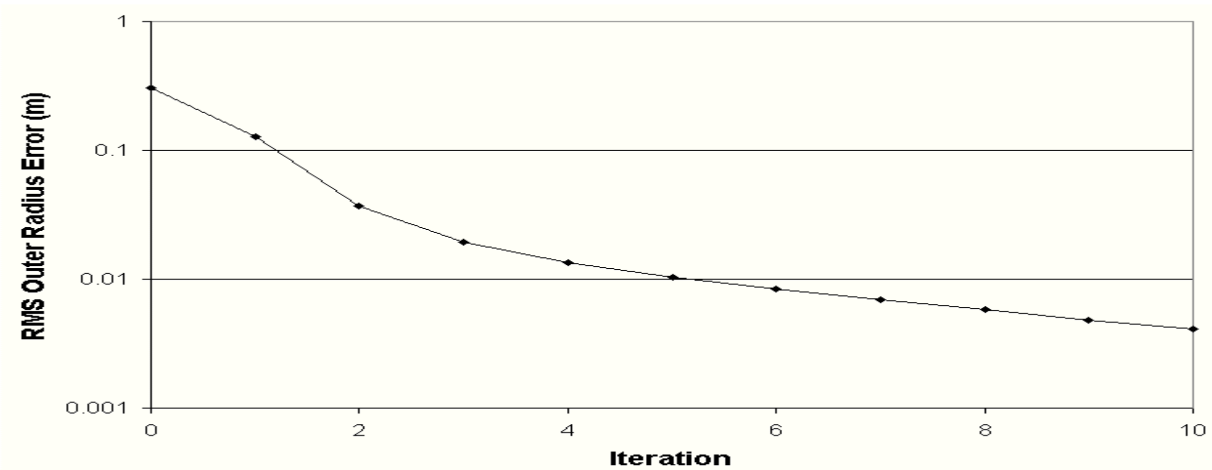

Fig. 6. Symmetric case: convergence history of the RMS error of inner surface geometry.

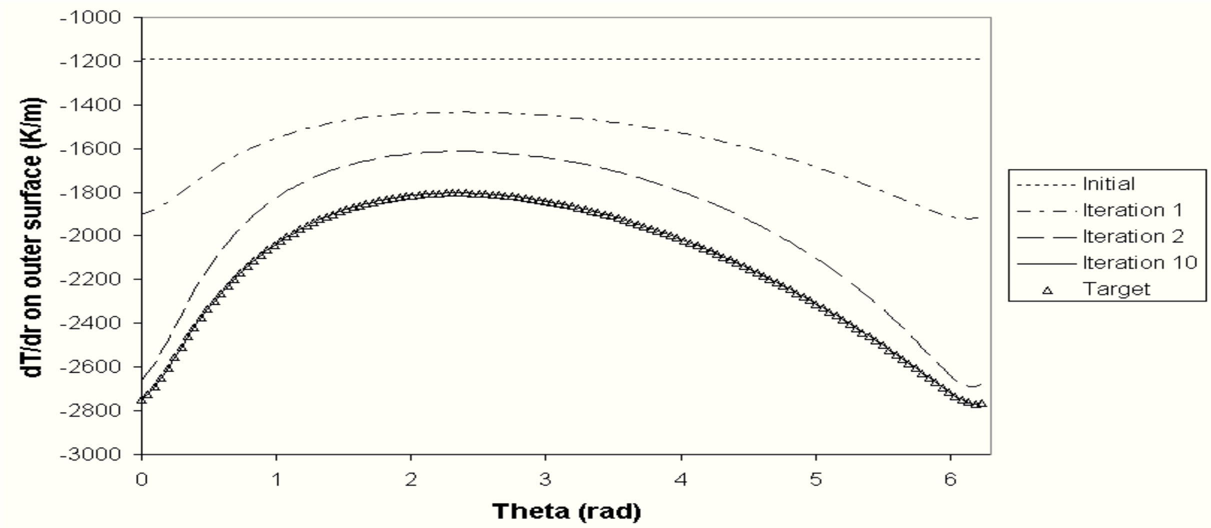

Fig. 7. Asymmetric case: convergence history of the outer surface heat flux.

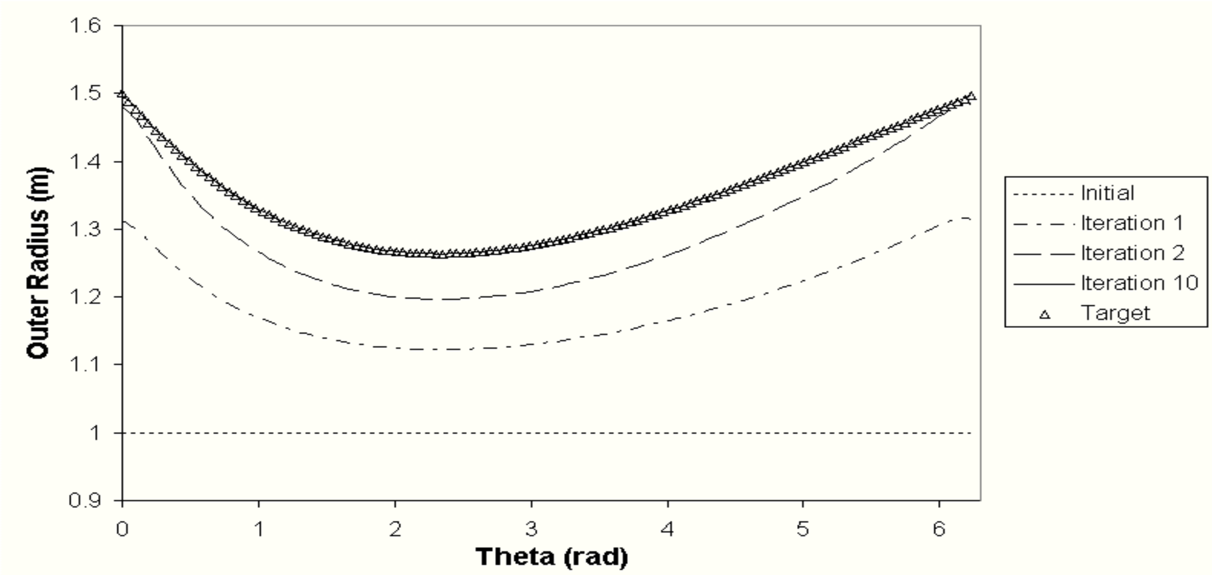

Fig. 8. Asymmetric case: convergence history of the inner surface geometry.

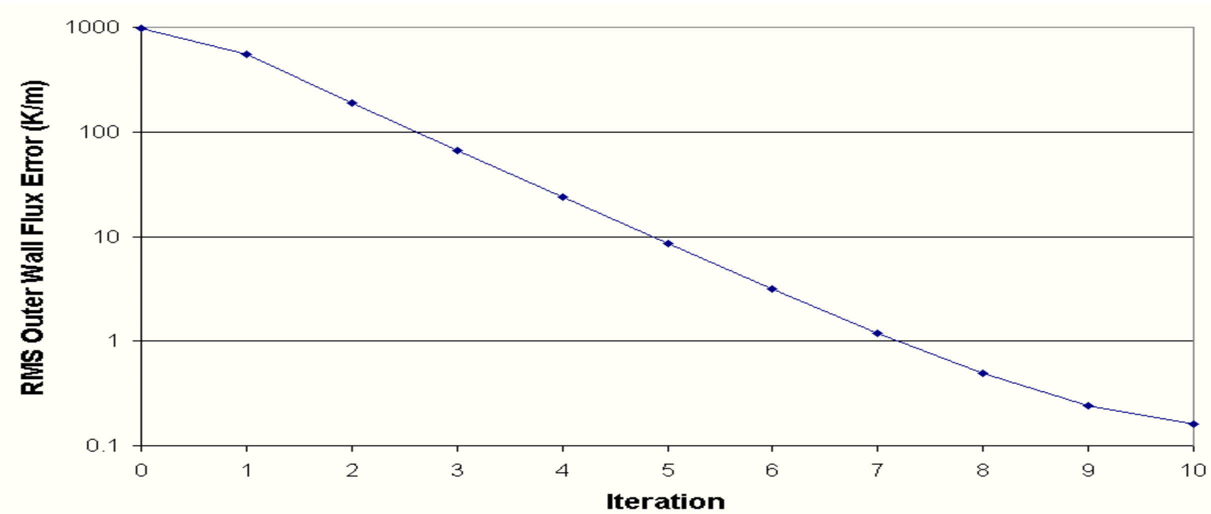

Fig. 9. Asymmetric case: convergence history of the RMS error of the outer surface heat flux.

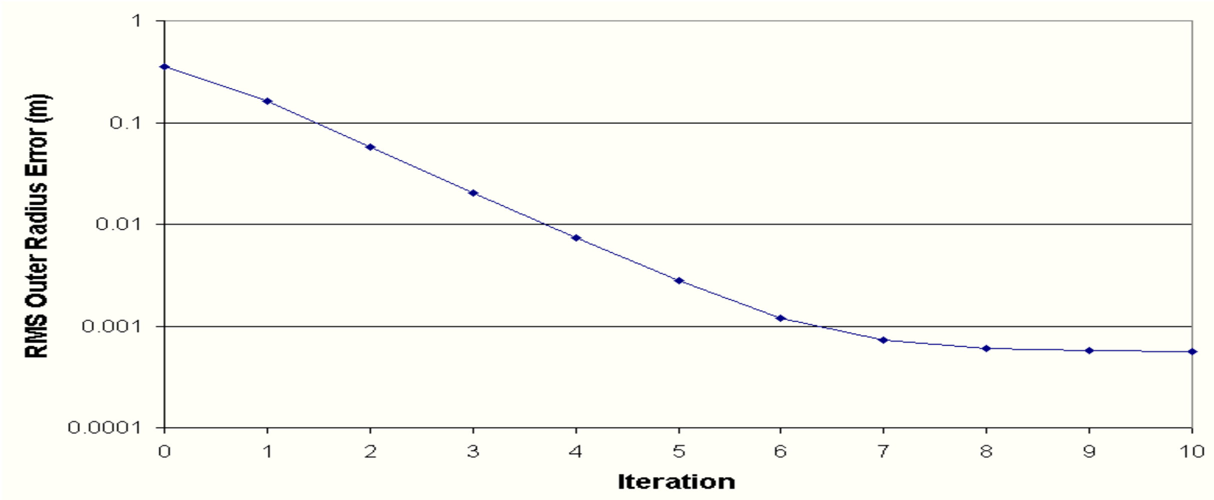

Fig. 10. Asymmetric case: convergence history of the RMS error of inner surface geometry.

EFFECT OF MEASUREMENT ERRORS

An actual furnace was not available to evaluate the accuracy of this inverse shape determination method. In actual field operation of the proposed method the thermocouples will read local values of temperature on the outer surface of the furnace wall. These readings will inevitably be in error and this error will be randomly distributed among the thermocouples. It is desirable to get maximum information out of as few thermocouples as possible. Consequently, we simulated measurement of the flux at only 8 points on the outer surface of the furnace (Fig. 1 and 2). Then, an unbiased error was applied to those 8 measurements by using a Gaussian probability distribution. Values of the randomly perturbed 8 flux values were then spline fitted and interpolated to the remainder of the outer surface of the wall. In a similar fashion the random error was applied to the external temperature thus simulating actual field measurements with errors. Then, the inverse shape determination procedure was performed while measuring the difference between the converged shape subject to such perturbed thermal boundary conditions and the correct shape. The entire process was repeated a number of times (20 in this case) and the average amount of error in the geometry of the predicted inner surface of the furnace wall was found to be quite small (Table 1).

Table 1. Errors in the predicted inner surface radius due to different levels of the simulated measurement errors of temperature and heat flux on the outer surface.

Simulated measurement errors		Expected RMS error in Ri (m)	
T_O error (percent)	$(dT/dr)_O$ error (percent)	Symmetric shape	Asymmetric shape
5.0	5.0	0.031540	0.049854
0.0	5.0	0.031074	0.047852
5.0	0.0	0.009099	0.006373
0.0	0.0	0.006273	0.006372

SUMMARY

A conceptually new method has been developed and tested for the automatic determination of wall thickness distribution in blast furnaces and smelters by utilizing external surface measurements of temperature and heat flux and employing a Fourier series formulation of the solution of an elastic membrane model for the evolution of the furnace inner surface shape.

The method accepts any available computer code capable of analyzing steady temperature field in the furnace wall. It also requires a relatively small number of inexpensive thermocouples. The entire procedure is computationally very efficient, highly accurate even under the simulated conditions of measurement noise, and could be extended to realistic three-dimensional furnace wall configurations with sections having different temperature-dependent thermal properties.

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