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MAXIMIZING MULTISTAGE AXIAL GAS TURBINE EFFICIENCY OVER A RANGE OF OPERATING CONDITIONS

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ABSTRACT

A new design system has been developed which optimizes hub and shroud geometry and inlet and exit flow-field parameters for each blade row of a multistage axial flow turbine. Very fast and accurate flow calculation and performance prediction of multistage axial flow turbines at design and off-design conditions were performed using a compressible steady state inviscid through-flow code with high fidelity loss and mixing models. The code is based on a stream function model and a finite element solution procedure. Optimization was performed using our hybrid constrained optimization code that includes the following modules: genetic algorithm, simulated annealing, modified simplex method, sequential quadratic programming, and Davidon-Fletcher-Powell gradient search algorithm where switching among the modules is performed automatically. By varying a relatively small number of geometric variables per each turbine stage it is possible to find an optimal radial distribution of flow parameters at the inlet and outlet of every blade row. Thus, an optimized meridional flow path is found that is defined by the optimized shape of the hub and shroud. The design system has been demonstrated using an example of an actual single-stage and a two-stage axial gas turbine. The comparison of computed performance of initial and optimized designs shows significant improvements in the turbine efficiency at design and significant off-design conditions. The entire design optimization process is feasible on a typical single-processor computer workstation.

NOMENCLATURE

c	absolute velocity
h	static enthalpy
h^0	total enthalpy
$\bar{h}$	relative blade height, fraction of span
Ma	Mach number
$\dot{m}$	mass flow rate
n	rotational speed (rpm)
n_S	number of stages
p	pressure
R	radius
Re	Reynolds number
s	entropy
T	temperature
t	blade pitch
u	tangential velocity
z	axial coordinate
α	absolute flow angle (from tangential direction)
β	relative flow angle (from tangential direction)
δ	thickness, clearance height
ζ	loss coefficient

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η	efficiency
Π	pressure ratio
ω	angular velocity (s^{-1})
ψ	stream function
θ	deflection angle

Subscripts

Cl	clearance
in	inlet
m	mean radius
out	outlet
r	radial component, reduced value
s	isentropic
tt	total to total
TE	trailing edge
u	tangential component
z	axial component
1	stator outlet
2	rotor outlet

1. INTRODUCTION

In the field of turbomachinery, efforts have been made to achieve high aerodynamic efficiency at design conditions, acceptable performance at reasonable partial loads, and reliable operation for all expected conditions. Design methods are currently developed that incorporate aerodynamic, thermal, and structural calculation procedures and numerical optimization to enable the variation of a large number of design parameters and an automatic determination of their best combination (Martin and Dulikravich, 1997; Dulikravich et al. 1998).

In recent years, a number of papers have been published on the application of optimization techniques in the design of turbomachines. Most of the early work focused on optimization of 1-D pitchline design (for example, Balje and Binsley, 1968). This technique gives the optimum geometry of the row at mean diameter and is widely used for turbomachinery preliminary design. Massardo et al. (1990) used a through-flow code to optimize spanwise distribution of axial compressor blade geometry. Cravero and Dawes (1997) presented a concept for optimizing the design conditions of an axial turbine stage. The objective was to find such radial distribution of absolute swirl velocity at the inlet and exit of stator blade row and rotor blade row that will minimize the overall single stage losses. They used a standard streamline curvature through-flow analysis code having standard loss correlations. A simple polynomial parameterization of the swirl velocity and the stator and rotor lean angles was used. For optimization, they used a rudimentary gradient search constrained minimization routine.

Petrovic, Dulikravich and Martin (1999) expanded on this general idea by demonstrating a highly accurate and fast method of maximizing efficiency of multistage axial gas turbines by utilizing a high fidelity through-flow analysis code and a robust hybrid constrained optimizer. The method automatically determines an optimal shape variation of hub and shroud without changing the original blade shapes. As a byproduct, it also automatically determines an optimal radial distribution of flow angles at inlet and exit of every blade row. The optimized solution provides for maximum multistage axial turbine efficiency. The design system will be demonstrated on the examples of experimentally tested single-stage and a two-stage axial gas turbine.

The final step in the multistage turbine design optimization could be a detailed 3-D inverse shape design or shape optimization of isolated stator and rotor blades' geometry subject to the optimized inlet and exit boundary conditions. Based on these new detailed blade shapes, it would be possible to repeat the hub/shroud and ilet/exit boundary conditions optimization thus optimizing the shape of the loss coefficient radial profile and further improving the turbine efficiency.

2. THROUGHFLOW CODE

Simultaneous detailed fully 3-D aerodynamic shape optimization of all blade rows in a complete multistage turbine by applying a fully 3-D multistage turbulent compressible flow-field analysis code would be excessively time consuming even on an advanced multiprocessor computer. Consequently, our design optimization approach is suggested as an attractive highly cost-effective quasi 3-D alternative.

Through-flow method developed by Petrovic (1995) is based on a classical through-flow theory (Hirsch and Deconinck, 1985), but includes significant improvements and extensions. It is basically an inviscid code with high fidelity distributed loss model. A finite element procedure with eight-node isoparametric quadrilateral elements and biquadratic interpolation functions was applied to solve the distribution of a stream function in a turbine meridional surface. The loss model developed by Traupel (1988) was adapted to 2-D calculations and applied in order to compute the loss coefficient and the entropy increase. New models for realistic radial distribution of losses, spanwise mixing, 2-D deviation and off design flow loss prediction have been developed and introduced in order to achieve high accuraty of the code. An important feature has been added to encompass far off-design flow phenomena (flow separation and flow reversal) and low load turbine operation with power consumption of part of bladings. All applied models are based on a large amount of experimental data thus enabling it to predict main flow losses very accurately. More details about the analytical background of the method and applied loss and mixing model are given in a paper by Petrovic and Riess (1997b).

The through-flow code is able to calculate axisymmetric flow-fields in axial multistage gas turbines and steam turbines at subsonic and transonic conditions. It can accurately and very quickly predict the flow-field and the turbine performance at the design load and over a wide range of partial loads. The reliability of the method was verified by comparing the calculations with the experimental results for several gas and steam turbines (Petrovic and Riess, 1997a; 1997b). The comparisons of numerical results with results of extensive experimental investigation show very close agreement.

The through-flow code can be used in two modes: analysis and design mode. In the analysis mode, the flow-field (distribution of p, T, h, s and velocity vector components) on a meridional surface and turbine overall performance are calculated for the specified turbine geometry and fixed values of blade row inlet and outlet angles. For optimization purposes, the through-flow code has been rewritten in a design mode, in which the radial distribution of the tangential velocity component is fixed. The flow angle and turbine efficiency can then be calculated.

3. CONSTRAINED HYBRID OPTIMIZATION ALGORITHM

The design space for a typical optimization problem of this type has a number of local minimums. A typical gradient based optimization algorithm would quickly terminate in the nearest available local minimum which might not even satisfy all of the specified constraints. In order to avoid the local minimums, it is advantageous to use a constrained evolutionary hybrid optimization approach (Foster and Dulikravich, 1997; Martin and Dulikravich, 1997; Dulikravich et al. 1998; Dulikravich, Martin and Han, 1998; Dulikravich et al. 1999). The hybrid constrained algorithm used in this work incorporated four of the most popular optimization approaches: a genetic algorithm, the Nelder-Mead simplex method, simulated annealing, and Davidon-Fletcher-Powell gradient search method. Each time when a local minimum was detected, an automatic switching logic was used to change to another optimization algorithm (Dulikravich et al. 1999). The hybrid optimizer treats the existence of constraints in three ways: Rosen's projection method, a feasible search, and random design generation. Rosen's projection method provided search directions which guided the descent direction tangent towards active constraint boundaries. In the feasible search, designs that violated constraints were automatically restored to feasibility via the minimization of the active global constraint functions. If at any time this constraint minimization failed, a number of random designs were generated using a Gaussian-shaped probability density cloud about a desirable and feasible design until a new design was reached. This hybrid constrained optimizer accepts an arbitrary number of equality and inequality constraints.

4. OPTIMIZATION

The goal of the present optimization is to find optimal shape of hub and shroud and radial distribution of inlet and outlet angles of every blade row giving the maximum turbine efficiency η_{tt} ($\eta_{tt} = (\Delta h + c_{in}^2 - c_{out}^2)/(\Delta h_s + c_{in}^2 - c_{out}^2)$). Since optimization algorithm is defined to minimize a function, the objective function in our case is: $f_{obj} = -\eta_{tt}$. During the geometry optimization process, turbine mass flow rate $\dot{m}$, total enthalpy drop Δh^0, and rotational speed n are kept constant. For computing losses in the through-flow code, we have utilized an updated version of Traupel's loss model that was adapted for 2-D flow calculation. In this model, the correlations for profile, secondary, and clearance losses are functions of geometrical and flow parameters.

The analysis of loss correlations indicates that turbine stage efficiency is a function of geometric parameters (mean radius R_m, blade height h, pitch t, clearance radius R_{Cl} and height δ_{Cl}, trailing edge thickness δ_{TE}, chord length) and flow parameters (inlet and outlet flow angles: β_1, β_2, deflection θ, Mach and Reynolds numbers Ma, Re, incidence angle, and velocity vectors). Some of these parameters (δ_{Cl}, δ_{TE}) have to be fixed. The others are not independent: velocities, Ma, and Re are functions of flow angles, mean radius and hlade height. Also, inlet angle is a function of the outlet angle of the previous row and of the incidence angle. Consequently, four parameters needs to be optimized for each stage of the multistage turbine: radial distribution of flow angles and axial distribution of mean radius and blade height.

However, it is easier to optimize the variation of the following four parameters per each stage of the multistage machine:

- $c_{u1}(R)$ – tangential component of velocity at stator outlet,
- $c_{u2}(R)$ – tangential component of velocity at rotor outlet,
- $R_{Hub}(z)$ – hub radius,
- $R_{Shroud}(z)$ – shroud radius.

From the optimized values of $c_{u1}(R)$ and $c_{u2}(R)$ it is easy to determine the corresponding optimized spanwise distribution of angles.

The axial variation of hub and shroud radii can be described by two spline functions, $R_{Hub}(z)$ and $R_{Shroud}(z)$. For simplicity, we decided to use only four geometrical parameters per every stage for hub and four geometrical parameters per every stage for shroud. This can be easily changed to an arbitrarily complex spline representation of the hub and shroud.

Tangential components of velocities at stator and rotor exits were described by the fourth degree polynomials as a function of relative blade height ($0.0 < \bar{h} < 1.0$). This representation needs the following 10 parameters:

$$c_{u1} = A_1 + B_1\bar{h} + C_1\bar{h}^2 + D_1\bar{h}^3 + E_1\bar{h}^4 \qquad (1)$$

$$c_{u2} = A_2 + B_2\bar{h} + C_2\bar{h}^2 + D_2\bar{h}^3 + E_2\bar{h}^4 \qquad (2)$$

One of these 10 parameters can be obtained using condition $\Delta h^0 = const.$ This means that for a single stage of a multistage machine the integral of Euler equation over mass flow rate has to be constant for every combination of flow and geometric parameters.

$$\Delta h^0 = \int_0^1 (u_1 c_{u1} - u_2 c_{u2}) d\psi = const. \qquad (3)$$

Replacing c_{u1} and c_{u2} with the polynomials (1) and (2), we obtain

$$\Delta h^0 = A_1 \omega (I_1 - I_2) \qquad (4)$$

$$I_1 = \int_0^1 R\left(1 + \frac{B_1}{A_1}\bar{h} + \frac{C_1}{A_1}\bar{h}^2 + \frac{D_1}{A_1}\bar{h}^3 + \frac{E_1}{A_1}\bar{h}^4\right) d\psi \quad (5)$$

$$I_2 = \int_0^1 R\left(\frac{A_2}{A_1} + \frac{B_2}{A_1}\bar{h} + \frac{C_2}{A_1}\bar{h}^2 + \frac{D_2}{A_1}\bar{h}^3 + \frac{E_2}{A_1}\bar{h}^4\right) d\psi \quad (6)$$

$$A_1 = \frac{\Delta h^0}{\omega(I_1 - I_2)} \quad (7)$$

Here, I_1 and I_2 are the values of the integrals in eqns. (5)-(6). Stream function, ψ, and radius, R, are functions of relative blade height, $\bar{h}$.

This means that there are finally 9 flow parameters to optimize the tangential components of velocities at stator and rotor outlets of a single stage:

$$\frac{B_1}{A_1}, \frac{C_1}{A_1}, \frac{D_1}{A_1}, \frac{E_1}{A_1}, \frac{A_2}{A_1}, \frac{B_2}{A_1}, \frac{C_2}{A_1}, \frac{D_2}{A_1}, \frac{E_2}{A_1} \quad (8)$$

In summary, the total number of parameters to optimize for every turbine stage is 17 (8 geometrical and 9 flow parameters). In case of a multistage turbine, the distribution of the turbine enthalpy drop over the stages can be also optimized regarding the following condition:

$$\Delta h^0 = \sum_{i=1}^{n_S} \Delta h^0_{Si} = const \quad (9)$$

where n_S is the number of turbine stages, Δh^0_{Si} is the enthalpy drop of the stage i, and Δh^0 is the turbine total enthalpy drop. In that case, there are additional $n_S - 1$ parameters to optimize.

<u>Optimization Strategy</u>

The first optimization cycle (iteration) starts with a detailed initial turbine geometry and input flow data: $\dot{m}$, T_{in}, p_{in}. Through-flow code in its analysis mode then evaluates overall performance of the initial configuration and initiates the distribution of c_{u1} and c_{u2}. The optimizer then delivers an improved set of geometrical (shapes of hub and shroud) and flow (distribution of c_{u1} and c_{u2}) parameters. A new finite element grid is then automatically generated for the meridional flow-field surface. The through-flow code in its design mode then runs with the new distributions of c_{u1} and c_{u2} thus creating a new set of flow parameters. The result is the spanwise distribution of flow angles at inlet and outlet of every blade row, and the overall turbine efficiency. Depending on the achieved η_{tt}, the optimizer enters the next iterative cycle by generating a new set of parameters and the iterative optimization process is repeated until the maximum of efficiency is achieved (Fig. 1).

By running the through-flow code in its analysis mode, the overall efficiency of the optimal configuration over a wide range of part loads is then calculated in order to check the optimized turbine off-design performance. If the overall efficiency of the optimal configuration at all expected operating conditions is acceptable, the optimization process is finished.

The iterative design process runs fully automatically. The designer's task is to prepare the input data and to set constraints

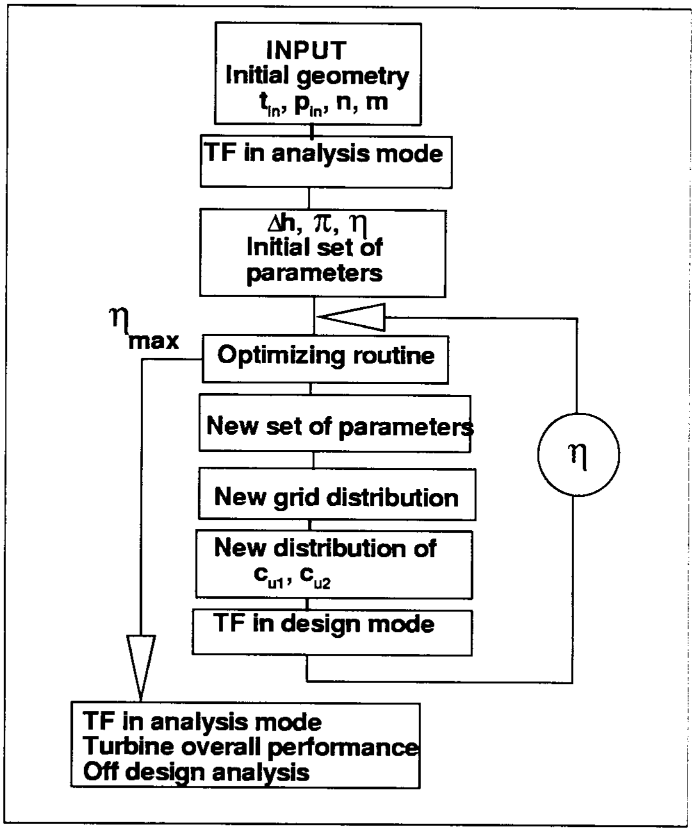

Fig. 1 Optimization algorithm

for design variables. The flow variables are usually allowed to vary without any limits. The variation of the rotor blade tip radius is often limited by the maximum stress at the rotor root, that is, by the maximum allowed tangential velocity if the rotating speed is fixed.

4. EXAMPLES

The first reported use of this design methodology was an example (Petrovic, Dulikravich, Martin, 1999) involving a single-stage uncooled transonic axial gas turbine that was experimentally tested by Foerster and Kruse (1990). At the design conditions, the calculated value of turbine efficiency was $\eta_{tt} = 0.8952$, while the experimentally measured value was $\eta_{tt} = 0.8940$ thus confirming the high accuracy of the through-flow code. Our multistage design optimization method was able to increase the maximum turbine efficiency to $\eta_{tt,opt} = 0.9154$.

This represents an absolute improvement of approximately $\Delta\eta_{tt} = 2.0\%$. Figure 2 shows the overall performance of the optimized configuration (OC) and a comparison with the performance of the initial configuration (IC). Figures 3-7 depict the initial and the optimized shapes of the hub and the shroud and the corresponding field distributions of temperature, enthalpy, pressure, Mach number, and entropy.

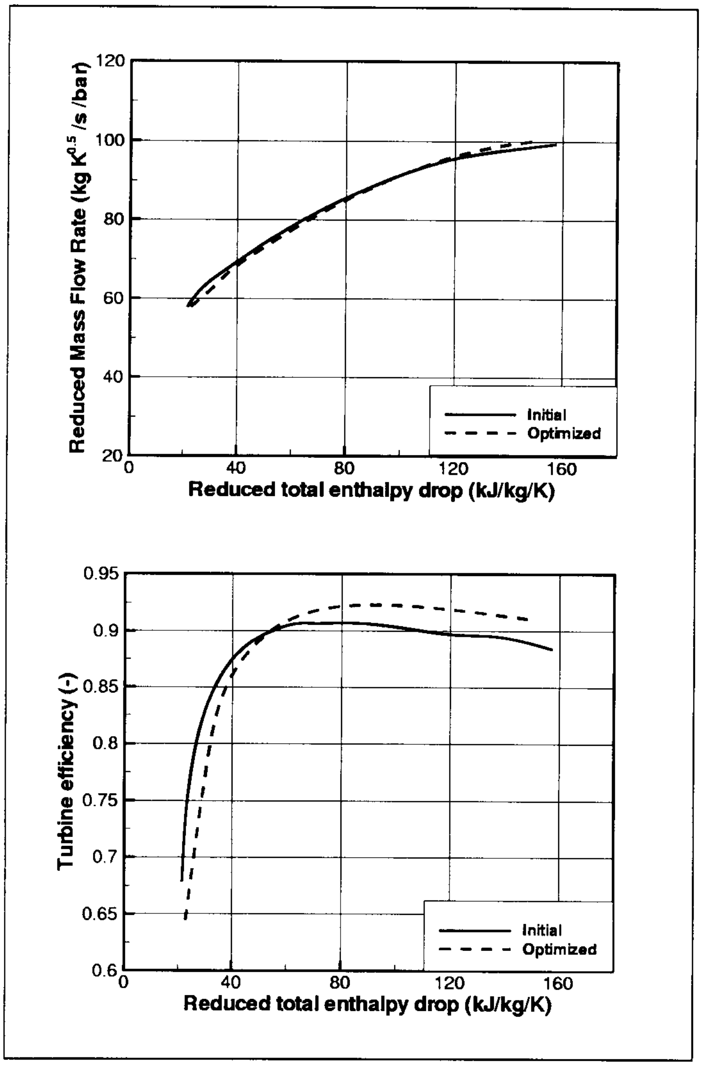

Fig. 2 Performance of initial and optimized single-stage turbine at design rotating speed (Petrovic, Dulikravich, Martin, 1999)

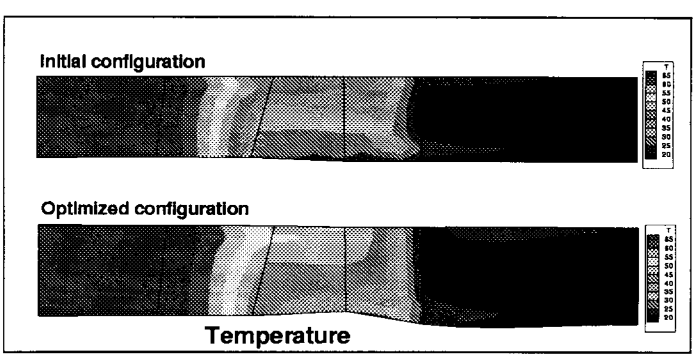

Fig. 3 Temperature field of initial and optimized single-stage turbine

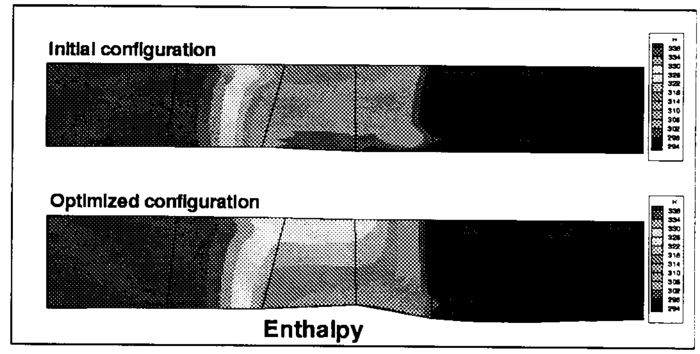

Fig. 4 Enthalpy field of initial and optimized single-stage turbine

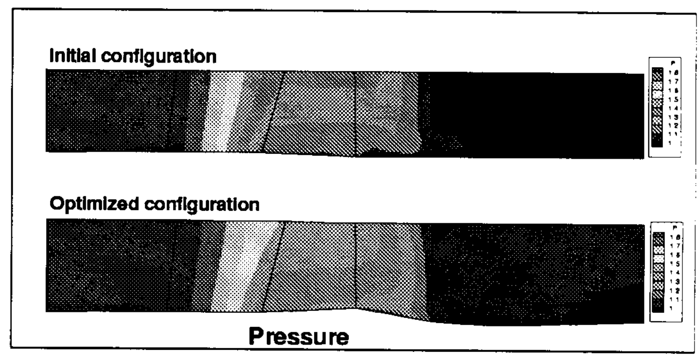

Fig. 5 Pressure field of initial and optimized single-stage turbine

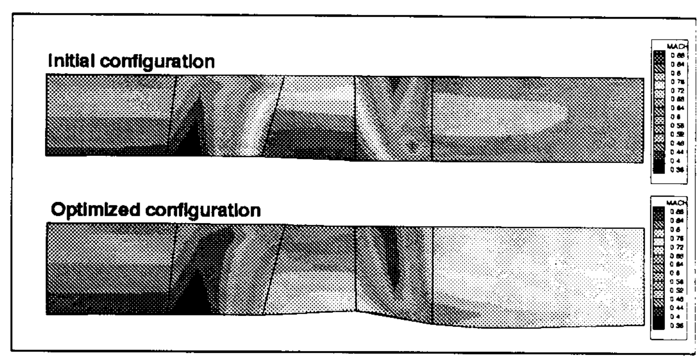

Fig. 6 Mach number field of initial and optimized single-stage turbin

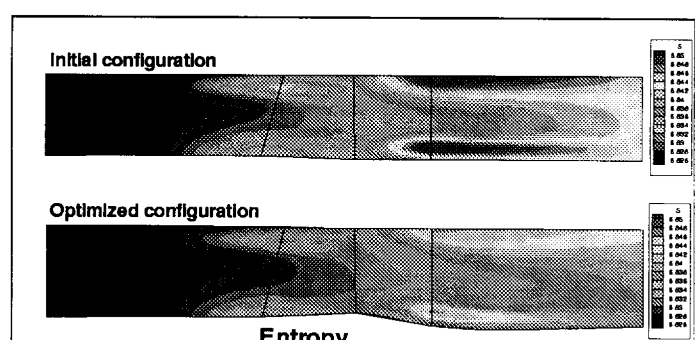

Fig. 7 Entropy field of initial and optimized single-stage turbine

A two-stage uncooled NASA axial turbine experimentally tested and described by Whitney et al. (1972), was selected for verification of the multistage optimization. The turbine was intended for high-temperature-engine application. The turbine had mean diameter of 660.4 mm. The mean blade hight in the first stage was 100 mm. The blade hight in the second stage was 134.6 mm at the stator outlet and 148.6 mm at the rotor outlet. Equivalent design speed, $N/\sqrt{\theta_{cr}} = 4407.36 rpm$ (θ_{cr} is squared ratio of critical velocity at turbine inlet to critical velocity of sea-level air. The turbine operated at an overall pressure ratio ranging from 1.4 to 4.0. The design pressure ratio was 3.2. The experimentally measured efficiency of the initial configuration was already extremely high: $\eta_{tt} = 0.932$. This well-documented set of experimental data has been used to check the accuracy of our numerical results and to demonstrate the possibility of the proposed multistage turbine design optimization system. To check the through-flow code in analysis mode, the calculation of the turbine flow at the nominal rotating speed has been performed. At the design conditions, our through-flow analysis code calculated the value of turbine efficiency as $\eta_{tt} = 0.9307$experimentally measured value was $\eta_{tt} = 0.9320$ (Whitney et al., 1972).

The optimization was performed while keeping constant rotational speed, mass flow rate, total enthalpy drop, number of blades, rotor tip clearance, blade chord lengths, and blade trailing edge thicknesses. Only small increase of rotors tip diameters was allowed. Polynomial spline discretization of the hub and shroud geometry was performed using the radii indicated in Fig. 9 as design variables.

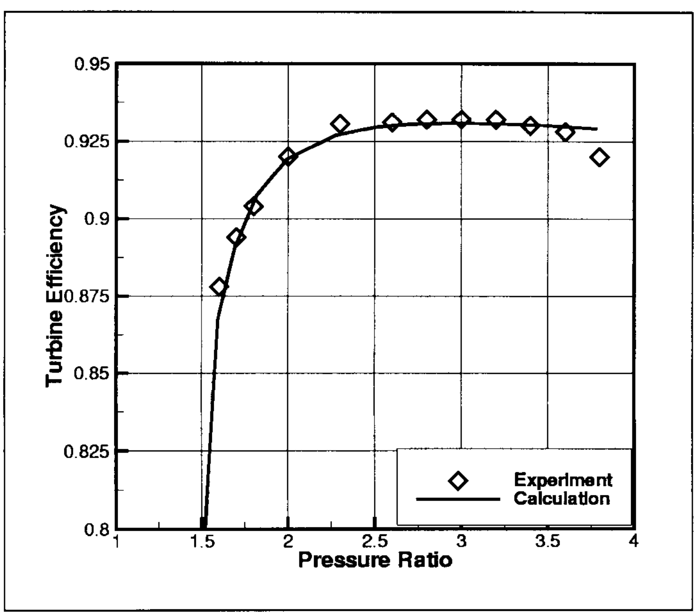

Fig. 8 Experimentally measured and calculated overall performance of the two-stage turbine

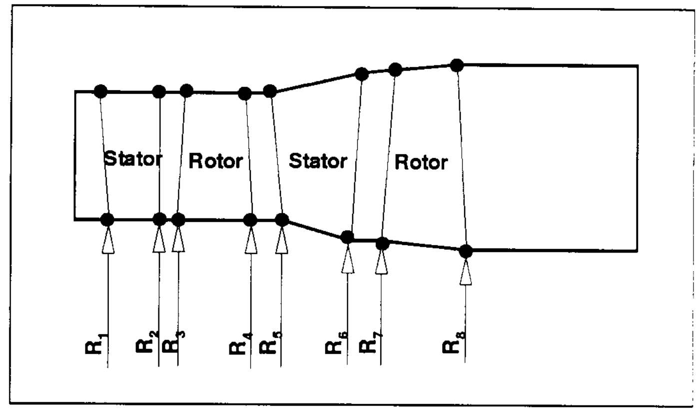

Fig. 9 Hub and shroud discretization of the two-stage turbine

The through-flow code in its design mode needed approximately 15 seconds to find each flow-field solution on a standard personal computer. In this particular example where the starting two-stage axial turbine already had an extremely high efficiency, about 2000 through-flow runs in the design mode were neccessary to find the turbine stage configuration that gives the maximum total-to-total efficiency. This would have consumed an unacceptable amount of computing time if a fully 3-D multistage Navier-Stokes flow-field analysis code was used instead of the fast augmented through-flow code. It was assumed that both blade count and chord lengths have been already determined by some preliminary design procedure (for example, meanline optimization). It is possible to include both of these parameters and other parameters deemed to be influential, although this would result in more than 17 design parameters per stage to be optimized.

The maximum achieved value for the turbine efficiency was $\eta_{tt,opt} = 0.9396$. Compared to the computed efficiency of the initial configuration ($\eta_{tt,init} = 0.9307$), the absolute improvement is approximately $\Delta\eta_{tt} = 0.9\%$ (Fig. 10). The efficiency of the optimized configuration is better over the entire range of loads.

Figure 11 shows relatively small changes of meridional flow paths since a very small increase in blade tip radius was allowed. The changes in tangential components of velocity are presented in Fig. 12.

Comparisons of spanwise distributions of blade exit metal angles, flow losses, reaction, stage efficiency for the initial and the optimized configuration are presented in Figs. 13 and Fig. 14. Figure 15 shows the computed flow-field entropy distribution in the initial and the optimized configuration.

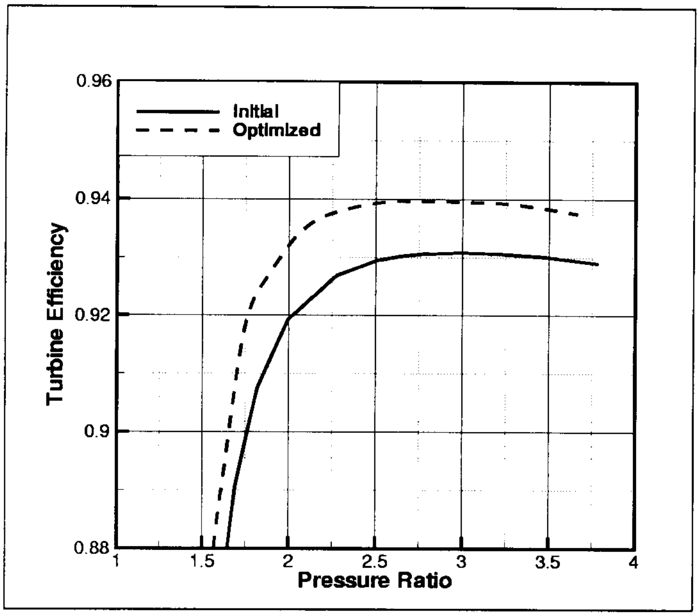

Fig. 10 Calculated initial and optimized overall performance of the two-stage turbine

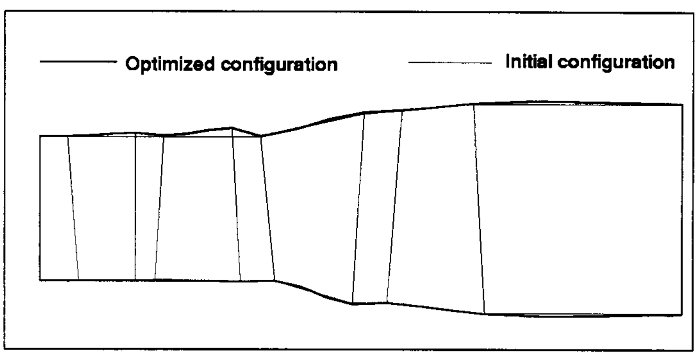

Fig. 11 Comparison of initial and optimized meridional flow paths in the two-stage turbine

It can be seen that it was not possible to reduce the flow loss coefficient in the turbine blade rows (Fig. 14). But, the flow losses and the entropy increase have been reduced (Fig. 15) by more suitable distribution of the enthalpy drop between stator and rotor in both stages. This was achieved by decreasing the tangential absolute velocity component at the exit of the blade rows (Fig. 13), that is, by changing the blades metal outlet angles (Fig. 14). The degree of reaction has been increased in the second stage and slightly decreased in the first stage. The optimized configuration has the degree of reaction at the blade mean radius of approximately 0.35 in the first stage and 0.60 in the second stage.

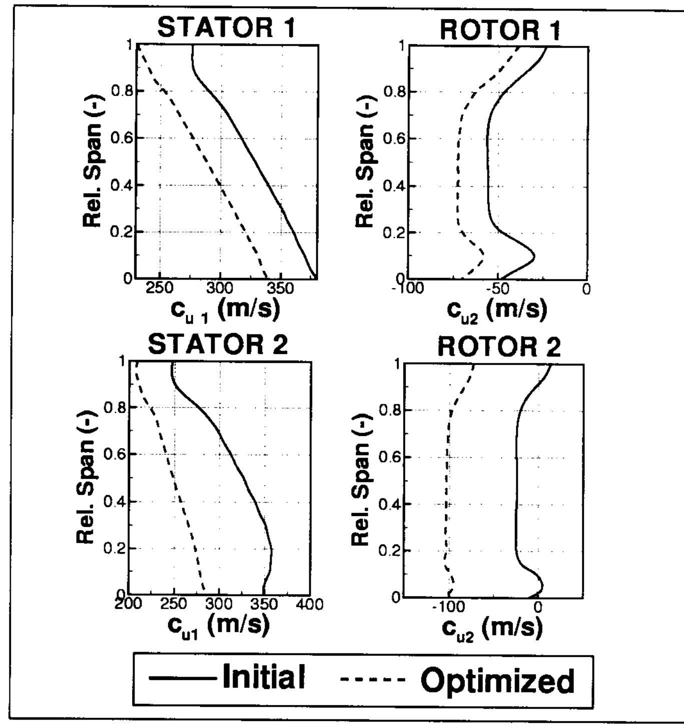

Fig. 12 Spanwise distribution of blade exit tangential velocities in the initial and the optimized two-stage turbine configuration.

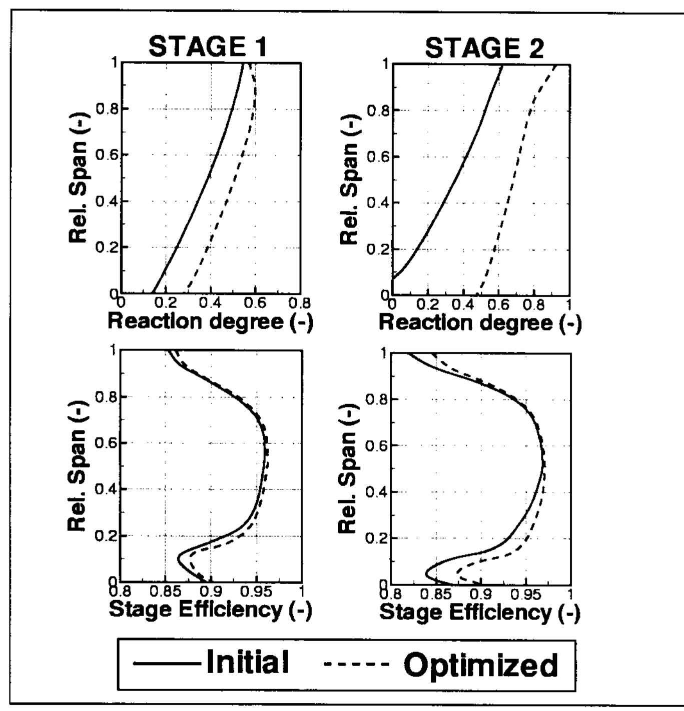

Fig. 13 Comparison of reaction and stage efficiency along the blade span in the initial and the optimized two-stage turbine configuration

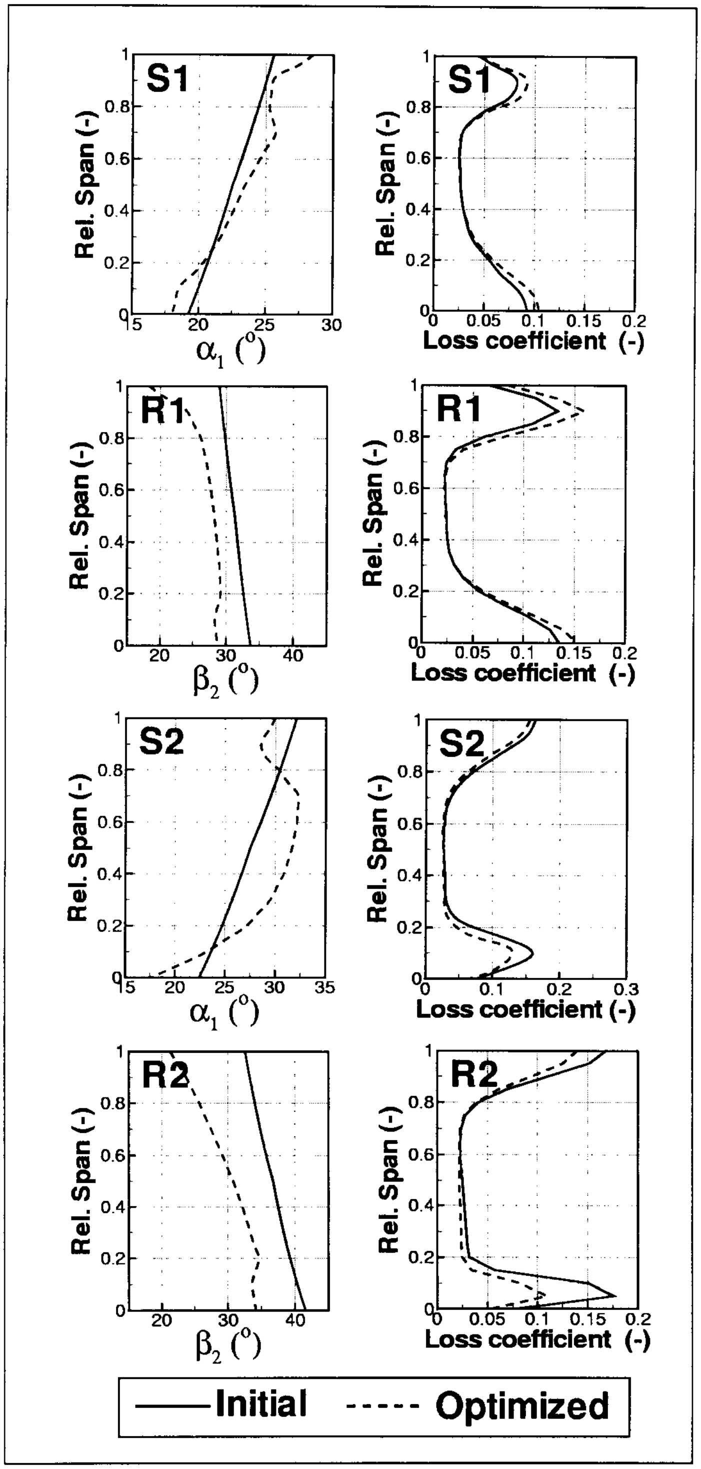

Fig. 14 Spanwise distribution of blade exit metal angles and flow losses, in the initial and the optimized configuration.

6. CONCLUSIONS

The design system for multistage axial turbine geometry optimization has been developed. The system uses a fast and accurate through-flow aerodynamics code and a robust constrained optimization package. The analysis of loss models has indicated that there are at least 17 parameters per every turbine stage that could be optimized: 8 geometrical parameters to describe the shapes of hub and shroud, and 9 flow-field parameters to describe the tangential component of velocity at stator and rotor exits. By varying these 17 parameters per turbine stage, the optimization code automatically searches the flow-field and the turbine hub and shroud geometry that gives the maximum efficiency. The design system has been successfully applied to optimization of the meridional flow path and radial distribution of circumferential mean flow angles in a two-stage axial gas turbine. The achieved efficiency of the optimized configuration was 0.9% better then the efficiency of the initial configuration at the design load. Through-flow code in an analysis mode was applied to analyze off-design behavior of the optimized configuration which was found to perform better over the entire range of loads compared to the initial configuration. The same design optimization procedure is applicable to multistage steam turbines. Further improvement of turbine efficiency is possible by 2-D sectional cascade optimization (Dennis, Dulikravich, and Han, 1999) or by a 3-D inverse shape design of isolated stator and rotor blades geometry subject to the optimized inlet and exit boundary conditions of each blade row.

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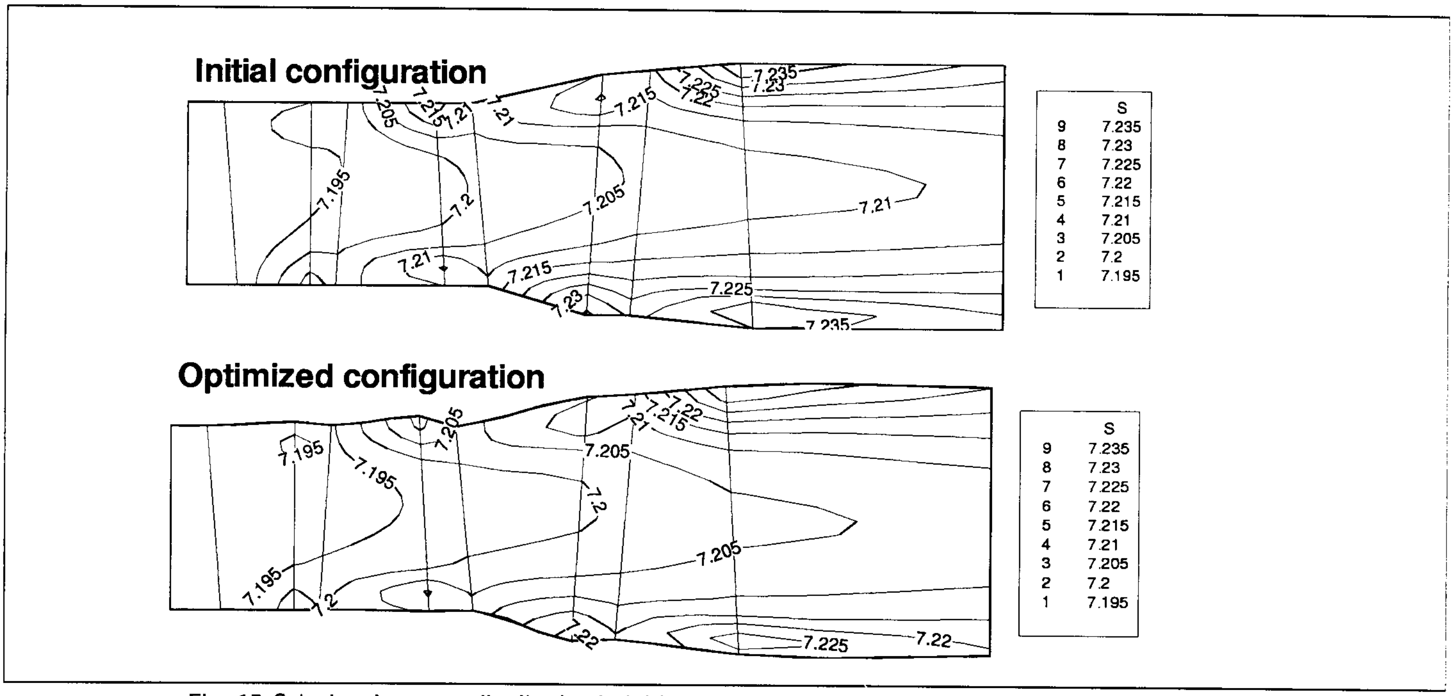

Fig. 15 Calculated entropy distribution in initial and optimized two-stage axial gas turbine configuration

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