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INVERSE DETERMINATION OF STEADY HEAT CONVECTION COEFFICIENT DISTRIBUTIONS

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ABSTRACT

An inverse Boundary Element Method (BEM) procedure has been used to determine unknown heat transfer coefficients on surfaces of arbitrarily shaped solids. The procedure is non-iterative and cost effective, involving only a simple modification to any existing steady-state heat conduction BEM algorithm. Its main advantage is that this method does not require any knowledge of, or solution to, the fluid flow field. Thermal boundary conditions can be prescribed on only part of the boundary of the solid object while the heat transfer coefficients on boundaries exposed to a moving fluid can be partially or entirely unknown. Over-specified boundary conditions or internal temperature measurements on other, more accessible boundaries, are required in order to compensate for the unknown conditions. An ill-conditioned matrix results from the inverse BEM formulation, which must be properly inverted to obtain the solution to the ill-posed problem. Accuracy of numerical results has been demonstrated for several steady two-dimensional heat conduction problems including sensitivity of the algorithm to errors in the measurement data.

NOMENCLATURE

[A] = coefficient matrix multiplying a vector of unknowns
B_i = Biot number (h_{conv} L / k)
{F} = vector of known sources and boundary conditions
[G] = BEM coefficient matrix multiplying nodal fluxes
h_{conv} = convective heat transfer coefficient
[H] = BEM coefficient matrix
k = coefficient of thermal conductivity
L = characteristic length
$\hat{n}$ = local outward unit normal vector
N_{BE} = number of boundary elements
N_{BN} = number of nodes on the boundary
q = normal temperature derivative or flux $\partial T / \partial n$
{Q} = vector of nodal fluxes
T = temperature
u^* = fundamental solution
{U} = vector of nodal temperatures
x = real space coordinate
{X} = vector of unknowns

Greek letters

Γ = boundary contour
δ = Dirac delta function
λ = Tikhonov regularization parameter
τ_{SVD} = SVD threshold value
Ω = domain

Subscripts

amb = ambient fluid quantities
INT = internal measurement
conv = convective heat transfer
perturb = perturbed value

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1. INTRODUCTION

A well-posed thermal boundary value problem requires either temperature or heat flux specified over the entire boundary of the solid region. When the surface is exposed to a moving fluid, convective heat transfer coefficients can be utilized as boundary conditions. Accurate values of the convective heat transfer coefficients are difficult to obtain experimentally because their values depend strongly on at least twelve variables or eight nondimensional groups (White, 1988). Typical semi-empirical expressions for prediction of heat convection coefficients represent curve fits through experimental data for very simple configurations covering only limited ranges of flow-field parameters. Consequently, in most practical situations the heat convection problems are solved by using a single value of the heat convection coefficient on the entire surface exposed to a moving fluid.

This paper explains an entirely different approach to a problem of predicting the surface variation of the heat convection coefficient. The most innovative aspects of this approach are that it does not require any information about the flow-field and that it is non-iterative. In other words, it is possible to treat the heat convection coefficient determination problem as an ill-posed boundary value heat conduction problem where no thermal data is available on parts of the boundary exposed to a moving fluid. This approach is capable of utilizing over-determined thermal measurements involving temperatures and heat fluxes where they are accessible. These data are then used to predict distributions of temperature, heat fluxes, and convective heat transfer coefficients on the boundaries where they are unknown.

We have developed a non-iterative algorithm that reliably and efficiently solves inverse (ill-posed) boundary condition problems governed by the Laplace equation in two-dimensional and three-dimensional multiply-connected domains having different thermal material properties (Martin and Dulikravich, 1996; Dulikravich and Martin, 1996). An extended version of this method was also successfully used in solving ill-posed problems in two-dimensional elasticity (Martin, Halderman and Dulikravich, 1994) as well as for the determination of heat sources (Martin and Dulikravich, 1996). This technique is based on the Green's function solution method, commonly referred to as the Boundary Element Method (BEM). It is an integral technique that generates a set of linear algebraic equations with unknowns confined only to the boundaries. For well-posed problems, the resulting solution matrix can be solved by Gaussian elimination or any other standard matrix inverter. When an ill-posed problem is encountered, the matrix becomes ill-conditioned. We have shown that the proper solution to this matrix provides accurate results to various steady inverse heat conduction boundary value problems (Martin and Dulikravich, 1996; 1997). This method has been shown to suppress the amplification in measurement errors in the input data while both minimizing the variance in the output and preventing output bias. The algorithm is applicable to complex, multiply-connected, two and three-dimensional configurations.

2. NUMERICAL FORMULATION

The governing partial differential equation for steady-state heat conduction in a two-dimensional solid with a constant coefficient of thermal conductivity is

$$k\nabla^2 T = 0 \tag{1}$$

This linear elliptic partial differential equation can be integrated subject to Dirichlet (temperature) boundary conditions, Neumann (heat flux) boundary conditions, and, when a boundary is exposed to a moving fluid, the Robin (convective heat transfer) boundary conditions given as

$$-k\left.\frac{\partial T}{\partial n}\right|_{\Gamma_{conv}} = h_{conv}\left(\left.T\right|_{\Gamma_{conv}} - T_{amb}\right) \tag{2}$$

When ill-posed boundary value problems are encountered, portions of the boundary must be over-specified with both temperatures and heat fluxes, while nothing is known on boundary Γ_{conv}. Since the domain Ω may be subdivided into a number of sub-domains, Ω_1, Ω_2, ..., Ω_M, each with a different coefficient of thermal conductivity, $k_1, k_2, \ldots, k_M$, the solutions to the partial differential equation for each sub-domain are combined using the compatibility relations on the interface boundaries. For example, for the interface between the sub-domains Ω_1 and Ω_2, the compatibility relations equate the temperatures and heat fluxes so that

$$T_{I1} = T_{I2} = T_I \tag{3}$$

and

$$-k_1\left(\frac{\partial T}{\partial n}\right)_{I1} = k_2\left(\frac{\partial T}{\partial n}\right)_{I2} = -k_1\left(\frac{\partial T}{\partial n}\right)_{I} \tag{4}$$

The BEM is the method of choice for many multidisciplinary and inverse design problems because it has certain distinct advantages over the more common finite element or finite difference methods. The BEM is a non-iterative solution procedure which, when used for linear boundary value problems, is significantly faster and more robust than the other numerical solution techniques. In addition, the non-iterative nature of the BEM eliminates stability, numerical dissipation and convergence problems, as well as removing any need for artificial dissipation. Analytical solutions to the partial differential equation, in the form of the Green's function, are part of the BEM solution. Therefore, high accuracy is expected because introducing the Green's functions does not introduce any error into the solution. This is valuable because iterative procedures tend to amplify errors thus requiring complex regularization (smoothing) algorithms (Tikhonov and Arsenin, 1977).

The BEM begins with the weighted residual statement remaining in its weak integral formulation. Green's second identity is equivalent to integrating the weighted residual statement by parts twice (Brebbia and Dominguez, 1984).

$$\int_{\Omega}\left(\nabla^2 T u^* - T\nabla^2 u^*\right)d\Omega = \int_{\Gamma}\left(\frac{\partial T}{\partial n}u^* - T\frac{\partial u^*}{\partial n}\right)d\Gamma \tag{5}$$

The fundamental solution, u^*, is the solution to the adjoint of the governing partial differential equation which satisfies the homogeneous boundary conditions in an infinite domain (Carslaw and Jaeger, 1948).

$$\nabla^2 u^*(x,\xi) + \delta(x-\xi) = 0 \quad (6)$$

After substituting the governing and the adjoint differential equation into Green's second identity, retaining the Cauchy principal value of the boundary integrals, and using the properties of the Dirac delta function, the following boundary integral equation (BIE) is obtained (Brebbia and Dominguez, 1989).

$$c(x)T(x) + \int_\Gamma q^*(x,\xi)T(\xi)d\Gamma = \int_\Gamma u^*(x,\xi)q(\xi)d\Gamma \quad (7)$$

Since we are strictly solving a boundary value problem, the unknown temperature T(**x**) is on the boundary Γ_{conv} which is a part of the overall boundary Γ. The boundary Γ can be discretized into N_{BE} isoparametric boundary elements connected at N_{BN} boundary nodes. In addition, N_{INT} internal measurement points could exist where temperature data is obtained. The T and q can vary between the neighboring end-nodes defining each boundary element, each boundary element is integrated numerically using a standard Gaussian quadrature integration formula. Boundary elements containing a singularity at one end-point can be integrated analytically resulting in a set of $N_{BN}+N_{INT}$ boundary integral equations, one for each boundary node plus one for every internal temperature measurement. The resulting discretized form of the BIE can be represented in matrix form as

$$[\mathbf{H}]\{\mathbf{U}\} = [\mathbf{G}]\{\mathbf{Q}\} \quad (8)$$

For a well-posed boundary value problem, every point on the boundary Γ is given one Dirichlet, Neumann or Robin boundary condition and no internal temperature measurements. These boundary conditions are then multiplied by their respective coefficient matrix and collected on the right hand side to form a vector of knowns, {**F**}. The left-hand side remains in the standard form [**A**]{**X**}. This well-posed system of linear algebraic equations can be solved for the vector of unknowns {**X**} on the boundary by any standard matrix solver such as Gaussian elimination or LU factorization.

If the boundary conditions (T, q, or h_{conv}) are unknown on parts of the boundary or if internal temperature measurements are included in the analysis, the problem becomes ill-posed. A solution may still be obtained by multiplying the known quantities in the vectors {**U**} and {**Q**} by their respective coefficient matrix columns and collecting them into the vector of knowns, {**F**}. The unknowns form a single vector, {**X**}, multiplied by a highly ill-conditioned coefficient matrix, [**A**], which is, in general, not square. The truncated Singular Value Decomposition (SVD) method (Press et al., 1992) has been often used to solve this ill-conditioned system of algebraic equations. Very small singular values of such ill-conditioned matrix [**A**] are zeroed out so that those algebraic terms that are dominated by noise and round-off error are eliminated from the matrix. In order to determine which singular values are to be truncated, we must choose a user-specified singularity threshold parameter, τ_{SVD}. Any singular value, whose ratio with the largest singular value is less than this singularity threshold, is zeroed out.

Tikhonov regularization (Tikhonov and Arsenin, 1977) is another type of single-parameter minimization where the solution vector {**X**} minimizes the weighted sum of the norm of the error vector plus a penalty term. Tikhonov regularization is a generalization of least-squares truncation, but instead of simply eliminating terms associated with small singular values, they are weighted by a factor $(1 + \lambda/w^2)$ where w is any of the singular values of the matrix [**A**]. The Tikhonov regularization parameter, λ, plays an important role. A low value drives the residual term [**A**]{**X**}-{**F**} smaller, approaching the least squares solution. Because of the destabilizing effect of the small singular values, the solution for an ill-conditioned matrix often oscillates erratically. Larger Tikhonov regularization parameters act as a filter to gradually reduce the effect of the singular values because w_j/w_{max} are less than the regularization parameter, λ. Thus, the optimal choice of the regularization parameter provides a balance between the accuracy and the smoothness of the solution and it should be chosen judiciously (Martin and Dulikravich, 1996).

In this work we attempted both SVD and Tikhonov's regularization in a number of test cases. We observed that Tikhonov's regularization produces unacceptable levels of global bias (Martin and Dulikravich, 1996). Therefore, all numerical results reported in this paper were obtained using SVD.

3. RESULTS

The equation for the heat flux from the Robin boundary condition was added directly into the linear BEM system. The unknown temperatures were factored together with the other nodal temperatures appearing on the left-hand side of the BEM matrix equation set. After the ill-conditioned coefficient matrix [**A**] has been inverted, the unknown boundary values of T and q were obtained from $\{\mathbf{X}\} = [\mathbf{A}]^{-1}\{\mathbf{F}\}$. Once these boundary values were known on the boundary Γ_{conv}, the convective heat transfer coefficients were determined from

$$h_{conv} = -k\frac{\partial T}{\partial n}\bigg|_{\Gamma_{conv}} \bigg/ \left(T\big|_{\Gamma_{conv}} - T_{amb}\right) \quad (9)$$

since T_{amb} is considered as known.

The following two test cases were used to assess the accuracy of the entire non-iterative BEM inverse algorithm and its sensitivity to measurement errors in boundary temperatures and heat fluxes.

A rectangular plate with side lengths a and b was subject to homogeneous Dirichlet boundary conditions (T = 0°C) on three boundaries and a Robin boundary condition (h_{conv} = 1.0 W/m^2 °C, T_{amb} = 1.0 °C) on the bottom boundary (Figure 1). Heat conduction coefficient was uniformly k = 1.0 W/m °C. The four boundaries of the rectangular plate were discretized with 40 equal-length linear boundary elements, 10 per side. The analysis or well-posed formulation consisted of three Dirichlet boundary conditions on the top, left and right boundary of the plate, and the Robin boundary condition was specified on the bottom boundary.

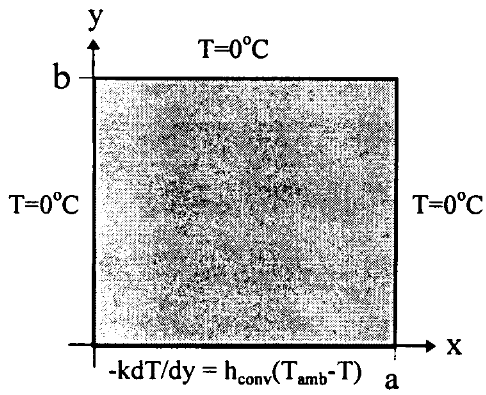

Figure 1. Rectangular plate with three homogeneous Dirichlet and one Robin boundary conditions.

Using separation of variables, an analytical solution for this test case can be found in the following form

$$T(x,y) = \frac{2h_{conv}}{a} \sum_{n=1}^{\infty} \left[\int_0^a T_{amb} \sin\left(\frac{n\pi x}{a}\right) dx \right] \tag{10}$$

$$\frac{\sinh\left(\frac{n\pi(b-y)}{a}\right) \sin\left(\frac{n\pi x}{a}\right)}{h_{conv} \sinh\left(\frac{n\pi b}{a}\right) + \frac{n\pi k}{a} \cosh\left(\frac{n\pi b}{a}\right)}$$

The analysis BEM predicted the heat fluxes on the four boundaries and temperature on the bottom boundary. When the entire computed temperature field is plotted, it can be seen that these numerical computations were practically identical to the analytical solution, having an error of less than 0.1% (Figure 2).

The inverse problem was then formulated by specifying nothing on the bottom boundary while one or more of the remaining boundaries were over-specified with temperatures and heat fluxes taken from the analytical solution. In order to check the performance of the code with respect to the amount of over-specified data, two variations of the numerical test case were performed.

3.1 Test Cases With Constant h_{conv}

In the first variation, only the top boundary was over-specified with temperature and heat flux and the side walls were specified with temperature only. The isotherms predicted by the inverse non-iterative BEM procedure are shown as dashed lines in Figure 2. The ambient fluid temperature was considered to be known (T_{amb} = 1.0 °C). Therefore, the convective heat transfer coefficients can be computed directly after both the temperature and heat flux on the bottom boundary have been predicted.

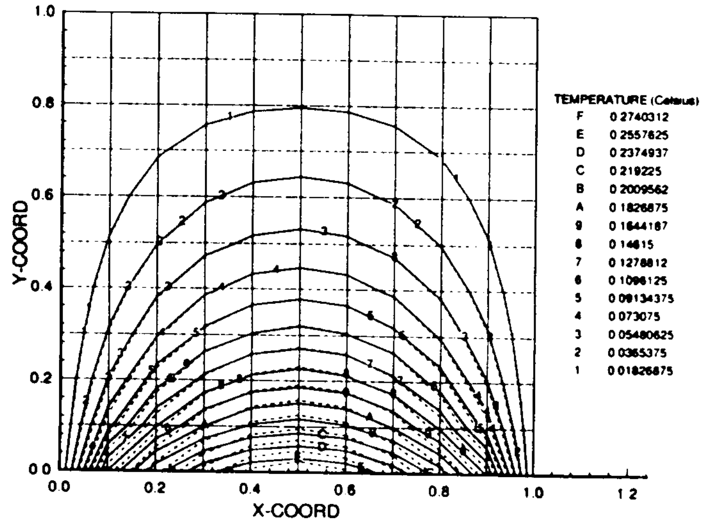

Figure 2. Isotherms predicted within the rectangular plate by the analytical (solid lines), direct BEM (dotted lines), inverse BEM with top boundary over-specified (dashed lines), and the inverse BEM with top and side boundaries over-specified (dash-dot lines).

The computed convective heat transfer coefficients on the bottom boundary, which in this test case should be h_{conv} = 1.0 W/m^2 °C, are plotted as square symbols in Figure 3. Although the average error in h_{conv} is very small in this test case when overspecified data were provided only on a single boundary farthest from the boundary with unknown thermal boundary conditions, the BEM predicted heat transfer coefficients that had a peak error of 30%.

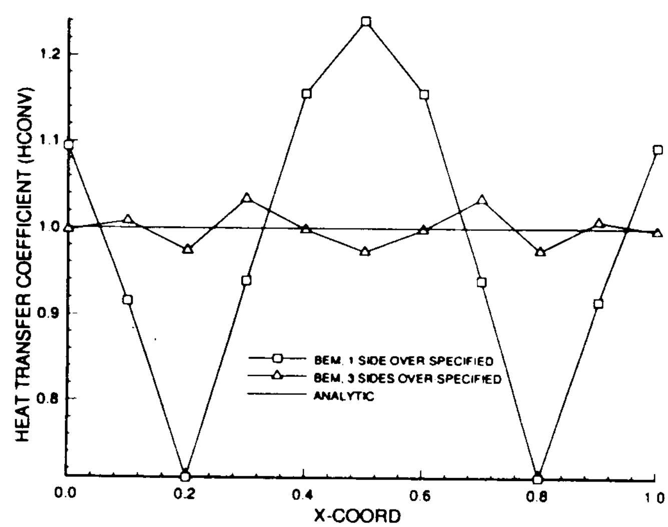

Figure 3. Convective heat transfer coefficient numerically predicted by the inverse BEM on the square plate when the opposite boundary (squares) and three boundaries (triangles) were over-specified.

In the second variation, the opposite (top) boundary as well as both side boundaries were over-specified with both temperature and heat flux from the analytical solution. The isotherms that were predicted by the inverse non-iterative BEM code are shown as dash-dot lines in Figure 2. In this figure, the isotherms are directly on top of the isotherms predicted by the forward BEM and the analytical solution. The numerically computed convective heat transfer coefficients from the inverse BEM procedure for this test case are plotted as triangle symbols in Figure 3. Notice that the prediction of heat transfer coefficients is now much more accurate, having a peak error of 4%. From these results we can conclude that the inverse prediction of unknown convective heat transfer coefficients is sensitive to the amount of over-specified data and the distance between the overspecified and unspecified boundaries.

3.2 Applicability to Different Values of Biot Number

The previous numerical results were obtained for unity Biot number ($B_i = h_{conv}\ L / k$). The second variation inverse problem was repeated for a variety of Biot numbers by utilizing values of heat conductivity coefficient $0.01 < k < 100.0$ W/m °C, thus varying the Biot number over the same range since L = 1 m and $h_{conv} = 1.0$ W/m^2 °C were kept constant. Threshold parameter in the SVD algorithm was $\tau_{SVD} = 10^{-6}$. From Figure 4 it can be concluded that standard deviation and maximum error of the predicted h_{conv} were very low for $0 < B_i < 20$, after which the maximum error increased until it reached 30% for $B_i = 100$. For example, a plasma coated gas turbine blade surface distribution of h_{conv} can be predicted quite accurately with this inverse BEM algorithm. In this case coating thickness is $L = 2 \times 10^{-4}$ m, k = 1.0 W/m °C, and if $B_i < 20$ this means that h_{conv} as high as $h_{conv} = 10^5$ W/m^2 °C can be predicted with maximum error of 2% and a standard deviation of less than 10^{-2} W/m^2 °C.

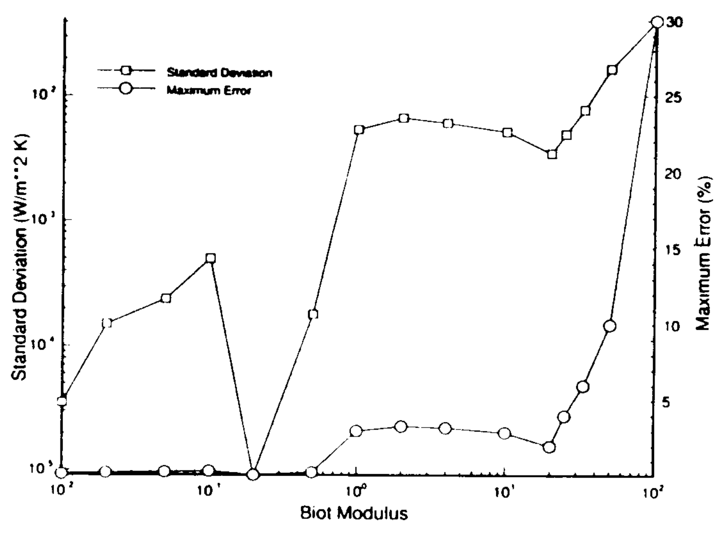

Figure 4. Influence of Biot number on maximum error and standard deviation of the predicted values of h_{conv}.

3.3 A Test Case With Variable h_{conv}

In order to evaluate this inverse BEM algorithm for the more realistic cases where the heat convection coefficient is not a constant, we have analyzed the temperature field in the same square domain with the same boundary conditions on the top and the side walls. The bottom wall was specified with variable heat convection coefficient $h_{conv} = 1.0 + \sin(2\pi x)$. This test case does not have an analytical solution. Therefore, we ran our BEM analysis code on this well-posed problem and treated the predicted isotherms as the accurate result (Figure 5).

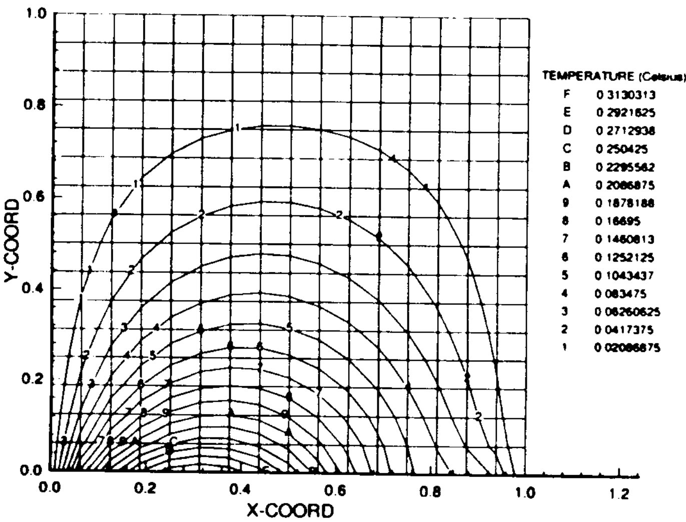

Figure 5. Isotherms predicted by a well-posed BEM (full lines) and inverse BEM (dashed lines) for the first test case, second variation, with $h_{conv} = 1.0 + \sin(2\pi x)$ on the bottom wall.

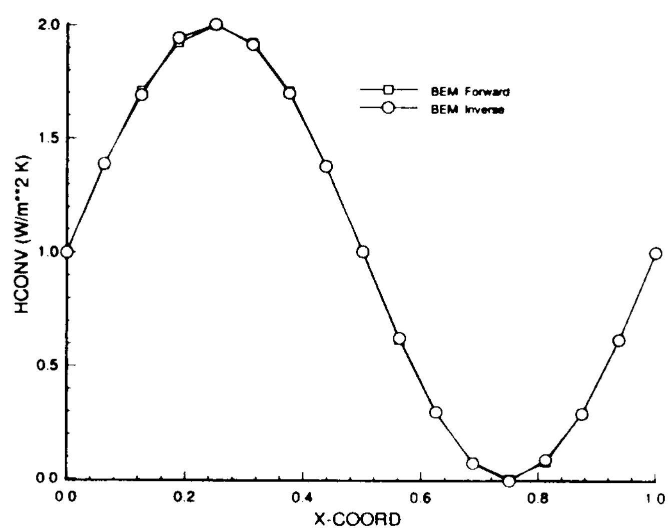

Figure 6. Exact (squares) and inverse BEM predicted (circles) variation of h_{conv} on the bottom boundary.

Then, an inverse problem was created by pretending that h_{conv} is unknown on the bottom boundary. These "unknown" values of h_{conv} were then predicted by over-specifying the vertical walls and the top wall with the T = 0 °C and the heat fluxes that were previously predicted by the BEM solution for the forward problem with sinus wave h_{conv} on the bottom boundary. Threshold parameter in the SVD algorithm was $\tau_{SVD} = 10^{-4}$. The result of inverse BEM code was a highly accurate temperature field (Figure 5) and the equally accurate prediction of the sine-wave h_{conv} variation on the lower boundary (Figure 6) thus confirming the applicability of this inverse BEM algorithm to prediction of variable h_{conv} values.

3.4 Test Cases With Asymmetric Boundary Conditions

For a second test case, geometry was a homogeneous square plate with each side of length L discretized with 10 linear isoparametric boundary elements. The boundary conditions were altered such that the right-side boundary had the Robin boundary condition (h_{conv} = 1.0 W/m^2 °C, T_{amb} = 0.0 °C). In the well-posed (analysis) problem, the top boundary was specified with a temperature T = 1.0 °C and the left-side and bottom boundaries were specified with a temperature T = 0.0 °C (Figure 7).

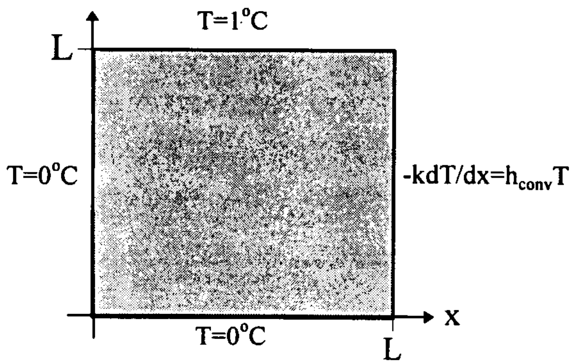

Figure 7. Square plate with one Robin, one inhomogeneous Dirichlet, and two homogeneous Dirichlet boundary conditions.

The analytical solution for this problem was found by separation of variables.

$$T(x,y) = \frac{2T_{amb} h_{conv} L}{k}$$

$$\sum_{n=1}^{\infty} \frac{1-\cos\alpha_n}{\alpha_n \left(h_{conv} \frac{L}{k} + \cos^2 \alpha_n \right)} \frac{\sinh \frac{\alpha_n y}{L}}{\sinh \alpha_n} \sin \frac{\alpha_n x}{L} \tag{11}$$

where

$$\tan\alpha_n = -\frac{\alpha_n k}{h_{conv} L} \tag{12}$$

As in the previous test case, the analysis BEM was compared to the analytical solution. The isotherms of the analytical test case and the numerical analysis or well-posed BEM numerical prediction are shown as solid and dotted lines in Figure 8. The isotherms are directly on top of each other. Again, two variations of the inverse problem were performed. One variation had only the opposite boundary over-specified. The other variation had all three boundaries over-specified. In both inverse variations nothing was specified on the right-side boundary where heat transfer coefficients are prescribed in the well-posed problem. The numerically predicted isotherms are shown in Figure 8. In the case where only the opposite boundary was over-specified, the isotherms (dashed lines) show an appreciable error.

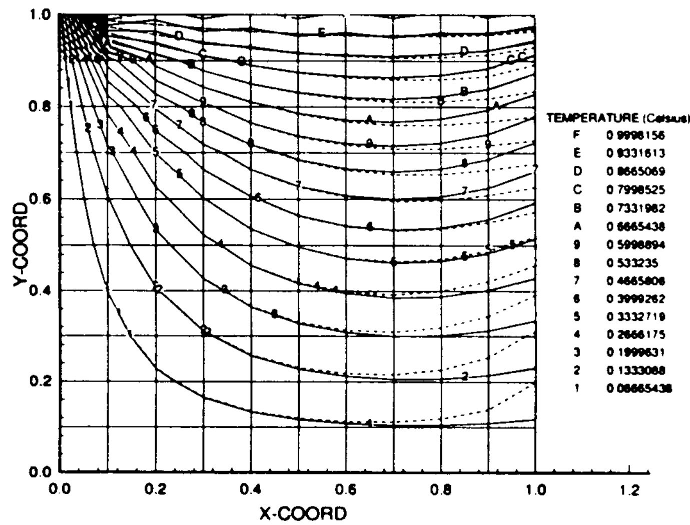

Figure 8. Isotherms predicted analytically (solid lines) and using analysis BEM (dotted lines), inverse BEM with opposite boundary over-specified (dashed lines), and the inverse BEM with three boundaries over-specified (dash-dot lines).

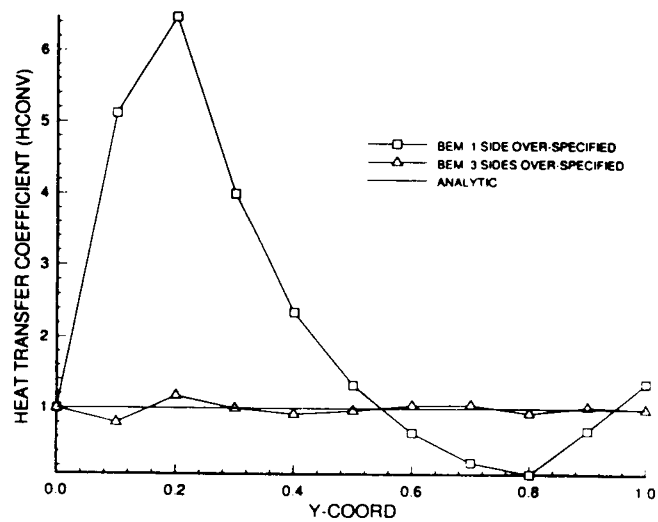

Figure 9. Convective heat transfer coefficient predicted by the inverse BEM on the square plate when the opposite boundary (squares) and three boundaries (triangles) were over-specified.

The error is more evident when the convective heat transfer coefficient on the boundary is obtained from the computed temperatures and heat fluxes (Figure 9). There was a large discrepancy in the computed h_{conv} values when only the opposite boundary was over-specified and when this boundary is far away from the unspecified boundary. The maximum error in predicted h_{conv} was dramatically reduced to about 4% when all three boundaries were over-specified (Figure 9).

3.5 Sensitivity to Errors in the Measurement Data

It is of utmost practical importance to access the influence of measurement errors of boundary values in any newly proposed inverse boundary value determination algorithm. This was numerically simulated by adding a random error based on the Gaussian probability density distribution to the temperature measurements. A random number $0 < R < 1$ was generated using a standard utility subroutine. The desired variance σ^2 was specified and error was added to the analytical temperature data points T according to

$$T_{perturb} = T \pm \sqrt{-2\sigma^2 \ln R} \qquad (13)$$

where addition and subtraction of the random error had a 50-50 chance of been chosen. This simple test of sensitivity was applied to the second variance of the first test case (Figure 1) discussed in this paper. The predicted values of h_{conv} had a maximum local error of 4% when $\sigma = 0.0$ (Figure 10). A maximum local error of 10.5% was realized when average perturbation was 0.1% of T_{max}. It increased to 15% when the average perturbation was 1% of T_{max}. The maximum local error in predicted h_{conv} reached 33% when the average perturbation of supplied (measured) boundary temperature was 10% of the maximum temperature in the field.

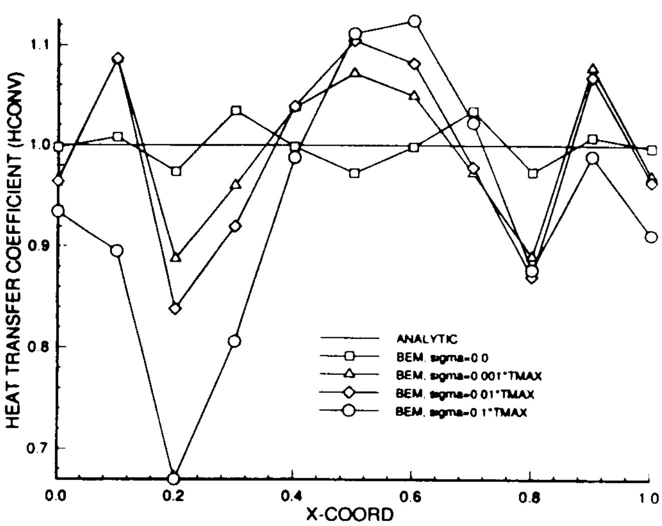

Figure 10. Sensitivity of the predicted convective heat transfer coefficient distributions for different standard deviations of the overspecified boundary temperatures.

At the same time, it can be seen (Figure 10) that the average error in the predicted h_{conv} is of the approximately same magnitude as the average level of the perturbations (errors) introduced in the boundary temperatures. This is a remarkable result since most of the iterative inverse boundary value problem determination algorithms cause significantly larger errors in the predicted values than the errors introduced in the specified boundary values; consequently, these errors need to be controlled using sophisticated regularization algorithms (Murio, 1993).

4. CONCLUSIONS

It has been demonstrated how a simple modification to any existing BEM analysis algorithm for the solution of Laplace's equation can transform it into an inverse non-iterative determination code for unknown distributions of steady convective heat transfer coefficients. This approach is applicable to arbitrarily shaped two and three-dimensional solids where at least part of a boundary can be overspecified with both temperatures and heat fluxes. It is extremely fast and robust since it requires inversion of a single fully populated matrix. The inversion must be performed using an algorithm suitable for almost singular matrices. This method is considerably less sensitive to the errors introduced in the boundary measurements of temperature than are the general iterative inverse methods.

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