HTD-Vol. 340

ASME Proceedings of the 32nd

NATIONAL HEAT TRANSFER CONFERENCE

VOLUME 2

• INVERSE PROBLEMS IN HEAT TRANSFER AND FLUID FLOW

presented at

THE 32nd NATIONAL HEAT TRANSFER CONFERENCE
BALTIMORE, MARYLAND
AUGUST 8–12, 1997

sponsored by

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, ASME
THE AMERICAN INSTITUTE OF CHEMICAL ENGINEERS, AIChE
THE AMERICAN INSTITUTE OF AERONAUTICS AND ASTRONAUTICS, AIAA
THE AMERICAN NUCLEAR SOCIETY, ANS

edited by

GEORGE S. DULIKRAVICH
PENNSYLVANIA STATE UNIVERSITY

KEITH A. WOODBURY
UNIVERSITY OF ALABAMA

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
United Engineering Center • 345 East 47th Street • New York, N.Y. 10017

HTD-Vol. 340, National Heat Transfer Conference
Volume 2
ASME 1997

NON-ITERATIVE DETERMINATION OF TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY

Thomas J. Martin[1] and **George S. Dulikravich**[2]
Department of Aerospace Engineering
The Pennsylvania State University
University Park, PA

ABSTRACT

A method has been developed for the non-iterative determination of arbitrary temperature-dependence of heat conductivity. The method is non-intrusive and is based on a boundary element formulation for the solution of an over-specified boundary value problem. The procedure is valid for arbitrary two- and three-dimensional solid objects. Given heat flux measurements taken everywhere on the surface and a range of temperature measurements specified at a small number of isolated surface points, this method can predict the variation of thermal conductivity over the same range of temperatures. The non-iterative solution procedure was compared with the more common method of least squares minimization. The present approach was found to be much more efficient, and it could obtain a solution where least squares minimization terminated in a local minimum. The effect of measurement errors has been evaluated and found to be comparable to those resulting from the iterative approach.

NOMENCLATURE

F = objective function
[G] = geometric coefficient matrix
[H] = geometric coefficient matrix
k = thermal conductivity
Q = heat flux
q = Kirchhoff's heat flux
{Q} = vector of Kirchhoff's heat fluxes
R = random number
T = temperature
u = Kirchhoff's heat function
{U} = vector of Kirchhoff's heat functions
z = coordinate

Greek letters
α = coefficient of steepness of k-T curve
β = coefficient of non-linearity of k-T curve
Γ = boundary or surface of an object
σ = standard deviation

Superscripts
meas = measured or specified value
c = computed or predicted value

Subscripts
an = analytical
cold = cold boundary
hot = hot boundary
max = maximum value
min = minimum value
0 = reference value
1,2 = end points of an interval

1. INTRODUCTION

In many practical applications it would be highly desirable to evaluate the temperature-dependence of thermal properties of a material so that a heating or cooling pattern can be adjusted accordingly to provide the desired temperature field throughout the object Similarly, since thermal conductivity and specific heat are practically impossible to measure directly within the thin, mushy region of a solidifying or melting medium, it would valuable to develop another approach to determine variation of these physical properties with temperature. Often, it is very

[1] Graduate Research Assistant. Student Member ASME.
[2] Associate Professor. Fellow ASME.

impractical and even impossible to take a part of an existing object in order to create a properly sized and shaped laboratory test specimen. Thus, the non-intrusive and non-destructive character of any method for the determination of temperature-dependent thermal properties are essential. Thermal tomography and inverse thermal design techniques offer attractive solution procedures for these types of problems.

Iterative solution procedures are the most common method of solving inverse parameter identification problems. They are often classified as inverse heat conduction problems, because the nature of these problems has to do with the iterative minimization of the difference between the computed and measured temperatures and/or heat fluxes at the boundary, or at a finite number of interior thermocouple points. Orlande and Ozisik (1994) have noted that most work on parameter identification problems involves the use of finite dimensional minimization techniques. Gradient type minimization algorithms such as the steepest descent and the conjugate gradient method (Vanderplaats, 1984) have been employed to minimize the difference between the computed and measured temperatures at a finite number of points within the solid or on the boundary of the solid. Methods involving the adjoint equation have been used to obtain fairly accurate solutions using a temperature history at a single measurement point (Dantas and Orlande, 1996). Nevertheless, this approach, like any minimization formulation, seems to be computationally intensive (Blackwell and Eldred, 1997) and prone to local minima.

In this work, we are presenting a formulation and a solution procedure which differs substantially from the iterative approaches based on those developed by Artyukhin (1993) ant others. We start by assuming that measured values of heat fluxes are available everywhere on the surface of an arbitrarily shaped and sized two-dimensional or three-dimensional singly or multiply-connected solid. The Boundary Element Method (BEM) is then used to solve the linearized steady-state heat conduction equation for the transformation of the Kirchhoff heat function on the boundary. Given several surface temperature measurements, these heat functions are converted to heat conductivities at the points where the temperature measurements are taken. Thus, we can obtain the thermal conductivity versus temperature function over a range of temperatures which are measured by isolated thermocouple readings at the boundary. The approach is non-iterative and robust. Its solution requires only a couple of seconds on a personal computer.

2. ANALYSIS

The governing equation for steady-state heat conduction in an isotropic medium with temperature-dependent thermal conductivity is an elliptic non-linear partial differential equation of the type

$$\nabla \bullet \left[k(T)\nabla T\right] = 0 \qquad (1)$$

This equation can be linearized by the application of Kirchhoff's transformation which defines the heat function, u, so that

$$u = \int_0^T \frac{k(T)}{k_0} dT \qquad (2)$$

Hence

$$\nabla u = \frac{k(T)}{k_0} \nabla T \qquad (3)$$

Thus, Kirchhoff's transform converts the governing steady-state heat conduction equation into Laplace's equation for the heat function, u.

$$\nabla^2 u = 0 \qquad (4)$$

Dirichlet boundary conditions can also be transformed by applying Kirchhoff's transformation. Neumann boundary conditions can be directly related to the heat flux; that is,

$$\overline{Q} = -k \frac{\partial T}{\partial n} = -k_0 \frac{\partial u}{\partial n} \qquad (5)$$

When Robin-type boundary conditions are applied, the BEM system becomes non-linear. Consequently, an iterative solution procedure, such as the Newton-Raphson method, would be required.

2.1 Solution to the Direct Problem Using the BEM

The Boundary Element Method has a significant advantage when solving linear and quasi-linear, steady-state boundary value problems. This has been demonstrated by its effectiveness when ill-posed boundary conditions are prescribed at the boundary or when temperature measurements are enforced at isolated interior points (Martin & Dulikravich, 1996; Dulikravich and Martin, 1996). The BEM system for steady-state heat conduction can be written as a system of boundary integral equations (BIE) (Brebbia & Dominguez, 1989).

$$c(x)u(x) + \int_\Gamma q^*(x,\xi)u(\xi)d\Gamma = \int_\Gamma u^*(x,\xi)q(\xi)d\Gamma \qquad (6)$$

This BIE is valid for both two- and three-dimensional configurations. In this equation, $u^*(x,\xi)$ is the fundamental Green's function solution of the adjoint partial differential equation. The functions q and q* are the derivatives in the direction of the outward unit normal to the boundary Γ acting upon the heat function, u, and Green's function, u*, respectively. The boundary is then discretized with N_{BE} elements connected between N_{BN} boundary nodes.

Although the entire procedure is equally valid in three dimensions, for the sake of simplicity we will demonstrate it in a two-dimensional problem only. The variation of u and q over each flat boundary element (segment) was assumed to be linear and isoparametric. The integration over each boundary element was carried out using a numerical integration scheme, such as Gaussian quadrature. In the case when a singularity exists at one of the end points of a boundary element, analytical integration was performed. The nodal quantities of u and q were factored into the following matrix form.

$$[H]\{U\} = [G]\{Q\} \tag{7}$$

3. ITERATIVE DETERMINATION OF k-T CURVE

We have first attempted a solution procedure for determining the conductivity versus temperature (k-T) curves which was based upon the usual iterative minimization technique where the difference between the computed and measured values of temperatures and heat fluxes was formulated into a single scalar function.

$$F(k(T)) = \sqrt{\Sigma\left[\left(T^{meas} - T^{c}\right)^{2} + \left(Q^{meas} - Q^{c}\right)^{2}\right]} \tag{8}$$

This function was then iteratively minimized using a hybrid constrained optimization scheme (Foster et al., 1996) involving the DFP gradient search method (Fletcher and Powell, 1963), a genetic algorithm (Goldberg, 1989), a Nelder-Mead simplex method (Nelder-Mead, 1965) and a simulated annealing algorithm (Press, et al., 1984). For arbitrary variations of thermal conductivity, a parametric representation of the conductivity versus temperature curve was modeled using β-splines (Barsky, 1988) which are piecewise, second-order continuous and are closely related to the more common v-splines, B-splines and Bernstein polynomials. The design variables of the minimization process were the coordinates of the control vertices of the β-spline. These vertices were an ordered sequence of points, generally not lying on the k-T curve, but instead forming a control polygon. The use of β-splines allows for more precise control of the k-T curve since local variations in the temperature produced only local variations in the conductivity around that temperature.

An initial guess to the k-T variation was required to start the optimization process. Subsequent iterations with the hybrid optimization algorithm perturbed the set of β-spline vertices, thus changing the temperature-dependency of conductivity. The resulting temperature field was analyzed for each perturbation using the BEM until a conductivity function was found that minimized the objective function, F. Although this iterative technique was successful in determining the temperature-dependency of conductivity, it was very slow and less than robust. When the target k-T variation was irregular, and when bad initial guesses were used, the k-T curve needed to be constrained to avoid excessive oscillations. Even with the smoothing of the conductivity function and the known capabilities of the genetic and simulated annealing algorithms to avoid local minimums, the program often terminated in a local minimum. Similar difficulties with the iterative approach have been reported by other researchers (Blackwell and Eldred, 1997).

3.1 Example of Iterative Determination of k-T Curve

As an example of the iterative procedure, the boundary of a rectangular plate test specimen 10.0 cm wide by 1.0 cm long was discretized with 24 linear boundary elements. The ends of the plate were kept at constant temperatures of 100.0 °C and 0.0 °C and the side walls were considered to be adiabatic. Temperatures and heat fluxes were taken from the analytical solution (Chapman, 1960)

$$\frac{\beta}{2}T^{2} + T = \left(T_{hot} + \frac{\beta}{2}T_{hot}^{2}\right) - \left(1 + \frac{\beta}{2}\left(T_{hot} + T_{cold}\right)\right) \times \frac{\left(z_{hot} - z_{cold}\right)}{\left(z_{cold} - z_{hot}\right)}\left(T_{hot} - T_{cold}\right) \tag{9}$$

where the conductivity versus temperature was a linear function.

$$k(T) = k_0\left(1 + \beta\left(T - T_0\right)\right) \tag{10}$$

The results of the BEM analysis using Kirchhoff's transformation compared very well with the analytical solution, averaging an error of less than 0.1% for a wide range of non-linearity parameters $C = k_0\ \beta$ (Dulikravich and Martin, 1996). The three-dimensional BEM analysis averaged an error of less than 0.5%.

The inverse (ill-posed) problem was formulated by over-specifying the entire boundary of the rectangular plate with both temperature and heat flux, while the conductivity function was treated as unknown. The iterative procedure provided good results for cases where the initial guess for the k-T curve was not far from the correct (target) k-T curve. The k-T curve often became very noisy when the optimization started with a poor initial guess or when the target k-T curve was varying significantly. It was typical for the entire iterative algorithm to stall in a local minimum, where the converged k-T curve was far from the target curve. For example, when a constant conductivity $k = k_0$ was used for the initial guess, Figure 1 shows a typical convergence history of the objective function for the simple two-dimensional problem using the β-splines. Notice that the convergence rate is very slow and terminates in a local minimum. In many cases the β-splines produced a tangled k-T curve which caused the failure to achieve the global minimum objective function value of zero.

Actually, the stalling of the program in a local minimum was not due exclusively to the freedom of the β-splines. To illustrate this, the β-splines were removed and the k-T curve was allowed only to vary linearly with different values of β and T_0. Thus, the optimization program needed only to minimize a scalar function of two design variables. The program was again extremely slow in minimizing the objective function and it eventually stalled in a local minimum.

The iterative solution procedure for determining k-T curves was found to produce accurate predictions of the temperature-dependency of thermal conductivity given over-specified temperature and heat flux boundary conditions on the surface of solid objects. Unfortunately, many times the program converged in a local minimum when the initial guess to the k-T curve was a poor estimate of the actual k-T curve. In conclusion, although this approach can be entirely non-intrusive, the authors have concluded that this style of iterative inverse parameter identification is not advisable.

4. NON-ITERATIVE SOLUTION METHOD

In light of the iterative results, the authors have deemed that a more effective solution strategy must be sought out. It was found that a direct, non-iterative and non-intrusive solution strategy is possible using the Kirchhoff transformation and the BEM. This method is valid for arbitrary two- and three-dimensional solid objects. If heat fluxes are known over the entire boundary via measurements taken on the surface of the object, the BEM can be used to solve for the transform of the Kirchhoff heat functions on the boundary. For example, given heat fluxes known on the entire boundary, the linear BEM system appears as follows.

$$[\mathbf{H}]\{\mathbf{U}\}=[\mathbf{G}]\{\mathbf{Q}\}=\{\mathbf{F}\} \tag{11}$$

In this linear algebraic system, the matrices **[H]** and **[G]** are geometrically-dependent and are known. With the application of the heat flux boundary conditions, the matrix **[G]** can be multiplied by the vector **{Q}** to form a vector of known quantities **{F}**. The matrix **[H]** can be inverted using either Gaussian elimination or Singular Value Decomposition (SVD) (Press et al., 1987) so that the values of **{U}** can be obtained at each boundary node. The matrix **[H]** appears to be well conditioned so that regularization methods are not required.

Now that the nodal boundary values of **{U}** are known, the entire field of heat functions is known. Values of the Kirchhoff heat function, u, can be obtained in a post-processing fashion at any interior point. Since the boundary-value problem is over-specified, a number of temperature measurements, taken either non-intrusively on the boundary, or intrusively, at isolated interior points, can be used to convert the heat functions, u, into the corresponding values of thermal conductivity, k. Thus, knowing both the value of u and T at the same point, we can determine the conductivity, k, at that point.

The first difficulty which must be overcome is the fact that the solution to the analysis problem for the heat functions **{U}** is non-unique when only Neumann-type (heat flux) boundary conditions are specified everywhere over the boundary. This is due to the existence of an arbitrary constant in the solution. This constant can be determined by specifying at least one Dirichlet boundary condition. Since the conductivity function is unknown, we can specify a single Dirichlet condition by modifying the Kirchhoff's transformation so that it reads

$$u = u_1 + \int_{T_1}^{T} \frac{k(T)}{k_0} dT \tag{12}$$

Here, k_0 is a reference conductivity value and T_1 is the minimum temperature reading. We can determine the value of u_1, which occurs at the minimum temperature by taking the limit.

$$\lim_{T \to T_1} u = T_1 \tag{13}$$

Thus, $u_1 = T_1$ makes one Dirichlet boundary condition. Now, the BEM can be used to solve for the values of the heat function **{U}** over the entire boundary except at the coordinate of the minimum temperature reading. At this point, the normal derivative $q_1 = (\partial u / \partial n)_1$ is computed since T_1 is specified there.

Given the value of the heat function and temperature at the same point on the boundary, the heat conductivity can be determined at that point via the inverse Kirchhoff's transform which can be evaluated numerically using a variety of integration schemes. The trapezoid rule was our first method of choice.

$$\begin{aligned} u_n &= u_1 + \int_{T_1}^{T} \frac{k(T)}{k_0} dT \\ &= T_1 + \sum_{n=2}^{N} \left(T_n - T_{n-1}\right)\left(\frac{k_n + k_{n-1}}{2k_0}\right) \end{aligned} \tag{14}$$

The values of temperature T_n are known at a finite number of locations. At these points, the values of the heat function u_n are also known. Therefore, the values of conductivity k_n at these points can be determined using the Kirchhoff's transformation. The inverse of the Kirchhoff's transformation can be expressed as a system of algebraic equations represented in the following matrix form

$$[\mathbf{C}]\{\mathbf{K}/k_0\}=\{\mathbf{U}-T_1\} \tag{15}$$

where the elements of the lower-triangular [C] matrix have been determined as follows.

$$C_{i1} = \frac{T_2 - T_1}{2} \qquad \text{when } j = 1 \tag{16}$$

$$C_{ii} = \frac{T_j - T_{j-1}}{2} \qquad \text{when } i = j \tag{17}$$

$$C_{ij} = \frac{T_{j+1} - T_{j-1}}{2} \qquad \text{when } j < I \tag{18}$$

By inverting the [C] matrix, the values of the thermal conductivity can be obtained at the same points where the temperature measurements were taken. The values of the temperature must be sorted in ascending order $\{T_1, T_2, \ldots, T_N\}$ and identical temperature readings must be discarded. In addition, for better accuracy, the temperature measurements should be equally distributed in the temperature range. That is, the difference between two temperature readings T_n and T_{n+1} should be approximately the same. This system represents N-1 equations for N unknowns. The additional equation arises from the knowledge of the conductivity at the minimum temperature point. At this point, $u = T_1$, the following limit also applies.

$$\lim_{T \to T_1} \frac{\partial u}{\partial n} = \left(\frac{\partial T}{\partial n}\right)_1 \qquad (19)$$

Since, at this point, we know both the value of the heat flux, Q_1, from a measurement, and the normal derivative of the heat function, $q_1 = (\partial u / \partial n)_1 = (\partial T / \partial n)_1$ from the BEM solution, the coefficient of heat conductivity at the point of minimum temperature is

$$k_1 = -\frac{Q_1}{q_1} \qquad (20)$$

The trapezoid rule provided good results, but the predicted values of the conductivity were often oscillatory. Simpson's rule of numerical integration was attempted to remove this oscillatory behavior. Specifically, we used

$$u_n = u_1 + \int_{T_1}^{T} \frac{k(T)}{k_0} dT = T_1 + \sum_{n=3}^{N} \left(T_n - T_{n-2}\right)\left(\frac{k_n + 4k_{n-1} + k_{n-2}}{6k_0}\right)$$

for n = 3,5,7,...,N-1. (21)

$$u_n = T_1 + \left(T_2 - T_1\right)\left(\frac{k_2 - k_1}{2k_0}\right) + \sum_{n=4}^{N} \left(T_n - T_{n-2}\right)\left(\frac{k_n + 4k_{n-1} + k_{n-2}}{6k_0}\right)$$

for n = 4,6,8,..,N. (22)

Although the Simpson's rule removed the oscillatory behavior, the k-T curve which it predicted often was very incorrect at the endpoints of the measured temperature range. Instead, very good results were obtained by simply averaging the results predicted by the trapezoid rule alone.

4.1 Regularization Approaches

Regularization was required to properly invert the [C] matrix when random error was introduced into the temperature measurements. In order to evaluate the sensitivity of the algorithm to errors in the measurement data, a random error based on the Gaussian probability density distribution was added to the temperature measurements. A random number $0 < R < 1$ was generated using a standard utility subroutine. The desired variance σ^2 was specified and error was added to the analytical temperature data points T_{an}.

$$T_n = T_{an} \pm \sqrt{-2\sigma^2 \ln R} \qquad (23)$$

The truncated Singular Value Decomposition (SVD) technique (Press et al., 1987) was initially employed to invert the [C] matrix resulting in

$$\left\{K / k_0\right\} = [E]\left[\operatorname{diag}\left(\frac{1}{w_m}\right)\right]\left([D]^T\left\{U - T_1\right\}\right) \qquad (24)$$

where

$$[C] = [D]\begin{bmatrix} w_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & w_N \end{bmatrix}[E]. \qquad (25)$$

In order to determine which singular values were to be truncated, we chose a parameter τ as a singularity threshold. Any singular value whose ratio with the largest singular value was less than this singularity threshold was zeroed out. The zeroing of a small singular value corresponds to throwing away one linear combination from the set of equations that is corrupted by round-off error. The choice of τ was based upon the information about the uncertainty in the BEM matrix computation, the machine's floating point precision, and the standard deviation of the measurement errors in the boundary condition data. In order to zero out a singular value, the associated $1/w_j$ value was replaced by zero.

The predicted values of thermal conductivity at the discrete temperature measurements were improved by using a different type of regularization scheme. Tikhonov's regularization (Tikhonov & Arsenin, 1977) is another type of single-parameter minimization where the solution vector $\{K/k_0\}$ minimizes the weighted sum of the norm of the error vector. A minimum error norm is found by differentiating this equation with respect to each component of the unknown vector $\{K/k_0\}$ and setting the result equal to zero. After substituting the singular value decomposition and solving for the unknown vector, the resulting formulation is as follows,

$$\left\{K / k_0\right\} = [E]\left([w]^T[w] + \lambda[I]\right)^{-1}[w]^T[D]\left\{U - T_1\right\} \qquad (26)$$

where [I] is the identity matrix. Tikhonov's regularization is a generalization of least-squares truncation, but instead of simply eliminating terms associated with small singular values, they are weighted by a factor $(1 + \lambda/w^2)$.

4.2 Results for a Rectangular Plate

Although the non-iterative BEM approach with the Kirchhoff's transform is directly applicable to three-dimensional problems, for the sake of simplicity we will demonstrate this method on a two-dimensional, rectangular plate test specimen 10.0 cm wide by 1.0 cm long. The four sides were discretized with 24 linear boundary elements; ten on each of the horizontal walls and two on each of the vertical walls. Figure 2 illustrates the geometry of the numerical test specimen. The ends of the plate were kept at constant temperatures of 100.0 °C and 0.0 °C and the side walls were considered to be adiabatic. Two different variations of the conductivity versus temperature function were used; the linear function described previously (Eq. 10) and an arctangent function of the type

$$k(T) = \frac{1}{2}(1-\xi)k_{min} + \frac{1}{2}(1+\xi)k_{max} \tag{27}$$

where

$$\xi = \frac{k_{max} - k_{min}}{2\pi}\arctan\left(\delta\frac{(T_{max} + T_{min})}{2}\right) \tag{28}$$

Here, δ is a parameter which sets the slope of the jump in arctangent k-T curve.

The heat fluxes that were supplied as boundary conditions for the BEM solution of Laplace's equation were taken from the well-posed BEM solution where k(T) was known. Temperatures were also taken from the results of the well-posed problem for the over-specified temperature measurements.

4.2.1 Linear Conductivity Variation. The actual variation of conductivity versus temperature was linear between the values of k(T = 0 °C) = 1.0 W/m°C and k(T = 100 °C) = 6.0 W/m °C. The top and bottom walls of the rectangular plate were specified to be adiabatic (Q = 0). The right and left end walls were specified with the heat flux taken from the analytical solution (Q = +/- 35 W/m^2). The BEM solved for the Kirchhoff's heat functions at each of the boundary nodes. These heat functions were converted into values of thermal conductivity at the nodes where the temperatures from the analytical solution were specified. Figure 3 shows the non-iteratively predicted values of thermal conductivity versus temperature. The average error in predicted conductivity from the inverse BEM without input measurement errors was less than 0.1%. Errors in the temperature measurements were then simulated by adding standard deviations of 0.1 °C and 1.0 °C. Figure 4 shows that the computed k-T curve was very good when the temperature measurements were accurate to within 3 significant figures.

4.2.2 Conductivity With Steep Jump. Next, the actual variation of thermal conductivity versus temperature was described by the arctangent function (Eqs. 27 and 28). The top and bottom walls of the rectangular plate were specified to be adiabatic and the right and left walls were specified with the heat flux taken from the well-posed BEM solution (Q = +/-15 W/m^2). The temperature measurements were taken from the well-posed BEM solution and prescribed to the program with varying degrees of error (σ = 0.0, 0.1 and 1.0 °C). Figures 4 and 5 show the computed k-T curves when δ = 0.1 and δ =1.0 in equation (28), respectively. Again, the results are very good when the input temperature measurements have errors with a standard deviation σ of less than 0.1°C.

5. SENSITIVITY TO ERRORS IN HEAT FLUX MEASUREMENTS

In order to evaluate sensitivity of this non-iterative technique to errors in the measured values of surface heat fluxes, we have introduced different levels of random error in the surface heat fluxes for the test case with linear conductivity variation described above. Specifically, the exact value of constant heat flux on the two opposite ends of the square plate were Q = +/- 35 W/m^2. Random errors of Q were introduced that had varying degrees of standard deviation (σ = 0.0, 0.1, 1.0, and 3.5 W m^{-2}). The resulting values of temperature-dependent thermal conductivity are shown in Figure 8. By comparing Figures 3 and 8 it may be concluded that this non-iterative inverse methodology is less sensitive to measurement errors of heat fluxes than it is to measurement errors in surface temperatures.

6. CONCLUSIONS

The BEM has been used to non-iteratively predict the temperature variation of thermal conductivity given over-specified thermal boundary conditions. The procedure is entirely non-intrusive and non-destructive. It requires the knowledge of the heat flux over the entire surface of an arbitrarily-shaped two- or three-dimensional solid object. Steady-state temperature measurements are also required on the over-specified part of the surface of the solid. The algorithm can rapidly predict the values of the thermal conductivity that correspond to the measured temperatures. The non-iterative inversion of the inverse Kirchhoff's transformation is very accurate when the errors in the surface temperature measurements are less than 0.1% of the maximum reading. The accuracy of the algorithm is improved when the temperatures are evenly distributed, that is, when the difference between two thermocouple readings sorted in an ascending order are approximately equal. The non-iterative solution procedure was compared to the more common iterative minimization techniques. The non-iterative BEM approach was found to be much faster, as well as being more reliable, than the iterative approach. The entire non-iterative inverse algorithm required only several seconds on a personal computer for test cases shown.

7. ACKNOWLEDGMENTS

The authors are grateful for the NASA-Penn State Space Propulsion Engineering Center Graduate Student Fellowship, the National Science Foundation Grant DMI-9522854 monitored by Dr. George Hazelrigg, the NASA Lewis Research Center Grant

NAG3-1995 supervised by Dr. John Lytle, and for ALCOA Faculty Research Fellow Award administered by Dr. Yimin Ruan.

8. REFERENCES

Artyukhin, E. A., June 1993, "Iterative Algorithms for Estimating Temperature-Dependent Thermophysical Characteristics", *Proceedings of Inverse Problems in Engineering: Theory and Practice,* Editor: N. Zabaras, Palm Coast, FL, pp. 101-108.

Barsky, B. A., 1988, *Computer Graphics and Geometric Modeling Using Beta-Splines*, Springer-Verlag, Berlin, Germany.

Blackwell, B. F. and Eldred, M. S., 1997, "Application of Reusable Interface Technology for Thermal Parameter Estimation", ASME National Heat Transfer Conference, *Symposium on Inverse Design Problems in Heat Transfer and Fluid Flow*, Editors: Dulikravich, G. S. and Woodbury, K. A., August 10-12, 1997, Baltimore, MD

Brebbia, C. A. and Dominguez, J., 1989, *Boundary Elements, An Introductory Course,* McGraw-Hill Book Company, New York.

Chapman, A. J., 1960, *Heat Transfer*, McMillan Co., New York.

Dantas, L. B. and Orlande, H. R. B., 1996, "A Function Estimation Approach for Determining Temperature-Dependent Thermophysical Properties", *Inverse Problems in Engineering*, Vol. 3, No. 4, pp. 261-280.

Dulikravich, G. S. and Martin, T. J., 1996, "Inverse Shape and Boundary Condition Problems and Optimization in Heat Conduction", Chapter 10 in *Advances in Numerical Heat Transfer*, Editors: W. J. Minkowycz and E. M. Sparrow, Taylor & Francis, pp. 324-367.

Eckert, E. R. G. and Drake, R. M., 1972, *Analysis of Heat and Mass Transfer,* McGraw-Hill Book Company, New York.

Fletcher, R. and Powell, M .J. D., 1963, "A Rapidly Convergent Descent Method for Minimization", *Computer Journal*, Vol. 6, pp. 163-168.

Foster, N. F. and Dulikravich, G. S., 1997, "Three-Dimensional Aerodynamic Shape Optimization using Genetic Evolution and Gradient Search Algorithms", *AIAA Journal of Spacecraft and Rockets*, December-February 1997, Vol. 33, No. 3. pp. 33-33; also AIAA Paper 96-0555, AIAA Aerospace Sciences Meeting, Reno, NV, January 1996.

Goldberg, D. E., 1989, *Genetic Algorithms in Search, Optimization and Machine Learning,* Addison-Wesley.

Nelder, J. A. and Mead, R., 1965, "A Simplex Method for Function Minimization", *Computer Journal*, Vol. 7, pp. 308-313.

Martin, T. J. and Dulikravich, G. S., August 1996, "Inverse Determination of Boundary Conditions in Steady Heat Conduction with Heat Generation", *ASME Journal of Heat Transfer*, Vol. 118, No. 3, pp. 546-554.

Orlande, H. R. B. and Ozisik, M. N., 1994, "Determination of the Reaction Function in a Reaction-Diffusion Parabolic Problem", *ASME Journal of Heat Transfer,* Vol. 116, pp. 1041-1044.

Press, W. H, Teukolsky, S. A., Vetterling, W. T. and Flannery, B. P., 1986, *Numerical Recipes in FORTRAN, The Art of Scientific Computing*, 2nd Edition, Cambridge University Press, Cambridge.

Tikhonov, A. N. and Arsenin, V. Y., 1977, *Solutions of Ill Posed Problems*, V. H. Wistom & Sons, Washington, D.C.

Vanderplaats, G. N., 1984, *Numerical Optimization Techniques for Engineering Design*, McGraw-Hill, New York.

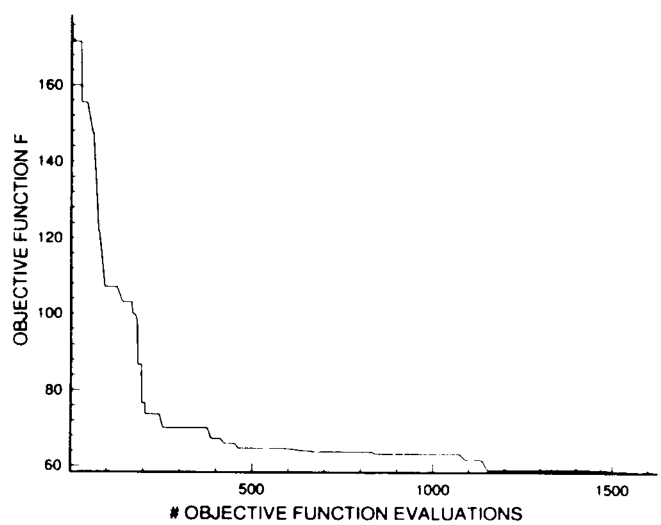

Figure 1. Convergence history of the objective function for the iterative method of predicting temperature-dependency of thermal conductivity.

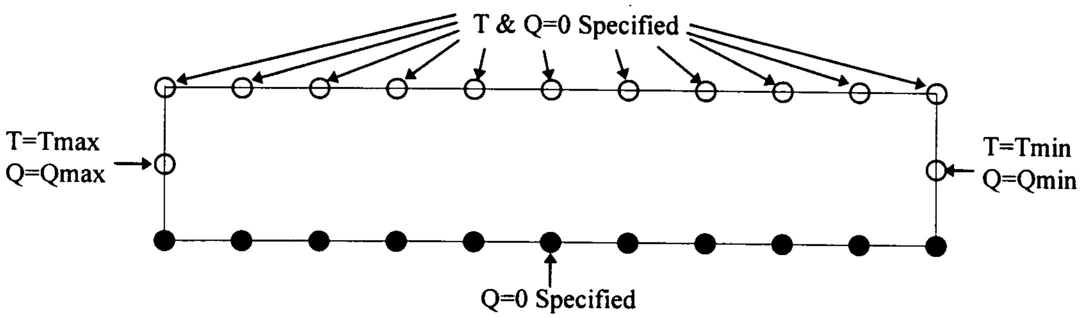

Figure 2. Rectangular plate test specimen and boundary discretization.

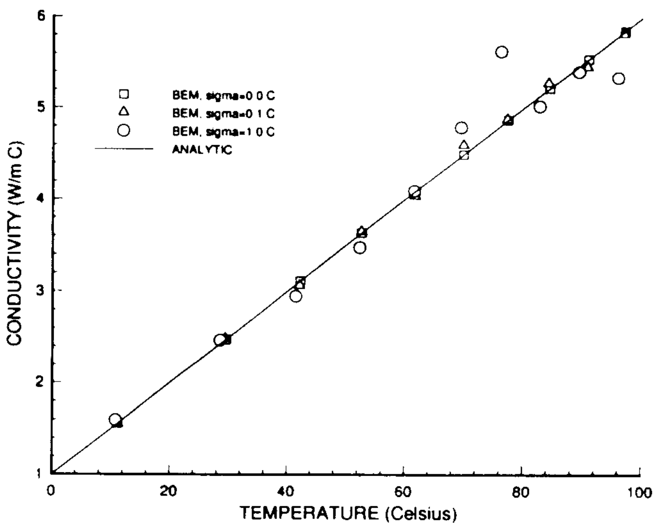

Figure 3. Variation of the thermal conductivity versus temperature for different levels of measurement error in surface temperature. The BEM results are compared to the analytical solution where the conductivity versus temperature function is linear and $\beta = 0.05\ ^{o}C^{-1}$, $T_0 = 0\ ^{o}C$ and $k_0 = 1.0$ W/m^{o}C.

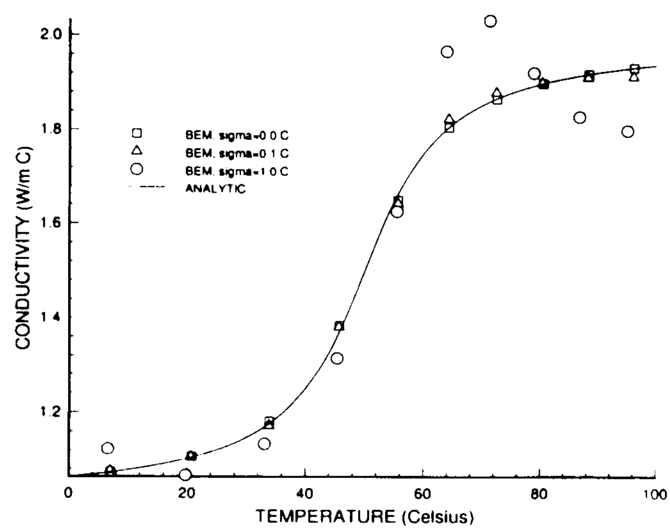

Figure 4. Variation of the thermal conductivity versus temperature for different levels of measurement error. The BEM results are compared to the analytical solution where the conductivity versus temperature function was an arctangent with $\delta = 0.1\ ^{o}C^{-1}$. An example with a small number of surface heat flux measurements.

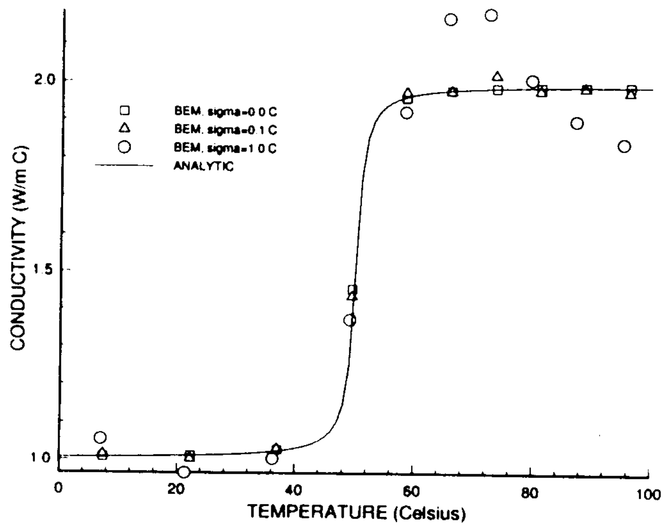

Figure 5. Variation of the thermal conductivity versus temperature for different levels of measurement error. The BEM results are compared to the analytical solution where the conductivity versus temperature function was an arctangent with $\delta = 1.0\ ^oC^{-1}$. An example with a small number of surface heat flux measurements.

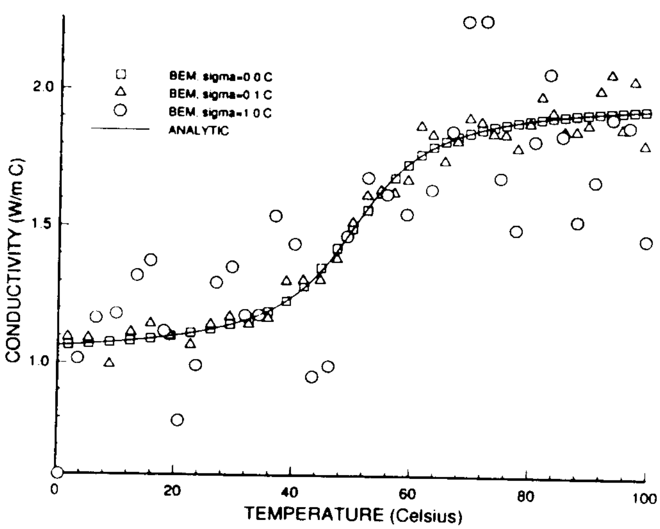

Figure 6. Variation of the thermal conductivity versus temperature for different levels of measurement error. The BEM results are compared to the analytical solution where the conductivity versus temperature function was an arctangent with $\delta = 0.1\ ^oC^{-1}$. An example with a large number of surface heat flux measurements.

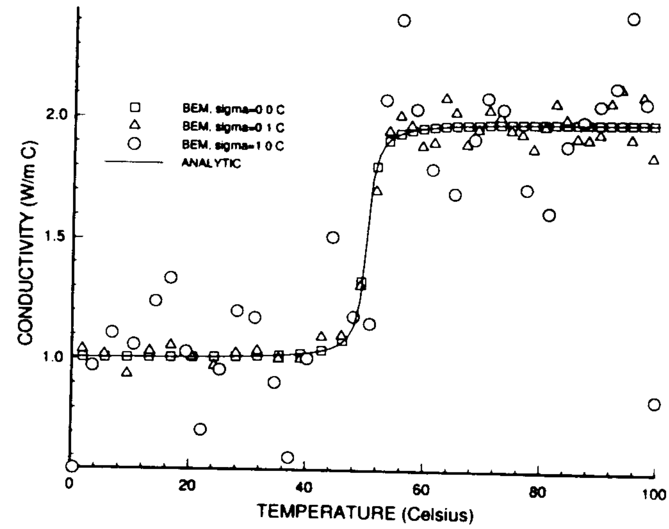

Figure 7. Variation of the thermal conductivity versus temperature for different levels of measurement error. The BEM results are compared to the analytical solution where the conductivity versus temperature function was an arctangent with $\delta = 1.0\ ^oC^{-1}$. An example with a large number of surface heat flux measurements.

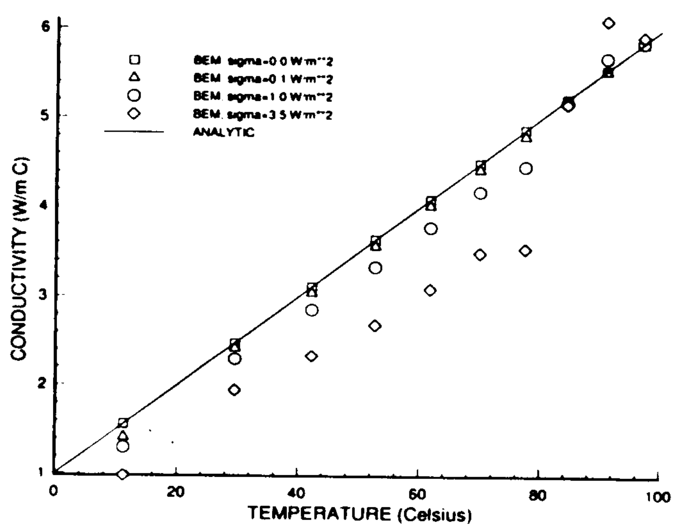

Figure 8. Variation of the thermal conductivity versus temperature for different levels of measurement error in heat fluxes. The BEM results are compared to the analytical solution where the conductivity versus temperature function is linear and $\beta = 0.05\ ^oC^{-1}$, $T_0 = 0\ ^oC$, $k_0 = 1.0$ W/m^{o}C and exact Q = +/-35 W m^{-2}.