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DETERMINATION OF THE PROPER NUMBER, LOCATIONS, SIZES AND SHAPES OF SUPERELLIPTIC COOLANT FLOW PASSAGES IN TURBINE BLADES

George S. Dulikravich and Thomas J. Martin
Associate Professor Graduate Assistant
Department of Aerospace Engineering, 233 Hammond Building
The Pennsylvania State University, University Park, PA 16802, USA

ABSTRACT

During the past several years we have developed an inverse design method that allows a thermal cooling system designer to determine the minimum proper number, and correct sizes, shapes, and locations of coolant passages (holes) in an internally cooled configuration. The methodology has been successfuly demonstrated on two-dimensional coated and non-coated turbine blade airfoils, scram jet combustor struts, etc. Using this method the designer can enforce the desired temperature and the desired heat flux distribution on the hot outer surface of the blade, while simultaneously enforcing the desired temperature distributions on the cooled interior surfaces of each of the coolant passages by automatically relocating, resizing, reshaping and reorienting the initially guessed holes. Hole shapes were allowed to vary according to an inclined Lame family of shapes. The designer only needs to guess the number, sizes, shapes, locations and orientations of the holes. Afterwards, the design process does not require any intervention on the part of the designer since it uses an automatic constrained optimization algorithm to minimize the difference between the specified and intermittently computed outer hot surface heat fluxes. Local minimas in the optimization process were successfully avoided by changing the formulation for the objective function whenever the local minimas were detected. All unnecessary holes were automatically eliminated, while honoring the specified minimal distances between the neighboring holes.

INTRODUCTION

When analyzing the hot gas flow field outside of a turbine blade, we usually specify either the blade hot surface temperature, T_{hot}^{spec}, or its surface heat flux, q_{hot}^{spec}, as one of the solid wall boundary conditions in order to account for the heat transfer between the blade and the hot gas. As a by-product of the hot gas flow field computation, we obtain the corresponding calculated heat flux, q_{hot}^{calc}, at the hot surface (if T_{hot}^{spec} was specified there) or we obtain the corresponding calculated temperature, T_{hot}^{calc}, at the hot surface (if q_{hot}^{spec} was specified there). Nevertheless, these calculated (or experimentally obtained) values for q_{hot}^{calc} (or T_{hot}^{calc}) are generally not compatible with the desired hot surface heat flux (or temperature). There are only two conceptual approaches to simultaneously achieving both desired hot surface thermal boundary conditions: either by optimizing the variation of thermal properties throughout the blade airfoil material of a given and fixed size and shape, or by allowing the coolant passages (holes) to be resized, reshaped, and relocated. The first option does not seem to be economically viable and readily physically realisable. Instead, the geometric shape optimization of the coolant passages (holes) represents the feasible approach to the thermal design of internally cooled turbine blades.

THEORETICAL FORMULATION

The inverse design method described in this paper deals strictly with steady heat conduction inside the thermally isotropic solid material of an internally cooled blade. Thermal expansion of the solid material and thermal stresses have not been included in the present work. The blade can be ceramically coated or otherwise made of regions having different thermal diffusivities (Chiang and Dulikravich, 1986). Then, Laplace's equation for the temperature, T, governs the temperature field in the solid

$$\nabla^2 T = 0. \tag{1}$$

The classical boundary value problem would be the one with a Dirichlet boundary condition meaning that we could specify the desired values of the temperatures, T_{hot}^{spec}, on a point-by-point basis along the entire outer hot surface of the turbine blade airfoil having a known and fixed geometry and at the same time specify the desired temperature, T_{cold}^{spec}, along the cold surfaces of the cooling passages (holes). This would be a rudimentary example of a well-posed elliptic problem of a Dirichlet type. Any version of a finite difference or a finite element temperature analysis code would require a considerable amount of computer time even in the case of a blade airfoil with a single hole. Obviously, only an accurate form of a boundary element method would be fast, reliable and versatile enough (Brebbia and Dominguez, 1989) to provide an economically feasible design procedure. We have performed the temperature field analysis using our accurate boundary integral element code with linearly varying temperature along surface panels.

In addition to the freedom of choosing the specified hot surface temperature distribution, T_{hot}^{spec}, we allow the designer to specify also the desired hot surface heat flux, q_{hot}^{spec}, on a point-by point basis. This combination of T_{hot}^{spec} and q_{hot}^{spec} represents an overspecified boundary value problem, meaning that, in general, it has no solution except, possibly, for a very special configuration of holes. In our past efforts we have demonstrated the ability to allow completely arbitrary shapes of single holes (Kennon and Dulikravich, 1985), multiple holes (Kennon and Dulikravich, 1986a), holes with partially fixed walls (Kennon and Dulikravich, 1986b), and circular holes (Chiang and Dulikravich, 1986). These efforts and inverse problems in unsteady heat conduction were summarized by Dulikravich (1988). Automatic elimination of unnecessary holes was for the first time demonstrated recently by Dulikravich and Kosovic (1991).

Since arbitrarily shaped holes are usually hard to manufacture, while the circular holes make for a very heavy blade, in this work we will allow hole shapes to belong to a general family of Lame curves (superelliptic functions). This means that we will describe each hole shape (Dulikravich, 1992; Dulikravich and Martin, 1992) and location via an equation of the type

$$((x - x_{0i})/a_i)^{n_i} + ((y - y_{0i})/b_i)^{n_i} = 1 \tag{2}$$

Here, for a given hole, i, x_{0i} and y_{0i} are the Cartesian coordinates of the hole center, a_i and b_i are the two semi-axis of the hole contour, and the exponent n_i can vary from $n_i<1$ (a four-pointed star), to $n_i=1$ (a diamond shape), to $n_i=2$ (a circle or an ellipse), to $n_i>>1$ (a square or a rectangle). Moreover, the local x,y Cartesian coordinate system can be inclined at an arbitrary angle α_i with respect to the global x',y' Cartesian coordinate system. After the desired surface temperatures T_{hot}^{spec} and T_{cold}^{spec} have been specified, the designer is free to guess the number of holes, N. Thus, per each of the N coolant passages (holes) we have to optimize the following six parameters: x_{0i}, y_{0i}, a_i, b_i, n_i , α_i in addition to the task of minimizing the total number of holes required. The Laplace's equation can be integrated numerically in this multiply connected domain subject to specified surface temperatures T_{hot}^{spec} and T_{cold}^{spec}. The solution of the Laplace's equation for heat conduction in the solid material will contain also the corresponding computed heat conduction fluxes including the values of the computed heat fluxes at the hot surface, q_{hot}^{calc}. Since we have integrated the Laplace's equation for the guessed values of N, x_{0i}, y_{0i}, a_i, b_i, n_i, and α_i, the computed hot surface heat flux q_{hot}^{calc} will not be the same as the desired hot surface heat flux, q_{hot}^{spec}. This difference in the computed hot surface heat fluxes and the specified hot surface heat fluxes can be either positive or negative. The objective is to minimize the squared and properly normalized difference of these fluxes by iteratively changing the locations of the centers of the holes, x_{0i}, y_{0i}, sizes of the holes, a_i, b_i, shapes of the holes, n_i , and orientations of the holes, α_i. That is, the objective function that needs to be minimized is the integrated and normalized global L_2 norm of the differences in the two hot surface heat fluxes.

$$F = \frac{\sum_i (q_{hot}^{spec} - q_{hot}^{calc})_i^2}{\sum_i (q_{hot}^{spec})_i^2 + \varepsilon} \times 100 \tag{3}$$

Here, ε is a very small number which should eliminate the singular behavior of F when $\sum (q_{hot}^{spec})_i^2 = 0$. An alternative objective function could be understood as the sum of the normalized local L_2 norms of the differences in the hot surface heat fluxes, that is

$$F = \sum_i \left\{ \frac{(q_{hot}^{spec} - q_{hot}^{calc})^2}{(q_{hot}^{spec})^2 + \varepsilon} \right\}_i \times 100 \tag{4}$$

Besides minimizing the heat flux error on the outer surface, the final configuration has to satisfy two constraints (Chiang and Dulikravich, 1986): a) specified minimum allowable distance between any two holes, d^{hole}, having radii r_i and r_k, and b) specified minimum allowable distance between any hole and the outer boundary, d^{surf}. The two constraints were incorporated into the objective function using a barrier function (Chiang and Dulikravich, 1986)

$$B(g(x),w_b) = \frac{1}{w_b} \sum_{i=1}^{N} \left[\sum_{j=1}^{M} \frac{d^{surf}}{\left(D_j^{surf} - d^{surf} - r_i\right)} + \sum_{k=1}^{N} \frac{d^{hole}}{\left(D_k^{hole} - d^{hole} - r_i\right)} \right] \tag{5}$$

Here, N is the total number of holes and w_b is the user specified barrier coefficient which was increased as the process kept converging. Thus, the composite objective function can have two forms depending on whether the global or local objective function is used for its evaluation.

$$F_m(g(x),w_b) = F_m(x) + B(g(x),w_b) \qquad m = 1, 2 \qquad (6)$$

It is also worth noting that casting core geometric constraints encountered in manufacturing such passages can be often accomodated by fixing one or more of the six optimization variables per hole. The minimization of F can be performed using any of a number of standard optimization algorithms. We chose to use a standard gradient search minimization algorithm of Davidon-Fletcher-Powell (Vanderplaats, 1984). All six optimization variables per each hole were optimized simultaneously. In order to find the gradient components, the variables were individually perturbed by the relative magnitude of 1.0E-6. The algorithm required on average between 15 and 40 complete solutions of the Laplace's equation per each of the $N \times (x_{0i} + y_{0i} + a_i + b_i + n_i + \alpha_i)$ design variables per each optimization cycle. Starting with a large number of arbitrarily located, sized, shaped and oriented holes, the optimization process is fully automatic and results in continuous reduction in sizes of all unnecessary holes until they were eliminated when their sizes reduced below a prespecified minimal allowable value.

RESULTS

A simple test case with a known analytical solution consisting of a disk shaped domain with a thick coating of another solid material was used to verify the accuracy of the design algorithm. The disk had a large centrally located circular hole (Fig. 1). For constant temperatures T_{hot}^{spec} and T_{cold}^{spec} on the outer (disk) boundary and the inner (hole) boundary, respectively, the analytical value of the heat flux on the outer boundary , q_{hot}^{spec}= const., is easily found. Thus, the desired solution is a circular disk with a centrally located circular hole having T_{cold}^{spec} on the surface of the hole, while having T_{hot}^{spec} and q_{hot}^{spec} on the outer surface.

Case 1: We attempted to satisfy these overspecified thermal boundary conditions by guessing that the hole should be elliptic, that is, of the Lame type (Table 1), located off the center and inclined at an angle with respect to the x-axis. Using our inverse design methodology, the analytical value of the outer surface heat flux, q_{hot}^{spec}= const., was enforced until the initially guessed hole has transformed into the correct solution represented by a centrally located circular hole. The process took 10 optimization cycles (Fig. 2) requiring 456 temperature field analysis calls and 410 seconds of CPU time on an IBM 3090. During the process the normalized global error in the outer surface heat flux reduced from 14% to 0.1%.

Hole	a	b	n	xo	yo	α
1	0.5	0.25	4.5	0.15	0.15	30

TABLE 1. Case no.1: Geometric Input Parameters

Case 2: Using the same thermal boundary conditions, instead of one centrally located circular hole we have then guessed three holes (Fig. 3) of elliptic, almost square, and almost rectangular shapes (Table 2), respectively. We used 36 panels on the outer surface of the unit disk and 16 panels on the surface of each hole. Using our inverse design methodology, the sizes of two holes have been iteratively reduced to zero (Figs. 3), while the third hole was enlarged and became practically indistinguishable from the analytically correct centrally located single circular hole. The convergence history of the optimization process (Fig. 4) indicates that this test case needed 55 optimization cycles. This required 1,226 calls to the temperature field analysis subroutine which amounts to 1,179 CPU seconds on IBM 3090 in order to reduce the global L_2 norm of the surface error in heat flux to 0.1%. Notice that this test case converged much slower and that it required switching between the two formulations of the objective function, F, in order to circumvent the local minimas.

Hole	a	b	n	xo	yo	α
1	0.5	0.25	2.0	-0.3	0.3	45.0
2	0.3	0.3	1.0	0.5	0.5	0.0
3	0.4	0.2	4.0	0.0	-0.5	0.0

TABLE 2. Case no.2: Geometric Input Parameters

Case 3: Next example involved an application of these techniques to a noncircular domain. The temperature field inside a non-coated turbine blade airfoil of a realistic shape (Fig. 5) having five different superelliptic holes was first obtained using our boundary element analysis code. The holes were of circular, elliptic, and almost rectangular shape, respectively (Table 3). The calculated outer surface heat flux (Fig. 5) was then used as the desired or specified value q_{hot}^{spec} in the optimization process, while guessing that there should be five circular (Fig. 6) holes (Table 4). The hot surface temperature T_{hot}^{spec} and the cold surface temperatures T_{cold}^{spec} of the actual five superelliptic holes were enforced. Outer surface of the blade was discretized with 49 panels that were clustered with respect to the leading and trailing edge of the blade. Each of the five holes were discretized with 16 panels that were automatically reclustered after each optimization cycle. After 277 optimization cycles (Figs. 7 and 8) requiring 8,755 calls to the temperature field analysis code, the integrated normalized hot surface heat flux error reduced from 82% to 4.2% in 36,780 CPU seconds. Notice that this test case involved frequent automatic switching between the two formulations for the objective function. Also, we made use of an exponential spline fitting and interpolation algorithm (Dulikravich, 1988) to find optimal values of the directional search parameter.

Hole	a	b	n	xo	yo	α
1	0.025	0.025	2.0	0.1	1.15	0.0
2	0.025	0.025	2.0	0.2	1.20	0.0
3	0.025	0.025	2.0	0.3	1.225	0.0
4	0.025	0.025	2.0	0.4	1.20	0.0
5	0.025	0.025	2.0	0.5	1.05	0.0

TABLE 3. Case no.3: Correct (Output) Geometric Parameters

Hole	a	b	n	xo	yo	α
1	0.05	0.05	2.0	0.1	1.175	0.0
2	0.075	0.075	2.0	0.275	1.225	0.0
3	0.05	0.05	2.0	0.45	1.15	0.0
4	0.10	0.05	2.0	0.575	0.90	-60.
5	0.10	0.025	8.0	0.675	0.65	-63.45

TABLE 4. Case no.3: Geometric Input Parameters

Case 4: The final example involved a partially constrained configuration of holes in a coated turbine blade airfoil. The initial guess consisted of differently sized and positioned (Fig. 9) rectangular holes (Table 5), except that the correct answer this time was the temperature field that corresponds to three differently sized, located and inclined (Fig. 9) unequal rectangular holes . The constraint was that the semi-axis and the Lame exponents were kept fixed for each of the holes throughout the optimization process (Table 6). The correct configuration (Fig. 9) was obtained after a relatively smooth optimization requiring 41 cycles (Fig. 10) which consumed 1,219 CPU seconds on an IBM 3090 and required 925 calls to the temperature field analysis routine.The integrated flux error was reduced from 30% to 0.01%.

Hole	a	b	n	xo	yo	α
1	0.5	0.75	12.0	3.5	12.25	15.0
2	0.5	1.0	12.0	8.0	13.0	0.0
3	0.5	0.75	12.0	6.75	13.0	-30.0

TABLE 5. Case no.4: Geometric Intput Parameters

Hole	a	b	n	xo	yo	α
1	0.5	1.0	12.0	4.0	13.0	0.0
2	0.5	1.5	12.0	5.5	13.0	-15.0
3	0.5	1.0	12.0	7.0	12.0	-30.0

TABLE 6. Case no.4: Correct (Output) Geometric Parameters

DISCUSSION

In any problem of the inverse type, there is a danger of accepting inadequately converged solutions. Chiang and Dulikravich (1986) have illustrated this point by terminating the iterative process in the case similar to Case no. 1 after only six iterations when the outer surface heat flux normalized error was seemingly acceptably low (1.868%). The sizes and locations of the three holes were clearly unacceptable. Thus, only a highly accurate analysis algorithm and a highly accurate optimization algorithm should be used in this inverse design procedure until a convergence level of less than 0.1% relative error is achieved.

In the test cases no. 1 and 4 the normalized thermal diffusivity coefficients for the outer region (coating) and the inner region were 1 and 5, respectively. Thus, the coating acted as a temperature gradient smoothing device creating a much smoother temperature and heat flux distributions at the coating/metal interface. On the other hand, in the test cases no.2 and 3 the thermal coating was non-existant so that the sharp variations of the temperature and the heat fluxes specified on the outer surface did not smooth much through the material. Consequently, the test cases no. 2 and 4 converged much faster than the other test cases.

We allow the designer to initially guess an arbitrarily large number of holes whereby all holes, except those that are absolutely necessary, should iteratively shrink to zero size. When the size of a hole becomes very small, its influence on the solution becomes negligible especially if it is far away from the boundaries. Consequently, the hole elimination-by-shrinking procedure is extremely time consuming and often terminates in a local minima (Dulikravich and Kosovic, 1991). Instead of wasting the computational time on recomputing the entire solution including these very small holes and further optimizing their sizes and locations, we explicitly eliminate any hole that reduced beyond a prespecified minimal size.

Whenever the convergence rate reduces to zero, this indicates that the proces has terminated in a local minima. In such situations we automatically switch from the integrated L_2 norm error (Eq. 3) to local L_2 norm error (Eq. 4). This sudden change in the way that the objective function is evaluated causes the solution to depart from a local minima (Fig. 8) and continue on its search for a global minima. Since these types of inverse design problems seem to involve very narrow minimas (Dulikravich, 1988), most of the standard optimization algorithms are unable of detecting them. To find such narrow global minimas automatically and reliably, we have incorporated our algorithm (Dulikravich, 1988) based on an exponential spline fitting and interpolation of the gradient search parameter.

The coolant flow passage wall temperature, T_{cold}^{spec}, cannot always be specified. It is limited by the geometry and the coolant flow field. Thus, this inverse design method should be coupled with the coolant flow field computations which is possible only in three dimensions.

SUMMARY

A technique for performing an automatic determination of proper sizes and locations of multiple coolant flow passages has been augmented by allowing the designer to also freely guess the number of holes needed. The algorithm automatically eliminates any guessed hole if it reduces in size below a user-specified minimum value. This feature allows even unexperienced designers to create a good thermal design in a single computer run. Local minimas in the minimization process are automatically avoided by switching from an integrated L_2 norm to a local L_2 norm of the hot surface heat flux error. Hole configurations are allowed to belong to a general family of inclined superelliptic (Lame) shapes. Manufacturing constraints are inforced using variable barrier function formulation. Narrow global minimas during the optimization process are detected automatically by utilizing an algorithm based on exponential spline fitting and interpolation of gradient search parameters.

REFERENCES

1. Brebbia, C.A. and Dominguez, J., Boundary Elements: An Introductory Course, McGraww-Hill Book Company, 1989.

2. Chiang, T.L. and Dulikravich, G.S., Inverse Design of Composite Turbine Blade Circular Coolant Flow Passages, ASME Journal of Turbomachinery, vol. 108, pp. 275-282, 1986.

3. Dulikravich, G.S., Inverse Design and Active Control Concepts in Strong Unsteady Heat Conduction, Applied Mechanics Reviews, vol. 41, No. 6, June 1988, pp. 270-277, 1988.

4. Dulikravich, G.S., Inverse Design of Proper Number, Shapes, Sizes and Locations of Coolant Flow Passages, Proceedings of the 10th Annual CFD Workshop, Editor: R. Williams, NASA MSFC, Huntsville, AL, April 28-30, 1992.

5. Dulikravich, G.S. and Kosovic, B., Minimization of the Number of Cooling Holes in Internally Cooled Turbine Blades, ASME paper 91-GT-103, ASME Gas Turbine Conf., Orlando, FL, June 2-6, 1991; Internat. Journal of Turbo & Jet Engines, 1992.

6. Dulikravich, G.S. and Martin, T.J., Determination of Void Shapes, Sizes and Locations Inside an Object With Known Surface Temperature and Heat Flux, Proceedings of the IUTAM Symposium on Inverse Problems in Engineering Mechanics, Editors: M. Tanaka and H.D. Bui, Tokyo, Japan, May 11-15, 1992.

7. Kennon, S.R., and Dulikravich, G.S., The Inverse Design of Internally Cooled Turbine Blades, ASME J. of Eng. for Gas Turbines and Power, pp. 123-126, 1985.

8. Kennon, S.R. and Dulikravich, G.S., Inverse Design of Multiholed Internally Cooled Cooled Turbine Blades, International Journal of Numerical Methods in Engineering, vol. 22, pp. 363-375, 1986a.

9. Kennon, S.R. and Dulikravich, G.S., Inverse Design of Coolant Flow Passages Shapes With Partially Fixed Internal Geometries, International Journal of Turbo & Jet Engines, vol. 3, (1), pp. 13-20, 1986b.

10. Vanderplaats, G.N., Numerical Optimization Techniques for Engineering Design, McGraw-Hill, New York, 1984.

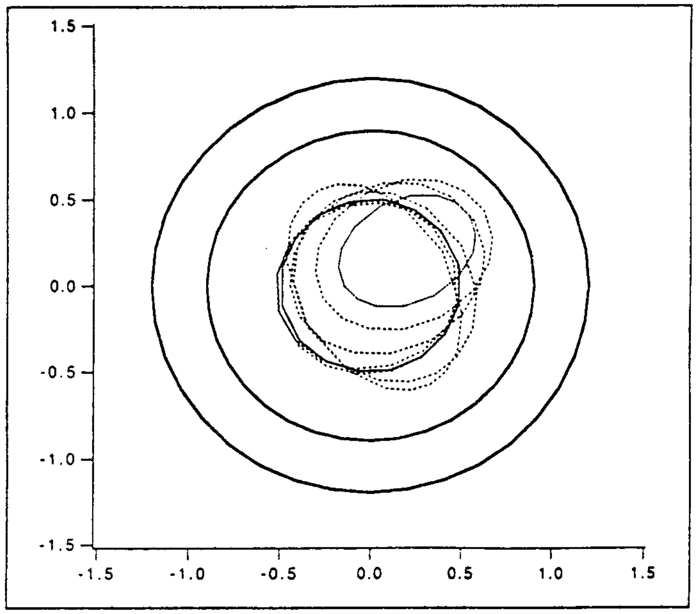

FIGURE 1. Case 1 - Coated circular disk: initial configuration consisting of an off-center, inclined, almost rectangular hole (designated with a full line) and optimized configuration (one large centrally located circular hole designated with a dashed line) for one-hole coated disk with intermediate hole shapes.

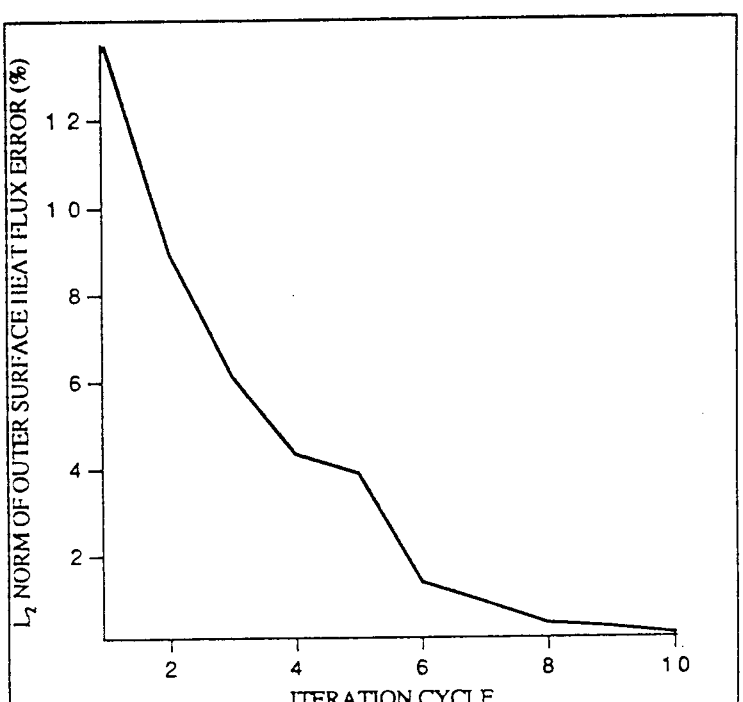

FIGURE 2. Case 1 - Coated circular disk: convergence history (percentage of the integrated heat flux error on the outer boundary versus optimization cycles) for one-hole configuration.

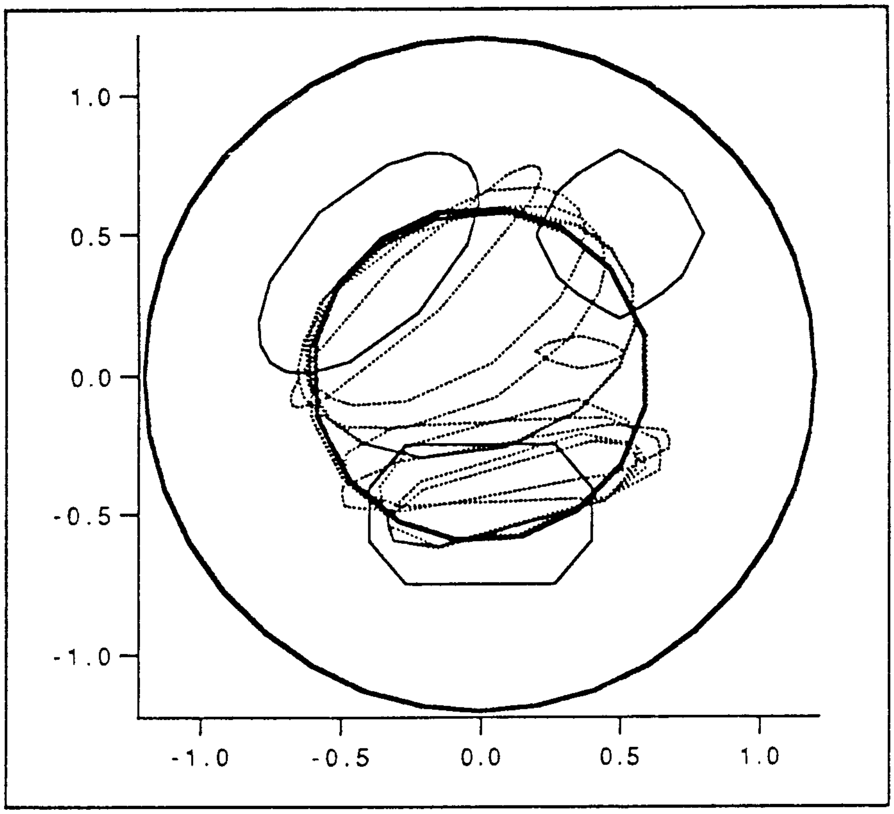

FIGURE 3. Case 2 - Non-coated circular disk: initial configuration consisting of three different superelliptic holes (solid lines), their intermediate shapes (dashed lines), and the final centered circular hole (thick line). Two unnecessary holes were reduced to zero size

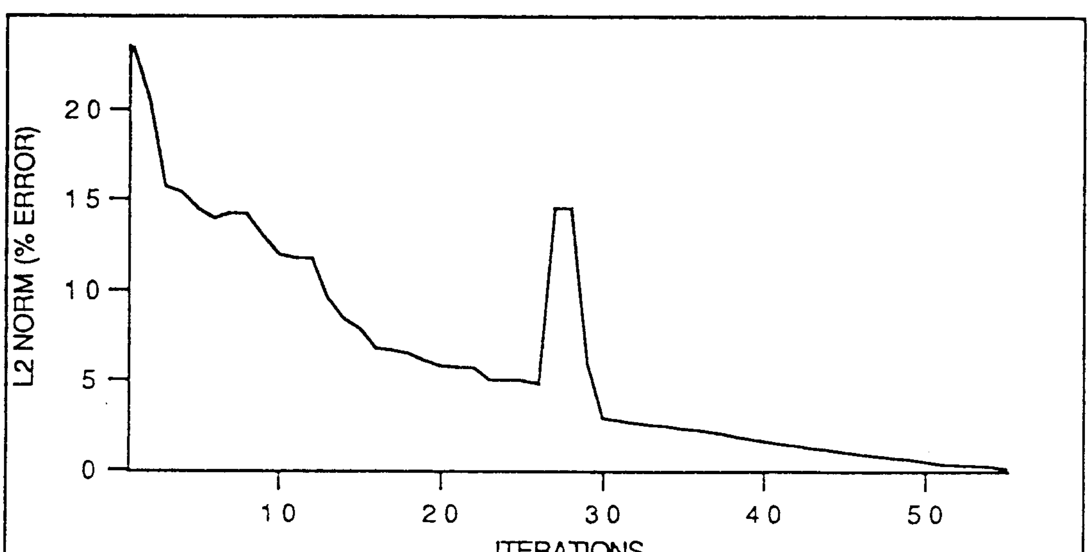

FIGURE 4. Case 2 - Non-coated circular disk: convergence history (percentage of the integrated heat flux error on the outer boundary versus optimization cycles) for the case with initially three-holes.

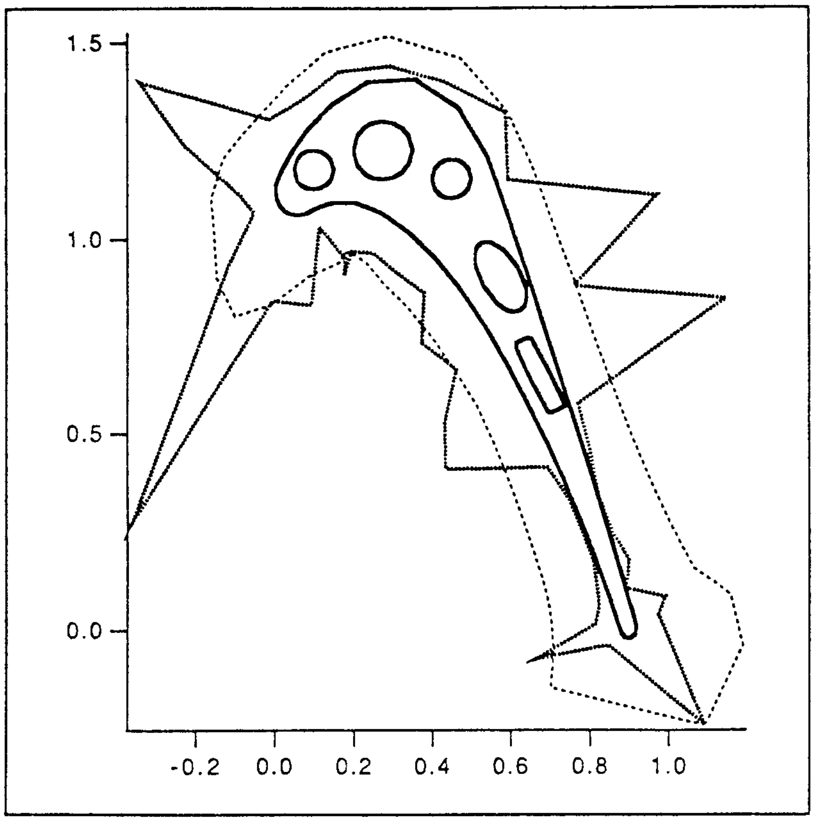

FIGURE 5. Case 3 - Turbine blade: specified temperature (dashed line) and heat influx (dotted line) distributions on the blade surface and the corresponding five-hole configuration.

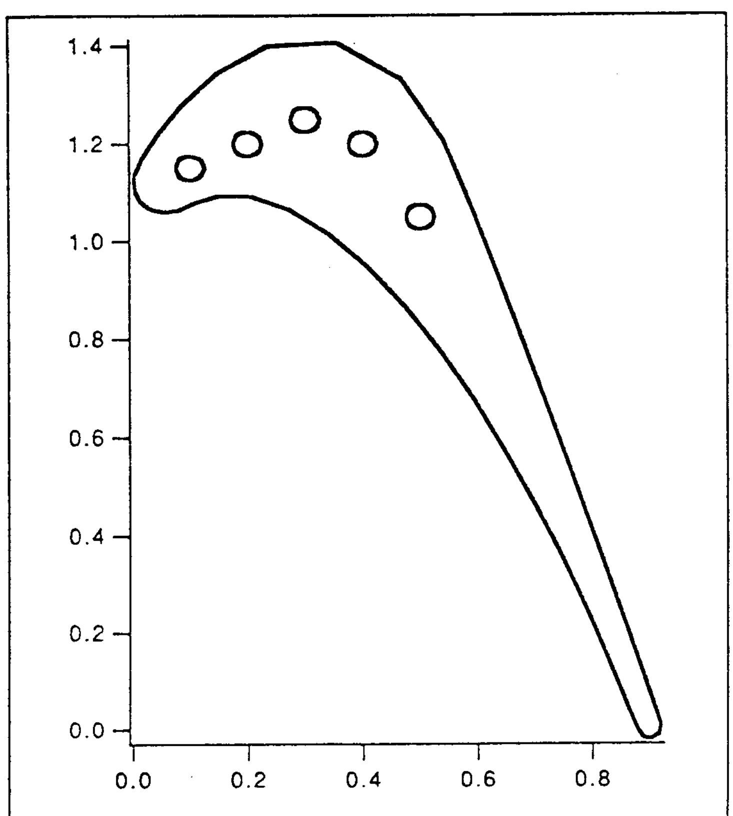

FIGURE 6. Case 3 - Turbine blade: initial configuration having five circular holes.

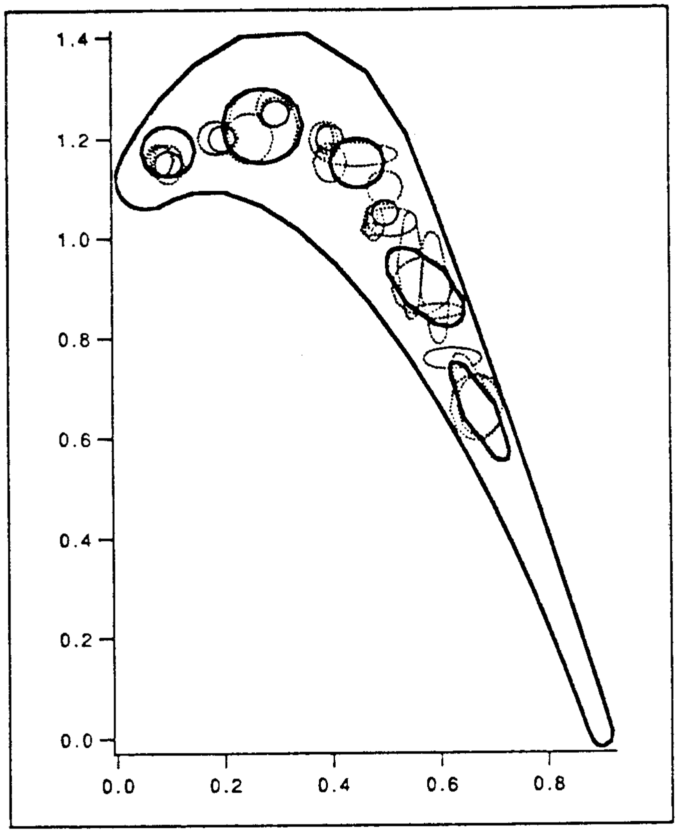

FIGURE 7. Case 3 - Turbine blade: initial configuration consisting of five circular holes (thin solid lines), their intermediate superelliptic shapes (dashed lines), and the final configuration consisting of five superelliptic holes (thick solid lines).

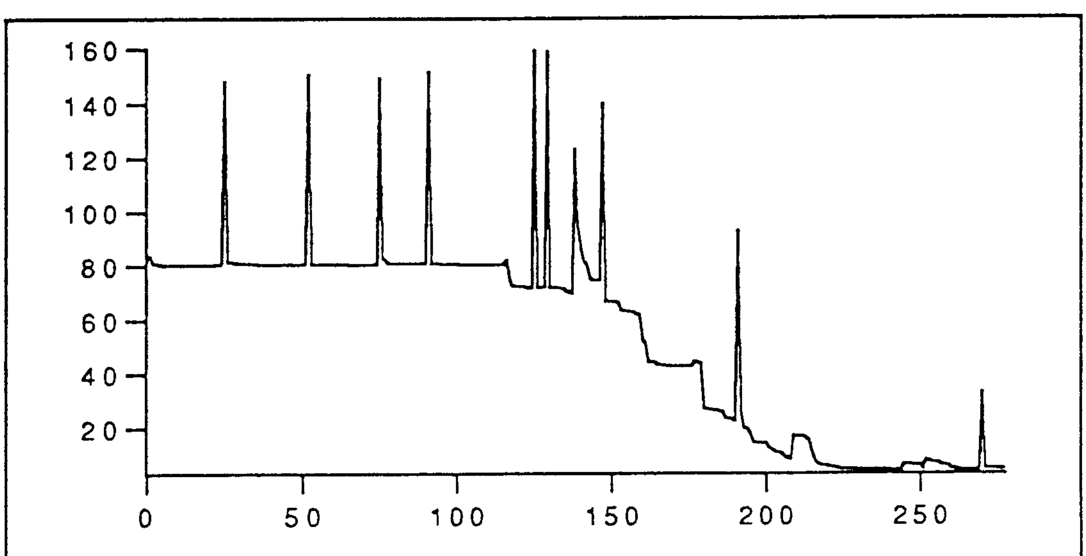

FIGURE 8. Case 3 - Turbine blade: convergence history (percentage of the integrated heat flux error on the outer boundary versus optimization cycles) for the turbine blade airfoil with initially five circular holes.

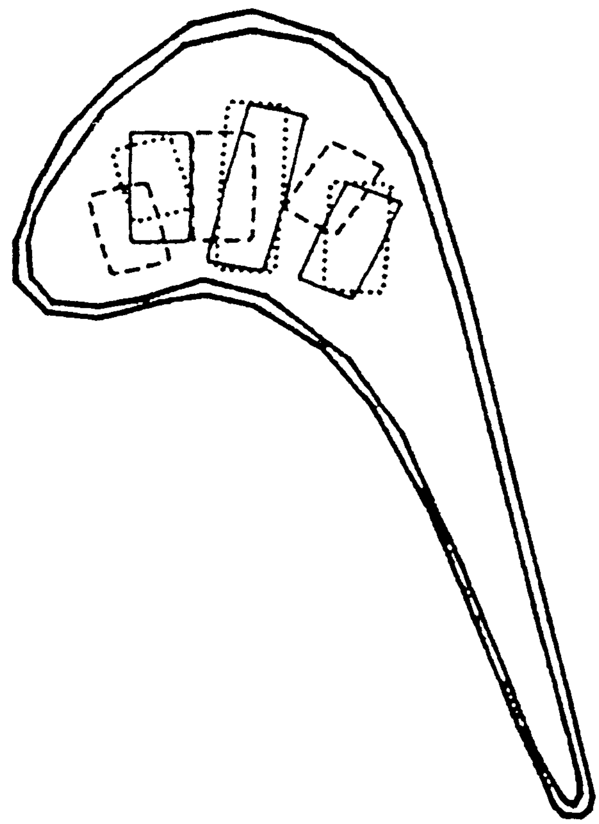

FIGURE 9. Case 4 - Coated turbine blade: initial configuration consisting of three unequal almost rectangular holes (dashed line), intermediate shapes (dotted line), and the optimized configuration (full line) having three differently sized, positioned almost rectangular partially constrained holes.

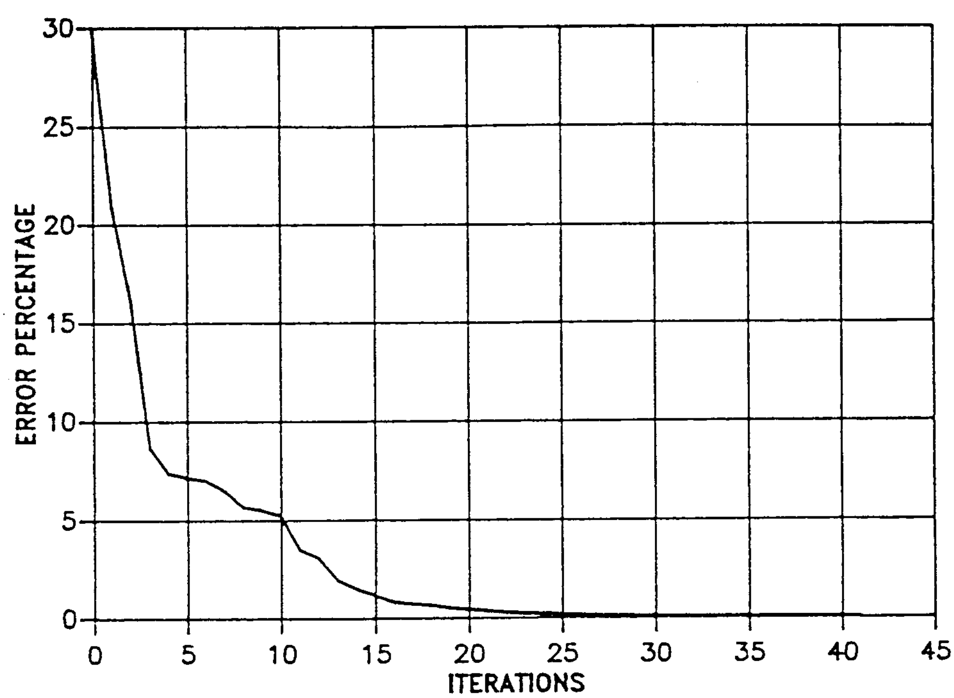

FIGURE 10. Case 4 - Coated turbine blade: convergence history (percentage of the integrated heat flux error on the outer boundary versus optimization cycles) with partially constrained optimization of three rectangular holes.