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Computation of Magnetohydrodynamic Flows With Joule Heating and Buoyancy

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A complete mathematical model has been developed governing steady laminar flow of an incompressible fluid subjected to a steady magnetic field including internal heating due to Joule effect, heat transfer due to conduction and convection, and thermally induced buoyancy forces. The thermal buoyancy was accounted for via Boussinesq approximation. The entire system of partial differential equations was solved iteratively by integrating sequentially a system of Navier-Stokes equations followed by a system of magnetic field equations and transferring the information through source-like terms before the next iteration. Explicit Runge-Kutta time-stepping and conservative finite differencing with artificial compressibility were used on structured non-orthogonal curvilinear boundary conforming coordinate grid. Results of test cases with and without Joule heating demonstrate significant differences in the thermally induced buoyancy flow patterns.

Background

With a rapidly growing interest in practical applications of ferromagnetic and electrorheological fluids [1] and in containerless solidification processes, it is important to fully understand the individual effects of the applied magnetic field on the fluid flow and temperature field. Of special interest is the effect of the electro-resistive (Joule) heating on the generation of thermally induced buoyancy and recirculating flow. It is known that the magnetic field can eliminate [2,3] vorticity from the flow field, while the electric field can enhance it. In this article we will restrict our attention to MHD flows, that is, flow fields with strong applied magnetic field and negligible applied electric field [4]. Such is the situation in, for example, Czochralski process of crystal growth where strong cooling of a liquid causes thermally induced buoyancy which could be modulated by an externally applied magnetic field.

The objective of this article is to examine the recirculating the temperature fields and the recirculating flow patterns resulting from the Joule heating effect. Specifically, influence of the magnitude of the magnetic field on the recirculating flow instability and the detrimental effect of the artificial (numerical) dissipation on the accuracy of the numerical results will be investigated.

Analysis

The following non-dimensional form of the flow field governing equations is obtained

$$v_{i,i} = 0 \tag{1}$$

$$v_{i,t} + \left(v_i v_j - \frac{Ht}{RmRe} H_i H_j\right)_{,j} = - p_{,i} + \frac{1}{Re} v_{i,jj} - \frac{Gr}{Re^2} e_i \theta \tag{2}$$

$$\theta_{,t} + \left(v_j \theta\right)_{,j} = \frac{1}{PrRe} \theta_{,jj} + \varepsilon_m \tag{3}$$

The magnetic field transport equations become

$$H_{i,t} + \left(v_j H_i - v_i H_j\right)_{,j} = \frac{1}{Rm} H_{i,jj} \tag{4}$$

Here, v is the velocity vector, H is the magnetic field vector, p is the combination of hydrostatic, hydrodynamic, and magnetic pressure, θ is the normalized temperature, $\frac{T - T_c}{\Delta T}$, and $\Delta T = T_h - T_c$, where T_h and T_c are the two reference temperatures. The unit vector in the direction of gravitational force is designated as e_i. The term due to Joule heating becomes

$$\varepsilon_m = \frac{EcHt^2}{RmRe^2} \varepsilon_{ijk} \varepsilon_{ilm} H_{k,j} H_{m,l} \tag{5}$$

and the non-dimensional numbers are defined in a standard way as

Eckert number	$Ec = \dfrac{v_r^2}{c_p \Delta T}$
Grashof number	$Gr = \dfrac{\rho \alpha g l_r^3 \Delta T}{\eta^2}$
Hartman number	$Ht = \mu l_r H_r \sqrt{\dfrac{\sigma}{c^2 \eta}}$
Magnetic Reynolds number	$Rm = RePm = \dfrac{4\pi\mu\sigma v_r l_r}{c^2}$
Hydrodynamic Reynolds number	$Re = \dfrac{\rho v_r l_r}{\eta}$
Magnetic Prandtl number	$Pm = \dfrac{4\pi\mu\sigma\eta}{\rho c^2}$
Prandtl number	$Pr = \dfrac{\eta c_p}{\kappa}$

The system of governing partial differential equations was transformed using a non-orthogonal, curvilinear, boundary conforming coordinate system [5] and iteratively integrated [3] using finite differencing in computational space [5], explicit Runge-Kutta time stepping, and an artificial compressibility [6] formulation.

Computational Results

Comparison of computational results [3] and known analytical solutions in two and three dimensions demonstrates high accuracy and smooth monotone convergence of the iterative algorithm. To demonstrate different aspects of the Lorentz force and the Joule heating, we have chosen a configuration consisting of a closed rectangular container (two-dimensional) filled with a homocompositional electrically conducting viscous incompressible Newtonian fluid. The domain was discretized with 60 x 30 clustered grid cells. In all test cases we used Gr = 3000 , Pr = 7.9, Pm = 1. Uniform steady magnetic field was applied vertically downwards. No explicit artificial dissipation [7] was added, except in the last test case when it was 0.01. The following test cases were numerically analyzed:

1. All four walls are at the same temperature ; Ec = 0.00001; Ht = 10.; horizontal container. Figure 1 depicts two recirculating cells that are caused strictly by the Joule heating. Isotherms in the same figure actually depict differences smaller than sixth decimal place (Fig. 1).
2. Top and bottom walls are at the same temperature, while side walls are thermally isolated; Ec = 0.01; Ht = 10.; horizontal container. Two recirculating symmetric zones developed that were produced solely by the Joule effect. Maximum temperature develops at the insulated side walls, although temperature differences are in the third decimal place only (Fig. 2).
3. Top and bottom walls are at the same temperature while side walls are thermally isolated; Ec = 0.1.; Ht =50.; horizontal container. Four asymmetric recirculating regions develop in this case of heat produced solely by the Joule effect. Notice that significantly higher Ecert number was used and that temperature differences are also much larger (Fig. 3).
4. Top wall is uniformly cold, bottom wall is uniformly hot ,while side walls are thermally isolated; Ec = 0.01; Ht = 10.; horizontal container.Four recirculating cells developed in this case of a combined wall heating/cooling and internal fluid heating by the Joule effect (Fig. 4).
5. Top wall is uniformly cold, bottom wall is uniformly hot ,while side walls are thermally insulated; Ec = 0.01; Ht = 10.; container inclined at 30 degrees. Two recirculating elongated cells developed in this case where the directions of the gravity force and the magnetic field differ by 30 degrees. The two zones practically look as a single recirculation zone (Fig. 5).

Convergence rates (Fig. 6) demonstrate that the computer code converges smoothly, although the convergence rate is low because of the source terms [3] in the Navier-Stokes equations. It is especially disturbing to see how the customary introduction of fourth order artificial dissipation can create an obviously incorrect solution (Fig. 7) although it is fully converged (Fig. 8). Hence, artificial dissipation should be avoided at any cost when computing highly sensitive flows like these depicted in this article.

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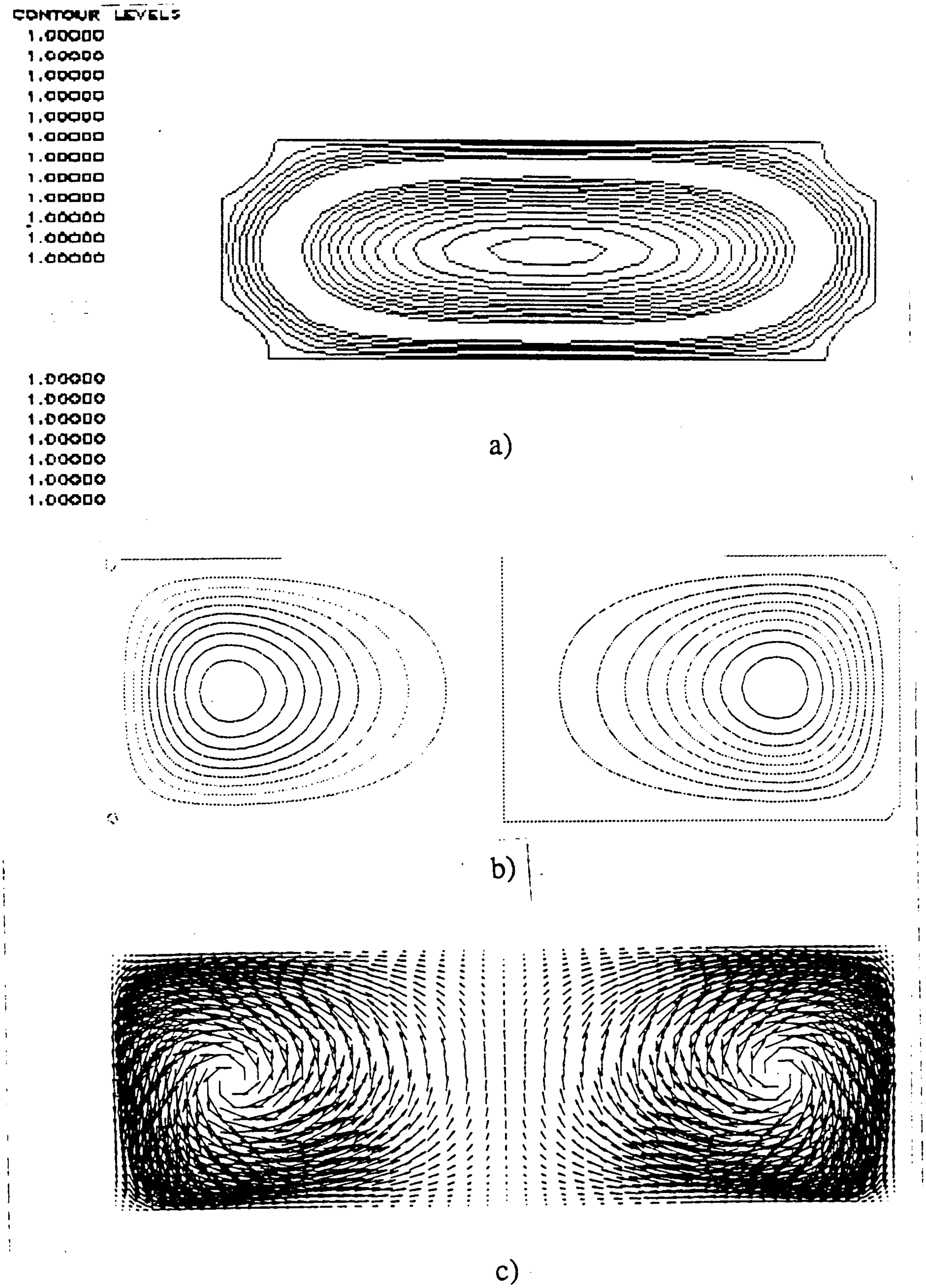

Fig. 1 All four walls are at the same temperature ; Ec = 0.00001; Ht = 10.; horizontal container: a) isotherms, b) streamlines, c) velocity vector field.

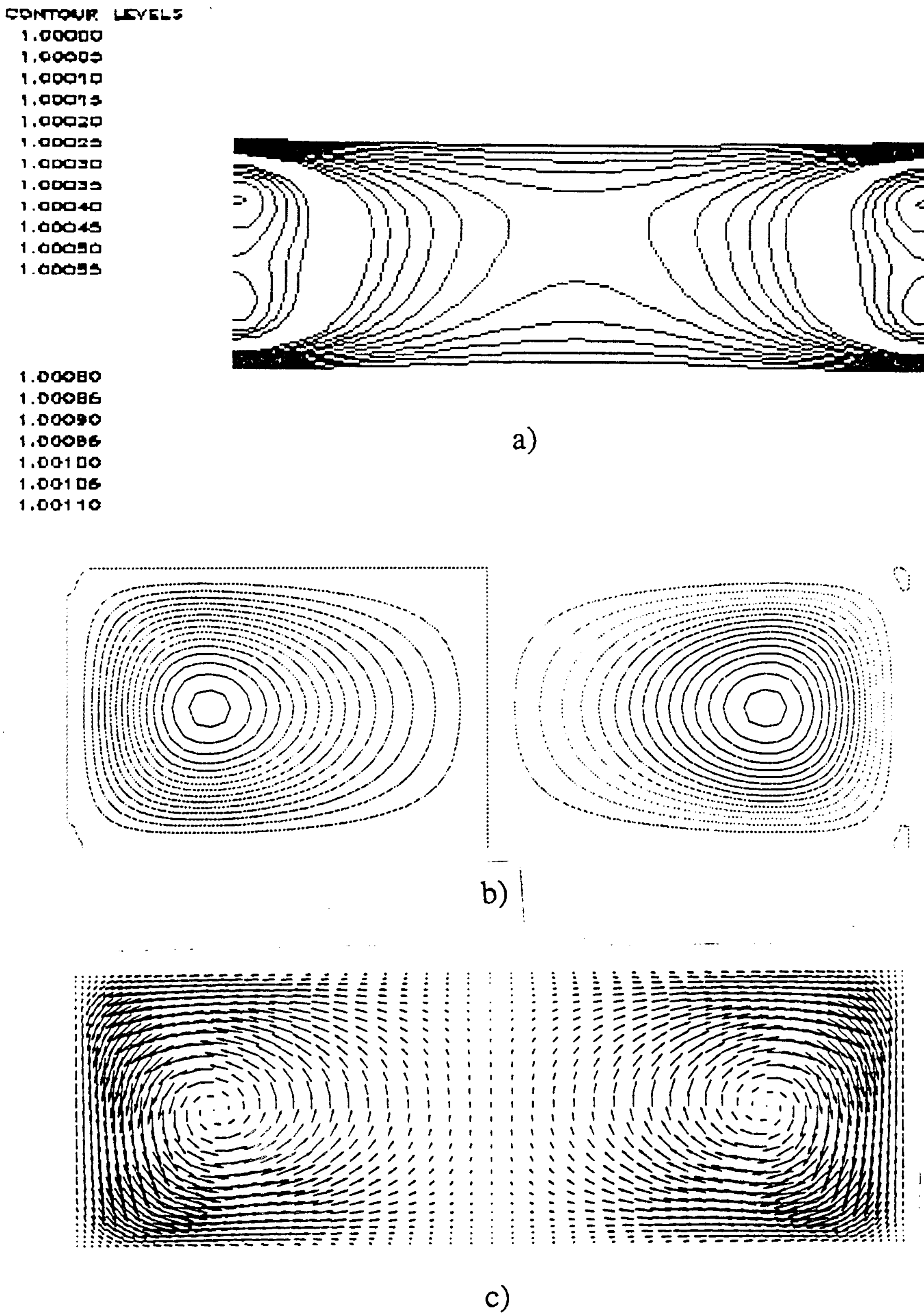

Fig. 2 Top and bottom walls are at the same temperature, while side walls are thermally isolated; Ec = 0.01; Ht = 10.; horizontal container: a) isotherms, b) streamlines, c) velocity vector field.

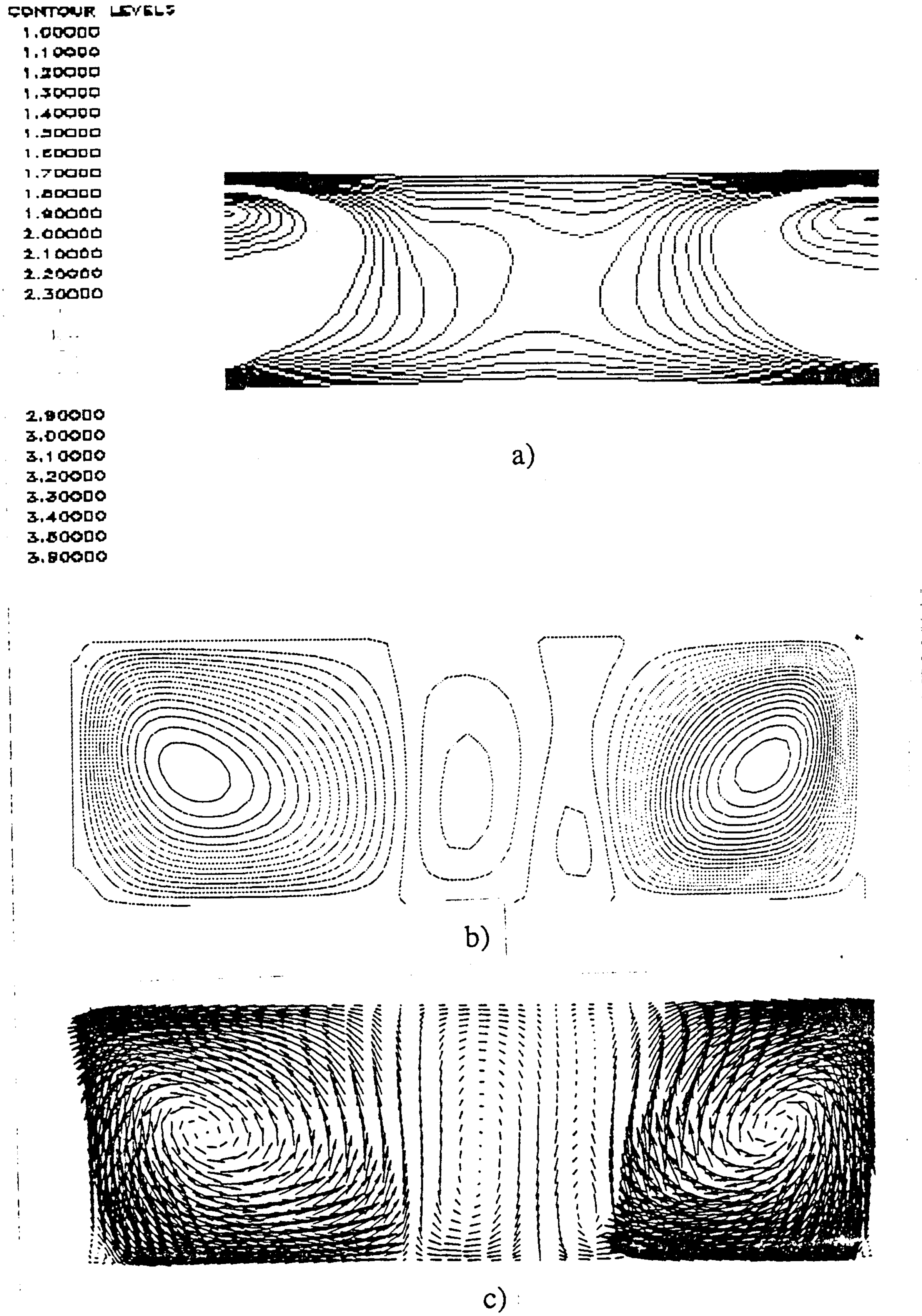

Fig. 3 Top and bottom walls are at the same temperature while side walls are thermally isolated; Ec = 0.1.; Ht = 50.; horizontal container: a) isotherms, b) streamlines, c) velocity vector field.

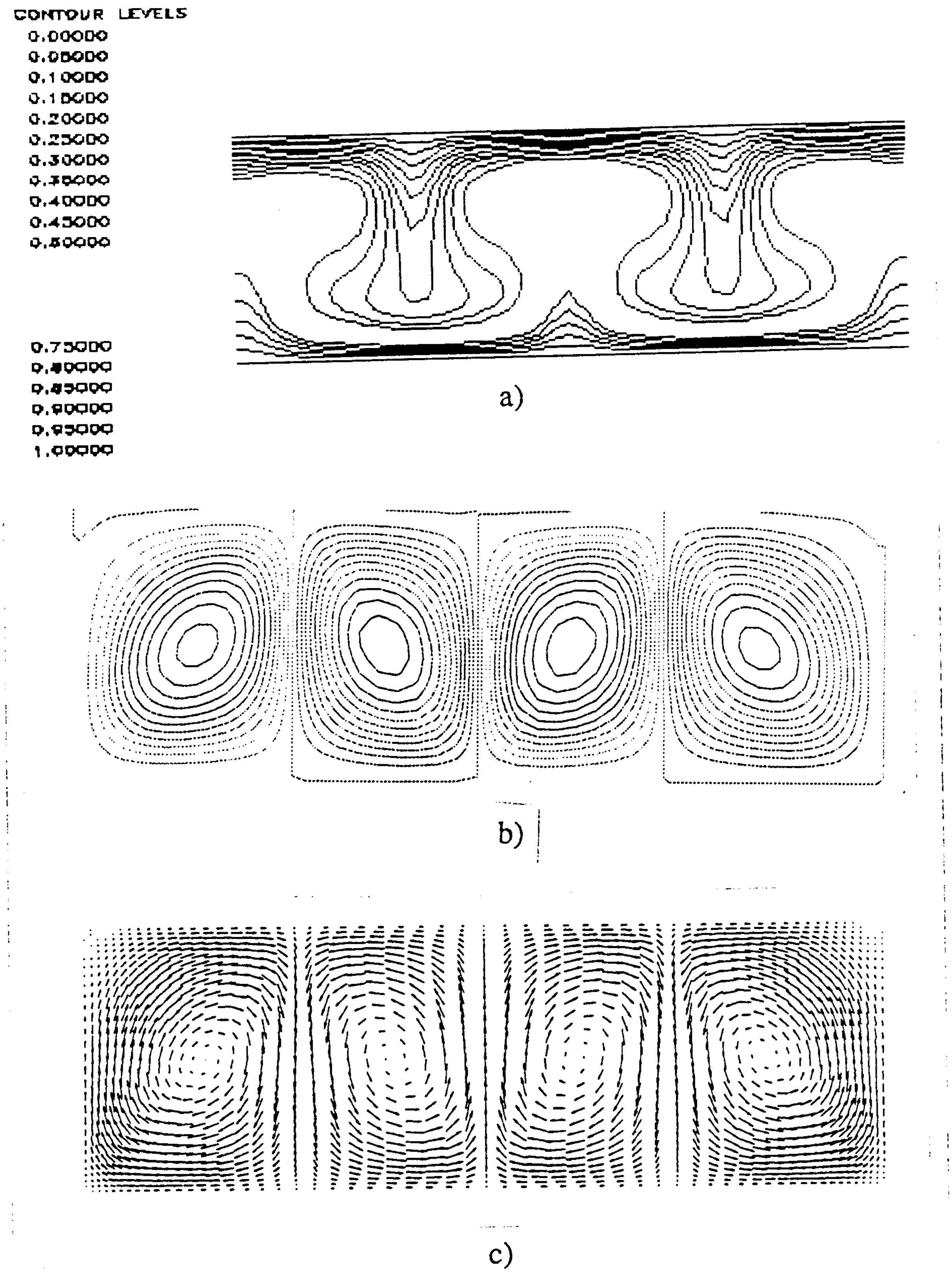

Fig. 4 Top wall is uniformly cold, bottom wall is uniformly hot ,while side walls are thermally insulated; Ec = 0.01; Ht = 10: a) isotherms, b) streamlines, c) velocity vector field.

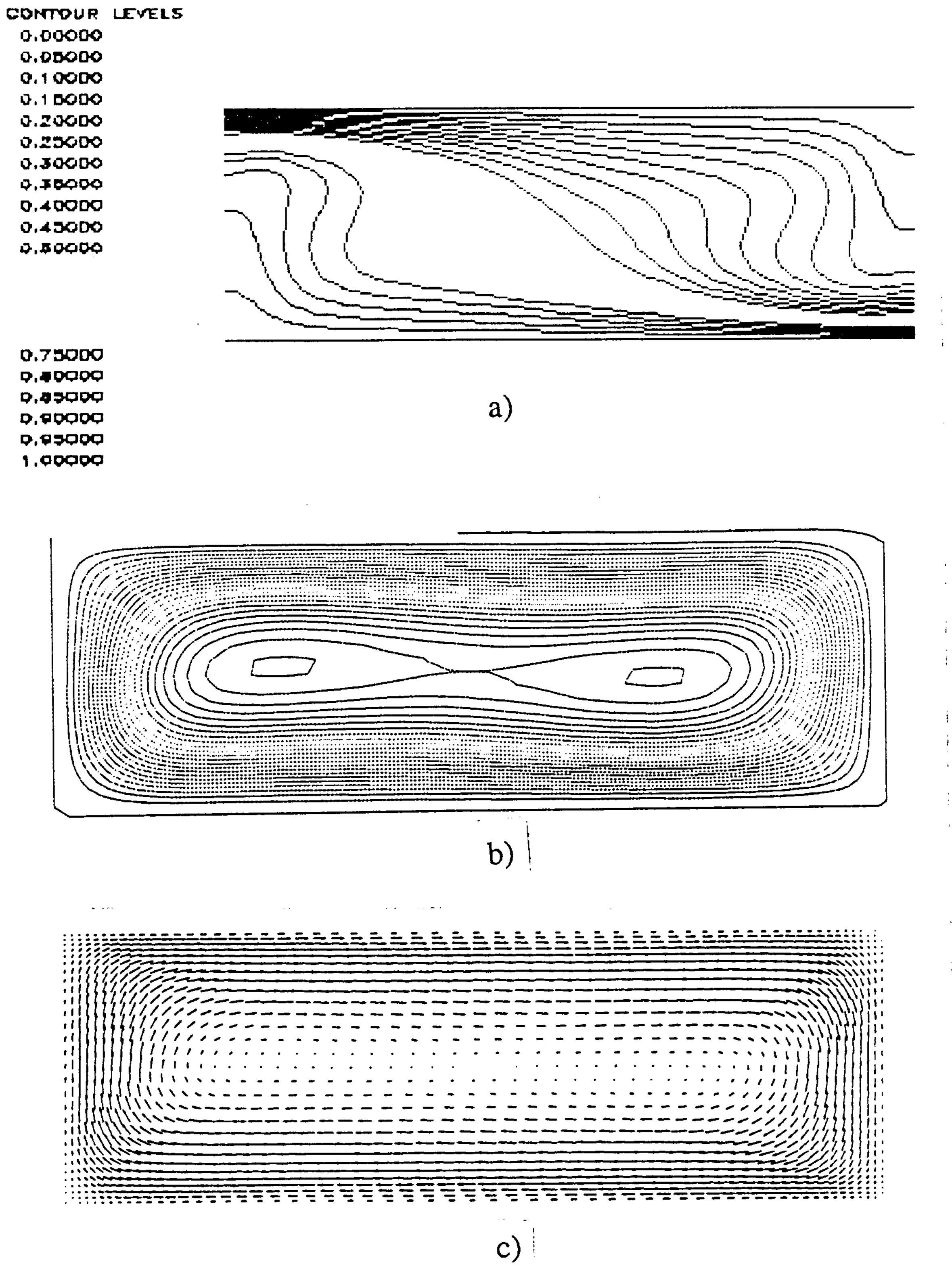

Fig. 5 Top wall is uniformly cold, bottom wall is uniformly hot ,while side walls are thermally insulated; Ec = 0.01; Ht = 10.; container inclined at 30 degrees: a) isotherms, b) streamlines, c) velocity vector field.

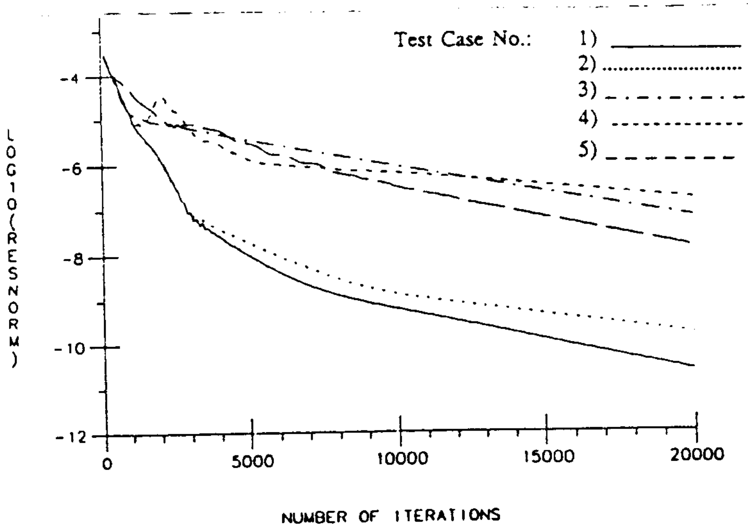

Fig. 6 Convergence histories for the five test cases

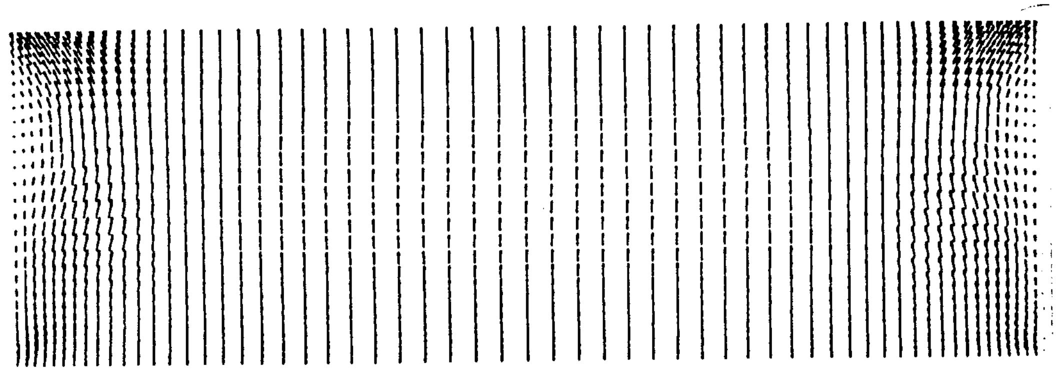

Fig. 7 All four walls are at the same temperature ; Ec = 0.00001; Ht = 10.; horizontal container. Fourth order artificial dissipation was added. Velocity vector field.

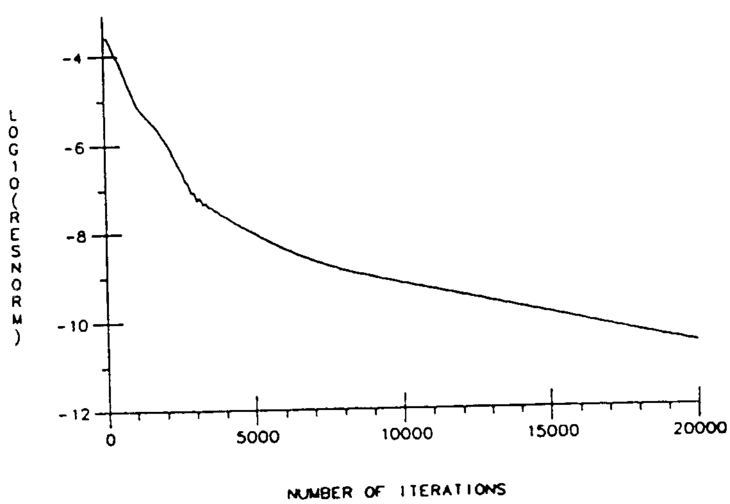

Fig. 8 All four walls are at the same temperature ; Ec = 0.00001; Ht = 10.; horizontal container. Fourth order artificial dissipation was added. Convergence history.