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UNIFIED ELECTRO-MAGNETO-FLUID DYNAMICS (EMFD): INTRODUCTORY CONCEPTS

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Abstract—Recent advances in numerical techniques and computing technology, as well as new, fully rigorous theoretical treatments, have made analysis of combined electro-magneto-fluid dynamic (EMFD) flows well within reach. A survey of electro-magnetics and the theory describing combined electro-magneto-hydrodynamic flows is presented. In Part 1 emphasis is placed on describing the sources and interactions of electric and magnetic fields described by Maxwell's equations. The natural and induced sources of material polarization and magnetization are described as well as the sources and transport of electric charge in the fluid. The paper concludes with a presentation of the unified EMFD system of equations, combining Maxwell's equations, the Navier–Stokes equations and material constitutive relations. Part 2 of this paper [*Int. J. Non-Linear Mechanics* **32**, 923–932 (1997)] will focus on a comparison of existing separate electro-hydrodynamic (EHD) and magneto-hydrodynamic (MHD) models with the unified EMFD model. © 1997 Elsevier Science Ltd.

Keywords: electromagnetics, fluid dynamics, electro-hydrodynamics, magneto-hydrodynamics

NOMENCLATURE

$\underline{\mathbf{B}}$	magnetic flux density vector, kg $\mathrm{A^{-1}\,s^{-2}}$
$\underline{\underline{\mathbf{d}}} = \frac{1}{2}(\nabla\underline{\mathbf{v}} + \nabla\underline{\mathbf{v}}^{\mathrm{T}})$	rate of deformation tensor, $\mathrm{s^{-1}}$
$\underline{\mathbf{D}} = \varepsilon_0\underline{\mathbf{E}} + \underline{\mathbf{P}}$	electric displacement field vector, $\mathrm{A\,s\,m^{-2}}$
e	internal energy per unit mass, $\mathrm{m^2\,s^{-2}}$
$e_0 = e + \frac{1}{2}\underline{\mathbf{v}}\cdot\underline{\mathbf{v}} + \underline{\mathbf{g}}\cdot\underline{\mathbf{r}}$	total energy per unit mass, $\mathrm{m^2\,s^{-2}}$
$\underline{\mathbf{E}}$	electric field vector, $\mathrm{kg\,m\,s^{-3}\,A^{-1}}$, or $\mathrm{V\,m^{-1}}$
$\underline{\mathscr{E}} = \underline{\mathbf{E}} + \underline{\mathbf{v}}\times\underline{\mathbf{B}}$	electromotive intensity vector, $\mathrm{kg\,m\,s^{-3}\,A^{-1}}$
$\underline{\mathbf{f}}$	mechanical body force vector per unit mass, $\mathrm{m\,s^{-2}}$
$\underline{\mathbf{f}}^{\mathrm{EM}}$	electromagnetic body force vector per unit volume, $\mathrm{kg\,m^{-2}\,s^{-2}}$
$\underline{g}$	acceleration due to gravity, $\mathrm{m\,s^{-2}}$
h	heat source or sink per unit mass, $\mathrm{m^2\,s^{-3}}$
$\underline{\mathbf{H}} = \underline{\mathbf{B}}/\mu_o - \underline{\mathbf{M}}$	magnetic field intensity vector, $\mathrm{A\,m^{-1}}$
$\underline{\mathbf{J}} = \underline{\mathbf{J}}_{\mathrm{c}} + \underline{\mathbf{J}}_{\mathrm{d}}$	electric current density vector, $\mathrm{A\,m^{-2}}$
$\underline{\mathbf{J}}_{\mathrm{c}}$	electric conduction current vector, $\mathrm{A\,m^{-2}}$
$\underline{\mathbf{J}}_{\mathrm{d}}$	electric drift current vector, $\mathrm{A\,m^{-2}}$
$\underline{\mathbf{J}}_{\mathrm{m}}$	apparent magnetization current vector, $\mathrm{A\,m^{-2}}$
$\underline{\mathbf{J}}_{\mathrm{p}}$	polarization current vector, $\mathrm{A\,m^{-2}}$
$\underline{\mathbf{M}} = \underline{\mathbf{M}}_{\mathrm{m}} + \underline{\mathbf{M}}_{\mathrm{p}}$	total magnetization vector per unit volume, $\mathrm{A\,m^{-1}}$
$\underline{\mathbf{M}}_{\mathrm{m}}$	rotational motion magnetization vector per unit volume, $\mathrm{A\,m^{-1}}$
$\underline{\mathbf{M}}_{\mathrm{p}}$	intrinsic or natural magnetization vector per unit volume, $\mathrm{A\,m^{-1}}$
$\underline{\mathscr{m}} = \underline{\mathbf{M}} + \underline{\mathbf{v}}\times\underline{\mathbf{P}}$	magnetomotive intensity vector per unit volume, $\mathrm{A\,m^{-1}}$
$\underline{\hat{\mathbf{n}}}$	outward normal unit vector to volume
p	pressure, $\mathrm{kg\,m^{-1}\,s^{-2}}$

Contributed by K. R. Rajagopal.

$\underline{\mathbf{P}} = \underline{\mathbf{P}}_e + \underline{\mathbf{P}}_p$	total polarization vector per unit volume, $\mathrm{A\,s\,m^{-2}}$
$\underline{\mathbf{P}}_e$	polarization vector per unit volume due to electric charge, $\mathrm{A\,s\,m^{-2}}$
$\underline{\mathbf{P}}_p$	polarization vector per unit volume due to total dipole moment, $\mathrm{A\,s\,m^{-2}}$
q_e	local free electric charge per unit volume, $\mathrm{A\,s\,m^{-3}}$
$q_e' = -\nabla\cdot\underline{\mathbf{P}}_e$	inhomogeneous electric charge per unit volume, $\mathrm{A\,s\,m^{-3}}$
$q_p = -\nabla\cdot\underline{\mathbf{P}}_p$	apparent electric charge per unit volume, $\mathrm{A\,s\,m^{-3}}$
$q_o = q_e - q_e'$	total or free electric charge per unit volume, $\mathrm{A\,s\,m^{-3}}$
$\underline{\mathbf{q}}$	heat flux vector, $\mathrm{kg\,s^{-3}}$
Q	point electric charge, A s
$\underline{\mathbf{r}}$	position vector, m
s	entropy per unit mass, $\mathrm{m^2\,kg^{-1}\,K^{-1}\,s^{-2}}$
$\underline{\underline{\mathbf{t}}} = -p\underline{\underline{\mathbf{I}}} + \underline{\underline{\tau}}^v + \underline{\underline{\tau}}^{EM}$	fluid stress tensor, $\mathrm{kg\,m^{-1}\,s^{-2}}$
$\underline{\mathbf{v}}$	fluid velocity vector, $\mathrm{m\,s^{-1}}$
$\mathcal{V}$	volume, $\mathrm{m^3}$

Greek symbols

ε	dielectric constant or electric permittivity, $\mathrm{kg^{-1}\,m^{-3}\,s^4\,A^2}$
$\varepsilon_o = 8.854 \times 10^{-12}$	vacuum dielectric constant or electric permittivity, $\mathrm{kg^{-1}\,m^{-3}\,s^4\,A^2}$
$\varepsilon_r = \varepsilon/\varepsilon_o$	relative electric permittivity, non-dimensional
κ	thermal conductivity coefficient, $\mathrm{kg\,m\,s^{-3}\,K^{-1}}$
λ_v	second coefficient of viscosity, $\mathrm{kg\,m^{-1}\,s^{-1}}$
σ	electric conductivity coefficient, $\mathrm{kg^{-1}\,m^{-3}\,s^3\,A^2}$
θ	absolute temperature, K
ρ	fluid density, $\mathrm{kg\,m^{-3}}$
$\underline{\underline{\tau}}^v = 2\mu_v\underline{\underline{\mathbf{d}}} + \lambda_v\,\mathrm{tr}(\underline{\underline{\mathbf{d}}})\underline{\underline{\mathbf{I}}}$	Newtonian viscous stress tensor, $\mathrm{kg\,m^{-1}\,s^{-2}}$
$\underline{\underline{\tau}}^{EM}$	electromagnetic stress tensor, $\mathrm{kg\,m^{-1}\,s^{-2}}$
μ	magnetic permeability coefficient, $\mathrm{kg\,m\,A^{-2}\,s^{-2}}$
$\mu_o = 4\pi \times 10^{-7}$	magnetic permeability of vacuum, $\mathrm{kg\,m\,A^{-2}\,s^{-2}}$
$\mu_r = \mu/\mu_o$	relative magnetic permeability, non-dimensional
μ_v	shear coefficient of viscosity, $\mathrm{kg\,m^{-1}\,s^{-1}}$
$\chi^E = \varepsilon_r - 1$	electric susceptibility, non-dimensional
$\chi^M = \mu_r - 1$	magnetic susceptibility, non-dimensional
$\Phi = \underline{\underline{\tau}}^v : \underline{\underline{\mathbf{d}}}$	viscous dissipation function, $\mathrm{kg\,m^{-1}\,s^{-3}}$
$\Psi = e - \theta s$	material free energy function, $\mathrm{m^2\,s^{-2}}$

1. INTRODUCTION

The ability of electro-magnetic fields to influence fluid flow and heat transfer has long been known and used with varying degrees of success. The equations governing the flow consist of the Navier–Stokes equations of fluid motion coupled with Maxwell's equations of electro-magnetics and material constitutive relations. The study of these flows is Electro-Magneto-Fluid Dynamics (EMFD). The full system of equations has, up until recently, been far too complex to solve generally. The Navier–Stokes relations alone become complex when analysis of flows of realistic interest are desired (that is, turbulent, chemically reacting, multi-constituent, and/or non-Newtonian). Coupled with Maxwell's equations, the complexity of the system is raised by orders of magnitude. Recently, rigorous theoretical continuum mechanics treatments of EHD [1] and unified EMFD [2] have allowed greater varieties of EMFD flows to be studied. These continuum mechanics approaches are limited, however, to non-relativistic, relatively low frequency ($< 10^3$ Hz) phenomena [3–5].

The objectives of this paper are to provide background resource and theory to allow implementation of numerical analysis for unified EMFD flows. The field is too vast to exhaustively cover the subject, so the intent is to provide an introductory survey of the field.

To accomplish this, Part 1 provides an overview of electro-magnetic theory with concentrated effort placed on descriptions of the electric and magnetic fields and electric charge and current. Effort is also made to provide a physical understanding of the field–material interactions causing polarization and magnetization. Finally, the system of equations governing the unified EMFD theory is presented [2]. Part 2 [6] will discuss the equations of motion in more detail and will compare the unified EMFD theory with classical EHD and MHD models.

2. ELECTRIC FIELD

The concept of an electric field is typically developed from the *Coulomb force* between two electric point charges in free space (vacuum). The force, $\underline{\mathbf{F}}_{12}$, between two point charges Q_1 and Q_2 is defined as [7, p. 15]

$$\underline{\mathbf{F}}_{12} = \frac{Q_1 Q_2(\underline{\mathbf{r}}_2 - \underline{\mathbf{r}}_1)}{4\pi\varepsilon_o |\underline{\mathbf{r}}_2 - \underline{\mathbf{r}}_1|^3} \tag{1}$$

where $\underline{\mathbf{r}}_1$ and $\underline{\mathbf{r}}_2$ are the position vectors of the stationary point charges Q_1 and Q_2. From this definition of electric force, the static *electric field*, $\underline{\mathbf{E}}_1$, at $\underline{\mathbf{r}}$ due to a single, stationary charge, Q_1, at position $\underline{\mathbf{r}}_1$ is defined as [8, p. 32]

$$\underline{\mathbf{E}}_1 = \frac{Q_1(\underline{\mathbf{r}} - \underline{\mathbf{r}}_1)}{4\pi\varepsilon_o |\underline{\mathbf{r}} - \underline{\mathbf{r}}_1|^3} \tag{2}$$

This concept can be expanded to define the electric field due to multiple charges, Q_i, through the superposition principle [7, p. 16]. In this case the static electric field in a vacuum at the location $\underline{\mathbf{r}}$ is defined as

$$\underline{\mathbf{E}} = \frac{1}{4\pi\varepsilon_o} \sum_i \frac{Q_i(\underline{\mathbf{r}} - \underline{\mathbf{r}}_i)}{|\underline{\mathbf{r}} - \underline{\mathbf{r}}_i|^3} \tag{3}$$

Thus, the electric field is a measure of the amount of force that can be exerted on a test charge by the electric charges surrounding it. Through this concept of the electric field, no longer must the effect of each charge be considered individually.

The electric field vector may be integrated over the surface, S, of a control volume, $\mathscr{V}$, resulting in the following expression

$$\int_S \underline{\mathbf{E}} \cdot \underline{\hat{\mathbf{n}}}\, \mathrm{d}S = \int_S \nabla\left(\frac{1}{4\pi\varepsilon_o} \sum_i \frac{Q_i}{|\underline{\mathbf{r}} - \underline{\mathbf{r}}_i|}\right) \cdot \underline{\hat{\mathbf{n}}}\, \mathrm{d}S \tag{4}$$

since [7, pp. 16–20]

$$\frac{(\underline{\mathbf{r}} - \underline{\mathbf{r}}_i)}{|\underline{\mathbf{r}} - \underline{\mathbf{r}}_i|^3} = \nabla\left(\frac{1}{|\underline{\mathbf{r}} - \underline{\mathbf{r}}_i|}\right) \tag{5}$$

After transforming the surface integral to a volume integral using the divergence theorem and rearranging, equation (4) becomes

$$\int_{\mathscr{V}} \nabla \cdot (\varepsilon_o \underline{\mathbf{E}})\, \mathrm{d}\mathscr{V} = \int_{\mathscr{V}} \sum_i \frac{Q_i}{4\pi |\underline{\mathbf{r}} - \underline{\mathbf{r}}_i|^3}\, \mathrm{d}\mathscr{V} \tag{6}$$

Since this relation must hold for any given control volume, the integrals may be dropped, resulting in the well known local form of Gauss's law of electrostatics in a vacuum

$$\nabla \cdot (\varepsilon_o \underline{\mathbf{E}}) = q_e \tag{7}$$

where q_e, the local free charge density accounting for all the free electrons and ions, is defined as

$$q_e = \frac{\sum_i Q_i}{\mathscr{V}} \tag{8}$$

3. POLARIZATION

Previously, the interactions of charges and electric fields in a vacuum were considered. The presence of material media affects these interactions. In order to define these effects it is necessary to introduce the concept of polarization. Charge polarization or *polarization* is created any time charges of opposite signs are separated by a distance.

The existence of polarization creates the *dipole moment* which may be defined as the distance from the net negative to the net positive charge multiplied by the net positive charge. For a simple dipole with a single positive charge, Q^+, separated from a single negative charge, Q^-, by a distance, $\underline{\xi}$, the dipole moment is defined as [2, p. 29]

$$\underline{\mathbf{p}} = Q^+ \underline{\xi} \tag{9}$$

An atomic dipole is created when the electron cloud of an atom is displaced from its nucleus through the action of an applied electric field (Fig. 1). A similar moment is produced in any atom, molecule, group of particles or continuum in which a net positive charge is separated from a net negative charge. In the case of a group of particles, the encountered moment is the sum of the individual particle dipole moments allowing the *continuum polarization* to be heuristically defined as the total dipole moment per unit volume [8, p. 117]

$$\underline{\mathbf{P}} = \frac{\sum_i \underline{\mathbf{p}}_i}{\mathcal{V}} \tag{10}$$

This definition holds regardless of whether the dipole moments arise from free electrons, ions, atoms, molecules or simply a charge gradient within a continuum.

Although many references define several sources of polarization [9], there are essentially two main sources of polarization: induced and natural [10]. *Induced polarization*, $\underline{\mathbf{P}}_\mathrm{p}$, is caused by an electric field acting on natural dipoles or neutral particles. The applied electric field induces an initial charge separation in neutral particles (Fig. 1, adapted from [1]). The applied electric field also creates greater charge separation within the molecules and causes molecular alignment with the applied field in natural dipoles (Fig. 1, adapted from [1]). The polarization shown in the first figure is wholly dependent on the electric field, whereas the second figure shows polarization which is dependent on both the electric field and on molecular geometry.

In contrast, *natural polarization*, $\underline{\mathbf{P}}_\mathrm{e}$, arises from natural dipoles and charged particles. An example of a natural dipole is water which has a geometry such that one end of the molecule is more positive than the other. Figure 2 shows natural dipoles. Ions, on the other hand are atoms or molecules whose overall charge is uneven; either positive or negative. At this point it is vital to realize that Fig. 2 illustrates both types of polarization: both the natural polarization caused by the molecular geometry and induced polarization from the alignment of the molecules with the electric field. This requires the further illustration of Fig. 2.

Although the molecules in the left half of Fig. 2 have polarization on a particle or molecular level, they do not have polarization on a continuum level. Consider water, H_2O, as a liquid or vapor for instance (a geometry similar to that in Fig. 2). On a molecular level water has polarization by virtue of its geometry. However, due to the fact that it is in a fluid state and the molecules are allowed to move freely and orient randomly, it will not have polarization on a continuum level—the sum of the molecular dipole moments throughout the continuum is zero. For water to be polarized on a continuum level an electric field must be applied as shown in the right half of Fig. 2.

From the above discussion it may seem that there is no reason, when dealing with fluids, to consider natural polarization. This, however, would be an erroneous assumption. Though the natural polarization may show no continuum effects without the presence of an electric field, in an electric field the overall polarization seen is both the induced polarization, $\underline{\mathbf{P}}_\mathrm{p}$, due to the electric field and the natural polarization, $\underline{\mathbf{P}}_\mathrm{e}$, of the molecules (which are now aligned by the electric field). The *total polarization*, $\underline{\mathbf{P}}$, is defined as [10, p. 22]

$$\underline{\mathbf{P}} = \underline{\mathbf{P}}_\mathrm{e} + \underline{\mathbf{P}}_\mathrm{p} \tag{11}$$

Both types of polarization can produce a charge density within the continuum. This may be seen by taking an infinitesimal control volume around a polarized continuum and

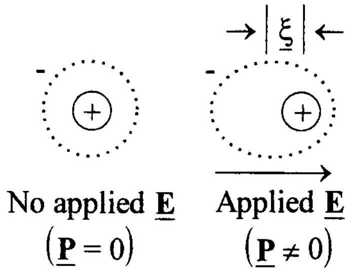

Fig. 1. Induced polarization on initially non-polar molecules.

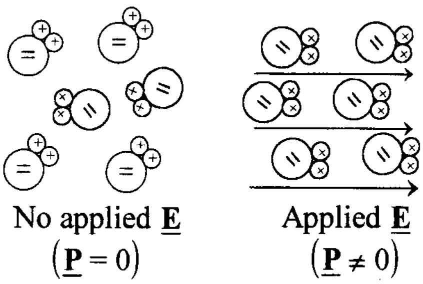

Fig. 2. Induced polarization on initially polar molecules.

performing a volumetric charge balance [8, p. 118]. Applying this definition to induced polarization, $\underline{\mathbf{P}}_p$, gives rise to the *apparent charge density*, q_p, which is defined as [10, p. 21]

$$q_p = -\nabla \cdot \underline{\mathbf{P}}_p \tag{12}$$

Similarly, natural polarization, $\underline{\mathbf{P}}_e$, gives rise to an *inhomogeneous charge density*, q_e', which is defined as [10, p. 22]

$$q_e' = -\nabla \cdot \underline{\mathbf{P}}_e \tag{13}$$

The inhomogeneous charge density is caused by a non-uniform distribution of charges within the continuum.

From the dipole moment [equation (9)] it can be seen that a time-varying distance, $\underline{\xi}$, will be due to charge motion; in effect, an electric current. This *polarization current*, $\underline{\mathbf{J}}_p$, is defined as the variation of the total polarization with respect to time [8, p. 121]

$$\underline{\mathbf{J}}_p = \frac{\partial \underline{\mathbf{P}}}{\partial t} \tag{14}$$

From this discussion it can be deduced that the degree to which material is polarized is related not only to the strength of the applied electric and magnetic fields, but to physical properties of the material expressed in terms of the material free energy, Ψ. The general constitutive relation for non-linear materials subject to electro-static or low frequency polarization is defined as [2, p. 173]

$$\underline{\mathbf{P}} = -2\rho\left(\frac{\partial \Psi}{\partial I_1}\underline{\mathscr{E}} + \frac{\partial \Psi}{\partial I_3}(\underline{\mathscr{E}} \cdot \underline{\mathbf{B}})\underline{\mathbf{B}}\right) \tag{15}$$

where,

$$\Psi = \Psi(I_1, I_2, I_3, \theta, \rho^{-1}) \tag{16}$$

$$\underline{\mathscr{E}} = \underline{\mathbf{E}} + \underline{\mathbf{v}} \times \underline{\mathbf{B}} \tag{17}$$

$$I_1 = \underline{\mathscr{E}} \cdot \underline{\mathscr{E}} \tag{18}$$

$$I_2 = \underline{\mathbf{B}} \cdot \underline{\mathbf{B}} \tag{19}$$

$$I_3 = (\underline{\mathscr{E}} \cdot \underline{\mathbf{B}})^2 \tag{20}$$

These relations, especially the free energy relation [equation (16)] can be quite complex. Fortunately a large number of materials are relatively linear, meaning that polarization is a function of one material property and the strength and direction of the applied electric field [11, pp. 220–222]. For linear materials the free energy of the material becomes [2, p. 178]

$$\Psi = \Psi_o - \frac{1}{2\rho}\left(\varepsilon_o \chi^E I_1 + \frac{\chi^M}{\mu_o(1+\chi^M)} I_2\right) \tag{21}$$

Thus, for linear materials only the first term in equation (15) remains and is usually written as [2, p. 178; 8, p. 164]

$$\underline{\mathbf{P}} = -2\rho \frac{\partial \Psi}{\partial I_1} \underline{\mathscr{E}} = \varepsilon_o \chi^E \underline{\mathscr{E}} \tag{22}$$

The dielectric susceptibility is often nearly constant, although the medium in use and external factors, especially electro-magnetic frequency and temperature, may influence its value [11, pp. 220–222].

4. MAGNETIC FIELD

The magnetic field is the second type of field of force originating from electric charges. However, a derivation of the magnetic field is not as intuitively straightforward as the derivation for the electric field. There are several analogies that may be drawn between the two. For example, both the electric field and magnetic field create forces.

The electric field, $\underline{\mathbf{E}}$, is obtained from the Coulomb force between electric charges while the magnetic field, $\underline{\mathbf{B}}$, is obtained from the ponderomotive force. The Coulomb and ponderomotive forces combine to form the Lorentz force, which is defined as [8, p. 28]

$$\underline{\mathbf{f}}^{EM} = q_o \underline{\mathbf{E}} + q_o \underline{\mathbf{v}} \times \underline{\mathbf{B}} \tag{23}$$

where the first term is the Coulomb force. The second term states that the velocity, $\underline{\mathbf{v}}$, of the free charge q_o, the magnetic field, $\underline{\mathbf{B}}$, and the resulting force are all mutually perpendicular.

Analogous to the electric field relation to electric charge, the magnetic field is related to charge movement or electric current, $\underline{\mathbf{J}}$. The fundamental statement for the case of steady magnetic field in a vacuum without polarization or magnetization is called Ampere's law and is expressed as [8, p. 85]

$$\nabla \times \frac{\underline{\mathbf{B}}}{\mu_o} = \underline{\mathbf{J}} \tag{24}$$

This law states that a steady electric current, $\underline{\mathbf{J}}$, will produce a circulating or rotational magnetic field, $\underline{\mathbf{B}}$ and *vice-versa*.

5. MAGNETIZATION

Magnetization is the magnetic field analogy of the electric field's polarization. Magnetization comes from two sources: particle circular motion and intrinsic or natural particle magnetism. To derive the magnetization due to circular motion of the particle, its velocity vector, $\underline{\mathbf{v}}$, may be expressed as a sum of the linear drift velocity, $\underline{\mathbf{v}}_d$, and a rotational velocity, $\underline{\omega} \times \underline{\mathbf{r}}$ [10, p. 25].

$$\underline{\mathbf{v}} = \underline{\mathbf{v}} + \underline{\omega} \times \underline{\mathbf{r}} \tag{25}$$

The charged particle drift velocity creates the *convection* or *drift current*, $\underline{\mathbf{J}}_d$, defined as [10, p. 67]

$$\underline{\mathbf{J}}_d = q_o \underline{\mathbf{v}}_d \tag{26}$$

The drift current is not associated with magnetization, but with the linear motion of charged particles. For a particle, the magnetization moment, $\underline{\mathbf{m}}$, due to the rotational velocity is defined as [10, p. 26]

$$\underline{\mathbf{m}} = \frac{Qr^2\underline{\omega}}{2}. \tag{27}$$

where r is the distance from the particle to its center of rotation, and $\underline{\omega}$ is the angular velocity vector. Similar to the way continuum polarization was defined, continuum *magnetization due to circular motion* is the volumetric density of the magnetization moment [8, p. 129].

$$\underline{\mathbf{M}}_{\mathrm{m}} = \frac{\sum_i \underline{\mathbf{m}}_i}{\mathscr{V}} \tag{28}$$

Intrinsic or *natural magnetization*, $\underline{\mathbf{M}}_{\mathrm{p}}$, is associated with electron and nuclear spins. Thus, intrinsic magnetization is a material property. The *total magnetization*, $\underline{\mathbf{M}}$, is then defined as

$$\underline{\mathbf{M}} = \underline{\mathbf{M}}_{\mathrm{m}} + \underline{\mathbf{M}}_{\mathrm{p}} \tag{29}$$

Neither type of magnetization gives rise to charge densities as polarization did, but magnetization due to circuital motion of the particles does create an electric current, the *apparent magnetization current*, $\underline{\mathbf{J}}_{\mathrm{m}}$, which is defined as [10, p. 26]

$$\underline{\mathbf{J}}_{\mathrm{m}} = \nabla \times \underline{\mathbf{M}}_{\mathrm{m}} \tag{30}$$

Intrinsic or natural magnetization does not have a related current because electric current is generally defined as the macroscopic or external movement of charges. Because intrinsic or natural magnetization is caused by internal spin of charges, that is electrons orbiting the nucleus, there is no electric current created [10, p. 26].

Magnetization, like polarization, depends not only on the electromagnetic field strengths, but on material properties as well. The constitutive definition of non-linear material magnetization for a relatively low frequency applied magnetic field is defined as [2, pp. 177, 130]

$$\underline{\mathscr{m}} = -2\rho\left(\frac{\partial\Psi}{\partial I_2}\underline{\mathbf{B}} + \frac{\partial\Psi}{\partial I_3}(\underline{\mathscr{E}}\cdot\underline{\mathbf{B}})\underline{\mathbf{B}}\right) \tag{31}$$

where

$$\underline{\mathscr{m}} = \underline{\mathbf{M}} + \underline{\mathbf{v}} \times \underline{\mathbf{P}} \tag{32}$$

The other variables have been previously defined in equations (16)–(20). If the material in question may be considered a linear function of one material property and the strength and direction of the applied magnetic field, then the magnetization is defined as [2, p. 178; 8, p. 164]

$$\underline{\mathscr{m}} = -2\rho\frac{\partial\Psi}{\partial I_2}\underline{\mathbf{B}} = \frac{\chi^{\mathrm{M}}}{\mu_{\mathrm{o}}(1+\chi^{\mathrm{M}})}\underline{\mathbf{B}} \tag{33}$$

Haus and Melcher [11, pp. 371–377] and Cottingham and Greenwood [7, pp. 92–96] discuss material aspects of magnetization in more detail. As with dielectric susceptibility, magnetic susceptibility may be a function of electro-magnetic frequency, temperature and other physical conditions.

6. ELECTRIC CHARGES

Electric charges appear in two types: free and bound. Free charges arise from electrons in the outer or free atomic shells and from ions. Bound charges are those arising from the molecular geometry and displacement of atomic inner electron shells. Polar molecules are one example of bound charges. By using the definition of apparent and inhomogeneous charge densities from equations (12) and (13), polarization may be explicitly shown in Gauss's law as

$$\nabla\cdot(\varepsilon_{\mathrm{o}}\underline{\mathbf{E}}) = q_{\mathrm{e}} - q_{\mathrm{e}}' - \nabla\cdot\underline{\mathbf{P}}_{\mathrm{e}} - \nabla\cdot\underline{\mathbf{P}}_{\mathrm{p}} \tag{34}$$

which, from equation (11) may be rewritten as

$$\nabla \cdot (\varepsilon_o \underline{\mathbf{E}}) = q_e - q_e' - \nabla \cdot \underline{\mathbf{P}} \tag{35}$$

Introducing the *total* or *free charge density* [10, p. 21] and grouping the polarization charge density with the electric field, Gauss's law becomes

$$\nabla \cdot (\varepsilon_o \underline{\mathbf{E}} + \underline{\mathbf{P}}) = q_o \tag{36}$$

Gauss's law for polarizable media then becomes [10, p. 22]

$$\nabla \cdot \underline{\mathbf{D}} = q_o \tag{37}$$

Note that if polarization is a linear function of the relatively low frequency electric field, $\underline{\mathbf{E}}$, then the displacement vector in the absence of a magnetic field becomes

$$\underline{\mathbf{D}} = \varepsilon_o \underline{\mathbf{E}} + \underline{\mathbf{P}} = \varepsilon_o (1 + \chi^E) \underline{\mathbf{E}} = \varepsilon_o \varepsilon_r \underline{\mathbf{E}} = \varepsilon \underline{\mathbf{E}} \tag{38}$$

7. ELECTRIC CURRENT

In addition to electric currents arising from direct charge motion and magnetization defined in equations (26) and (30), other conduction currents, $\underline{\mathbf{J}}_c$, have been observed and must be taken into account [2, pp. 161–163]. These currents are caused by the Seebeck, Hall, Nernst and a host of other effects which will be described in greater detail in Part 2 of this paper [6]. With this in mind, the Ampere–Maxwell's law with no magnetization or polarization states that [8, p. 86]

$$\nabla \times \frac{\underline{\mathbf{B}}}{\mu_o} = \frac{\partial (\varepsilon_o \underline{\mathbf{E}})}{\partial t} + \underline{\mathbf{J}}_d + \underline{\mathbf{J}}_c \tag{39}$$

Introducing the effects of magnetization and polarization, the Ampere–Maxwell's law of electrodynamics may be rewritten as [10, p. 30]

$$\nabla \times \frac{\underline{\mathbf{B}}}{\mu_o} = \frac{\partial (\varepsilon_o \underline{\mathbf{E}})}{\partial t} + \underline{\mathbf{J}}_p + \underline{\mathbf{J}}_m + \underline{\mathbf{J}}_d + \underline{\mathbf{J}}_c \tag{40}$$

At times it is necessary to define the *total current*, $\underline{\mathbf{J}}$, as the sum of the apparent magnetization, drift and phenomenological currents [10, p. 26]

$$\underline{\mathbf{J}} = \underline{\mathbf{J}}_d + \underline{\mathbf{J}}_c \tag{41}$$

since the contribution to the magnetization current by intrinsic magnetization is zero. Consequently [8, p. 132]

$$\nabla \times \frac{\underline{\mathbf{B}}}{\mu_o} = \frac{\partial \underline{\mathbf{D}}}{\partial t} + \underline{\mathbf{J}}_m + \underline{\mathbf{J}} \tag{42}$$

Additionally, the magnetization and magnetic field intensity vectors are often combined to form the magnetic field vector, $\underline{\mathbf{H}}$, as $\underline{\mathbf{B}} = \mu_o (\underline{\mathbf{H}} + \underline{\mathbf{M}})$. Hence, the Ampere–Maxwell's law for electrodynamics of polarizable and magnetizable media can be written as [8, p. 147]

$$\nabla \times \underline{\mathbf{H}} = \frac{\partial \underline{\mathbf{D}}}{\partial t} + \underline{\mathbf{J}} \tag{43}$$

8. EMFD GOVERNING SYSTEM OF EQUATIONS

Maxwell's equations are the system of linear partial differential equations governing electro-magnetic fields. They are given as [7, p. xv]

Gauss's law

$$\nabla \cdot \underline{\mathbf{D}} = q_o \tag{44}$$

Ampere–Maxwell's law

$$\frac{\partial \underline{\mathbf{D}}}{\partial t} - \nabla \times \underline{\mathbf{H}} = - \underline{\mathbf{J}} \tag{45}$$

Conservation of magnetic flux

$$\nabla \cdot \underline{\mathbf{B}} = 0 \tag{46}$$

Faraday's law

$$\frac{\partial \underline{\mathbf{B}}}{\partial t} + \nabla \times \underline{\mathbf{E}} = 0 \tag{47}$$

Conservation of electric charges [from equations (44) and (45)]

$$\frac{\partial q_o}{\partial t} + \nabla \cdot \underline{\mathbf{J}} = 0 \tag{48}$$

Detailed descriptions of these equations can be found in any number of texts [7, 8, 10, 11].

The equations of motion governing EMFD flow are the Navier–Stokes relations into which electromagnetic effects have been included. A summary of these equations is given below with a derivation from the global conservation law governing continuum mechanics given in Part 2 of this paper [6]. A rigorous derivation of these equations for compressible electro-magnetic fluids with non-linear physical properties is completed by Eringen and Maugin [2, p. 439] and results in

Conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{\mathbf{v}}) = 0 \tag{49}$$

Conservation of momentum

$$\frac{\partial (\rho \underline{\mathbf{v}})}{\partial t} + \nabla \cdot (\underline{\mathbf{v}} \rho \underline{\mathbf{v}} - \underline{\underline{\mathbf{t}}}) - \rho \underline{\mathbf{f}} - \underline{\mathbf{f}}^{\mathrm{EM}} = 0 \tag{50}$$

where the electromagnetic force per unit volume is

$$\underline{\mathbf{f}}^{\mathrm{EM}} = q_o \underline{\mathbf{E}} + \underline{\mathbf{J}} \times \underline{\mathbf{B}} + (\nabla \underline{\mathbf{E}}) \cdot \underline{\mathbf{P}} + (\nabla \underline{\mathbf{B}}) \cdot \underline{\mathbf{M}} + \nabla \cdot (\underline{\mathbf{v}\mathbf{P}} \times \underline{\mathbf{B}}) + \frac{\partial}{\partial t} (\underline{\mathbf{P}} \times \underline{\mathbf{B}}) \tag{51}$$

Conservation of energy

$$\frac{\partial (\rho e_0)}{\partial t} + \nabla \cdot (\rho e_0 \underline{\mathbf{v}}) - \nabla \cdot (\underline{\underline{\mathbf{t}}} \cdot \underline{\mathbf{v}}) + \nabla \cdot \underline{\mathbf{q}} - \rho h - \rho \underline{\mathscr{E}} \cdot \frac{D\left(\dfrac{\underline{\mathbf{P}}}{\rho}\right)}{\mathrm{D}t} + \underline{\mathscr{M}} \cdot \frac{\mathrm{D}\underline{\mathbf{B}}}{\mathrm{D}t} - \underline{\mathbf{J}}_{\mathrm{c}} \cdot \underline{\mathscr{E}} = 0 \tag{52}$$

Entropy Generation (Clausius–Duhem Inequality)

$$\rho \frac{\mathrm{D}s}{\mathrm{D}t} \geqslant \frac{\rho h + \Phi}{\theta} - \nabla \cdot \left(\frac{\underline{\mathbf{q}}}{\theta}\right) - \frac{\underline{\mathbf{q}} \cdot \nabla \theta}{\theta^2} + \frac{\rho \underline{\mathscr{E}} \cdot \dfrac{D\left(\dfrac{\underline{\mathbf{P}}}{\rho}\right)}{\mathrm{D}t} - \underline{\mathscr{M}} \cdot \dfrac{\mathrm{D}\underline{\mathbf{B}}}{\mathrm{D}t} + \underline{\mathbf{J}}_{\mathrm{c}} \cdot \underline{\mathscr{E}}}{\theta} \tag{53}$$

where the conduction current [2, p. 53] is given as

$$\begin{aligned} \underline{\mathbf{J}}_{\mathrm{c}} = {} & (\sigma_1 \underline{\mathscr{E}} + \sigma_2 \underline{\underline{\mathbf{d}}} \cdot \underline{\mathscr{E}} + \sigma_3 \underline{\underline{\mathbf{d}}}^2 \cdot \underline{\mathscr{E}}) + (\sigma_4 \nabla \theta + \sigma_5 \underline{\underline{\mathbf{d}}} \cdot \nabla \theta + \sigma_6 \underline{\underline{\mathbf{d}}}^2 \cdot \nabla \theta) \\ & + (\sigma_7 \underline{\mathscr{E}} \times \underline{\mathbf{B}} + \sigma_8 (\underline{\underline{\mathbf{d}}} \cdot (\underline{\mathscr{E}} \times \underline{\mathbf{B}}) - (\underline{\underline{\mathbf{d}}} \cdot \underline{\mathscr{E}}) \times \underline{\mathbf{B}})) \\ & + (\sigma_9 \nabla \theta \times \underline{\mathbf{B}} + \sigma_{10} (\underline{\underline{\mathbf{d}}} \cdot (\nabla \theta \times \underline{\mathbf{B}}) - (\underline{\underline{\mathbf{d}}} \cdot \nabla \theta) \times \underline{\mathbf{B}})) \\ & + \sigma_{11} (\underline{\mathbf{B}} \cdot \underline{\mathscr{E}}) \underline{\mathbf{B}} + \sigma_{12} (\underline{\mathbf{B}} \cdot \nabla \theta) \underline{\mathbf{B}} \end{aligned} \tag{54}$$

with an identical expression for the heat flux vector, $\underline{\mathbf{q}}$, except that physical constants σ_i ($i = 1, \ldots, 12$) now represent the heat flux coefficients. This is discussed further in Part 2 [6].

9. CONCLUSION

The objective of this paper was to survey background theory to allow initial implementation of a unified EMFD theory presented. To accomplish this, Part 1 presented introductory concepts in electro-magnetic field theory. The sources of both the electric and magnetic fields were derived with emphasis placed on describing physical aspects of polarization and magnetization. Polarization was shown to arise from both natural and induced sources and magnetization was shown to come about from material intrinsic and particle circular motion. The non-linear relations of both polarization and magnetization were presented. However, many materials have linear material properties, considerably reducing the complexity of calculating polarization and magnetization. The sources of electric charge and electric current were discussed. Finally, the equations governing unified EMFD flows as derived by Eringen and Maugin were presented. Part 2 will discuss these equations of motion in more detail and show a detailed comparison of classical EHD and MHD models with the unified EMFD model.

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