INTERACTION OF MAGNETIC FIELD WITH BLOOD FLOW

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Abstract

A mathematical model governing steady laminar flow of an incompressible fluid subjected to a steady magnetic field including internal heating due to Joule effect, heat transfer due to conduction and convection, and thermally induced buoyancy forces, has been developed. The thermal buoyancy was accounted for via Boussinesq approximation. Results of test cases with and without applied magnetic field demonstrate influence of the magnitude of the magnetic field on the recirculating blood flow instability.

Introduction

Understanding the individual effects of the magnetic field on the fluid flow and temperature field is crucial to numerous applications. Of special interest is the effect of the Lorentz force due to the imposed magnetic field on the recirculating blood flow. It is known that the magnetic field can eliminate [1,2,4] vorticity from the flow field, while the electric field can enhance it. Our attention will be restricted to MHD flows, flows with strong applied magnetic field and negligible applied electric field [3].

The objective is to examine the flow patterns resulting from the influence of the applied magnetic and thermal fields.

Analysis

At shear rates below 10 sec^{-1} in vessels smaller than 100μm blood behaves like a non-homogeneous, non-Newtionian fluid, but in many cases of practical interest particulate nature of blood can be neglected. The influence of particles such as red cells can be neglected and blood can be modeled as a homogeneous, Newtonian fluid. This assumption is valid when blood flows through sufficiently large vessels (characteristic length, l_r, should be greater than 100 μm) [5]. It should be noted that red cells contain hemoglobin which defines the magnetic properties of blood.

The following non-dimensional form of the flow field governing equations is obtained

$$v_{i,i} = 0 \quad (1)$$

$$v_{i,t} + \left(v_i v_j - \frac{Ht}{RmRe} H_i H_j\right)_{,j} = - p_{,i} + \frac{1}{Re} v_{i,jj} - \frac{Gr}{Re^2} e_i \theta \quad (2)$$

$$\theta_{,t} + \left(v_j \theta\right)_{,j} = \frac{1}{PrRe} \theta_{,jj} + \varepsilon_m \quad (3)$$

The magnetic field transport equations are

$$H_{i,t} + \left(v_j H_i - v_i H_j\right)_{,j} = \frac{1}{Rm} H_{i,jj} \quad (4)$$

Here, v is the velocity vector, H is the magnetic field vector, p is the combination of hydrostatic, hydrodynamic, and magnetic pressure, θ is the normalized temperature, $\frac{T - T_c}{\Delta T}$, and $\Delta T = T_h - T_c$, where T_h and T_c are the two reference temperatures. The unit vector in the direction of gravitational force is designated as e_i. The term due to Joule heating becomes

$$\varepsilon_m = \frac{EcHt^2}{RmRe^2} \varepsilon_{ijk} \varepsilon_{ilm} H_{k,j} H_{m,l} \quad (5)$$

Non-dimensional numbers: Reynolds number Re, Prandtl number Pr, Grashof number Gr, Eckert number Ec are defined in a standard way [2], while characteristic magnetic non-dimensional numbers are defined as:

Hartman number $$Ht = \mu l_r H_r \sqrt{\frac{\sigma}{c^2 \eta}}$$

Magnetic Reynolds number $$Rm = RePm = \frac{4\pi\mu\sigma v_r l_r}{c^2}$$

Magnetic Prandtl number $$Pm = \frac{4\pi\mu\sigma\eta}{\rho c^2}$$

where μ, σ, η, ρ, c are the magnetic permeability electrical conductivity, blood viscosity, blood density, and the speed of light, respectively. The entire system of partial differential equations was solved iteratively by integrating sequentially a system of Navier-Stokes equations followed by a system of magnetic field equations and transferring the information through source-like terms before the next iteration [2]. Explicit Runge-Kutta time stepping and finite difference scheme were used on a non-orthogonal, curvilinear, boundary conforming coordinate system [6]. Artificial compressibility [7] concept was applied to make the given system of partial differeantial equations hyperbolic in time.

Computational Results

To demonstrate effects of the Lorentz force and Joule heating on the blood flow, we have chosen two configurations. The first configuration consists of a closed rectangular container (two-dimensional), and the second is a three-dimensional pipe with rectangular cross section.

Two-dimensional chamber was discretized with 60×30 clustered grid cells. In all test cases Gr = 3000 , Pr = 3.7, Pm = 1 were used. Uniform steady magnetic field was applied vertically downwards. The following test cases were numerically analyzed:

1. Top and bottom walls are at the same temperature, while side walls are thermally isolated; Ec = 0.01; Ht = 20.; horizontal container. Two recirculating symmetric zones developed that were produced solely by the Joule effect. Maximum temperature develops at the insulated side walls, although temperature differences are in the fifth decimal place. Velocity vectors are scaled by 10^6 (Fig. 1).
2. Top wall is uniformly cold, bottom wall is uniformly hot, while side walls are thermally insulated. Four asymmetric recirculating regions develop in this case produced solely by the effect of thermal buoyancy force. (Fig. 2).
3. Top wall is uniformly cold, bottom wall is uniformly hot ,while side walls are thermally insulated; Ec = 0.01; Ht = 20.; Six recirculating cells developed in this case of a combined wall heating/cooling, Lorentz force, and internal fluid heating by the Joule effect . Velocity vectors are scaled by 10^{-4}(Fig. 3).

The second test case is Hartman flow in three dimensions. The computational domain was discretized with 50×20×20 rectangular clustered cells. Both hydrodynamic and magnetic Reynolds numbers were 10. When the uniform vertical magnetic field is applied the velocity profile flattens out (Fig. 4) and the streamwise pressure gradient is increased.

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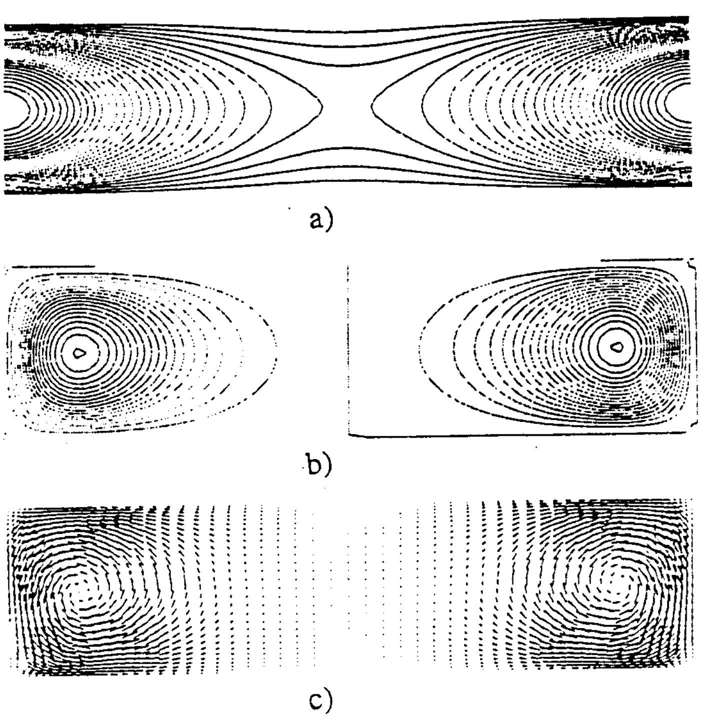

Fig. 1 All four walls are at the same temperature ; Ec = 0.01; Ht = 20.; horizontal container: a) isotherms, b) streamlines, c) velocity vector field.

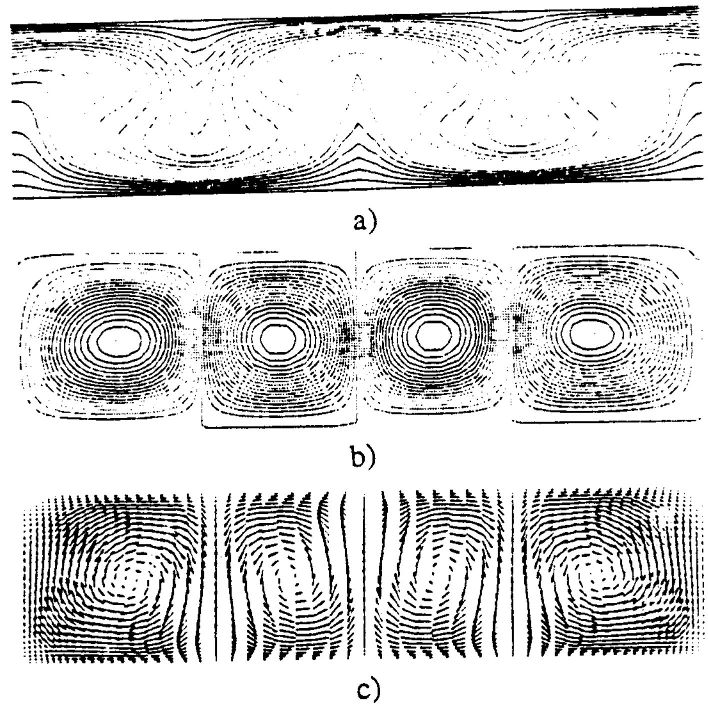

Fig. 2 Top wall is uniformly cold, bottom wall is uniformly hot, while side walls are thermally insulated; no magnetic field; a) isotherms, b) streamlines, c) velocity vector field.

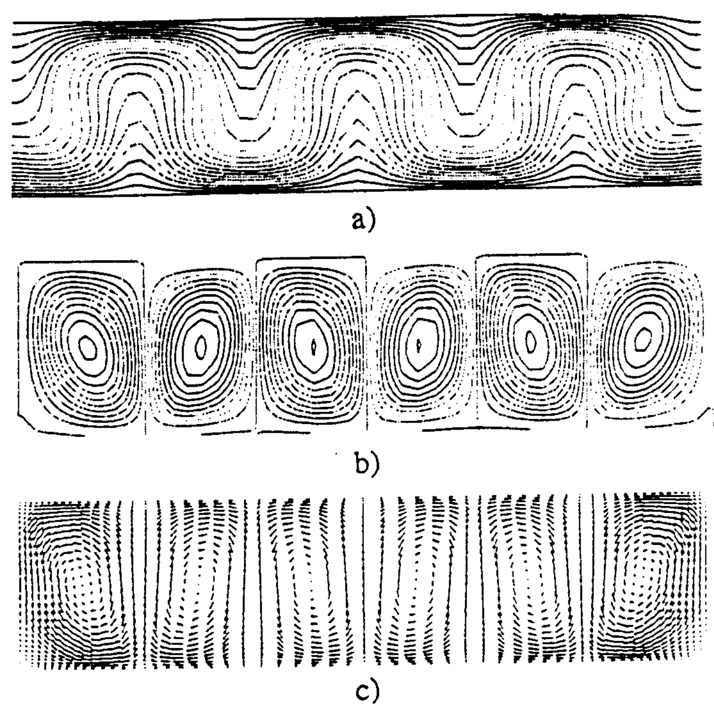

Fig. 3 Top wall is uniformly cold, bottom wall is uniformly hot, while side walls are thermally insulated; Ec = 0.01; Ht = 20.; a) isotherms, b) streamlines, c) velocity vector field.

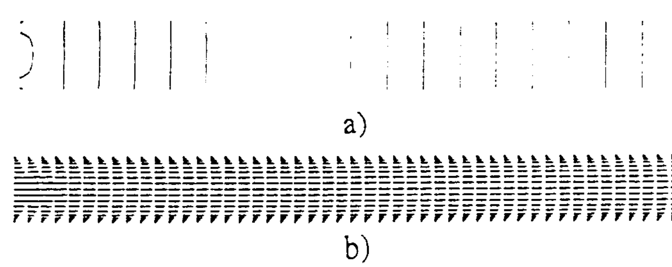

Fig. 4. Hartman flow; Re=10; Rm=10; Ht=10; a) pressure distribution; b) velocity vector field.