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RELIABILITY ENHANCEMENT OF NAVIER-STOKES CODES THROUGH CONVERGENCE ACCELERATION

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INTRODUCTION AND BACKGROUND

Reduction of total computing time required by iterative algorithms for numerical integration of Navier-Stokes equations for three-dimensional, turbulent flows with heat transfer is an important aspect of making the existing and future analysis codes widely acceptable as main components of design tools. Reliability of flow field analysis codes is an equally important item especially when varying input parameters over a wide range of values. Although a variety of methods have been tryed, it remains one of the most challenging tasks to develop and extensively verify new concepts that will guarantee substantial reduction of computing time over a wide range of grid qualities (clustering, skewness, etc.), flow field parameters (Mach numbers, Reynolds numbers, etc.), types and sizes of systems of partial differential equations (elliptic, parabolic, hyperbolic, etc.). The existing techniques are known to have certain drawbacks. Specifically, residual smoothing [1], although simple to implement, is a highly unreliable method, because it can offer either substantial reduction of number of iterations or it can abruptly diverge due to a poor choice of smoothing parameters [2,3,4]. Enthalpy damping [1,5] assumes constant total enthalpy which is incompatible with viscous flows including heat transfer. Multigridding in three-dimensional space is only marginally stable [5] when applied to non-smooth and non-orthogonal grids. GMRES method [6] based on conjugate gradients requires between 20 and 80 solutions to be stored which is intractable in three-dimensional viscous flow computations. Power method [7] which is practically identical to our GNLMR method [8,9] works well with a multigrid code. Without multigridding, it is highly questionable if it would offer any acceleration when applied to a system of nonlinear partial differential equations (PDE). Superstep method [10] seems to offer a good performance for elliptic and parabolic problems. It remains to be developed and tested for hyperbolic systems of PDE. Dominant eigenvalue annihilation [11,12] approach is very simple, yet its performance and especially reliability need to be improved. Time-inclination method [13] does not provide any noticeable acceleration at low and very high Mach numbers. Our own DMR method [14-16] in its present form has the same problem in transonic range. Numerous other methods have been published that are considerably more complex, while less reliable and effective. While a number of methods are capable of reducing the total number of iterations required to reach the converged solution, they require more time per iteration so that the effective reduction in the total computing time is often negligible.

The main commonality to all of these methods is that they all experience loss of their ability to reduce the computing time on highly clustered grids that are needed for turbulent flow computations. Consequently, this will be one of the main objectives of the proposed work which can be briefly summarized in the following the categories.

ZONAL DISTRIBUTED MINIMAL RESIDUAL (ZDMR) METHOD

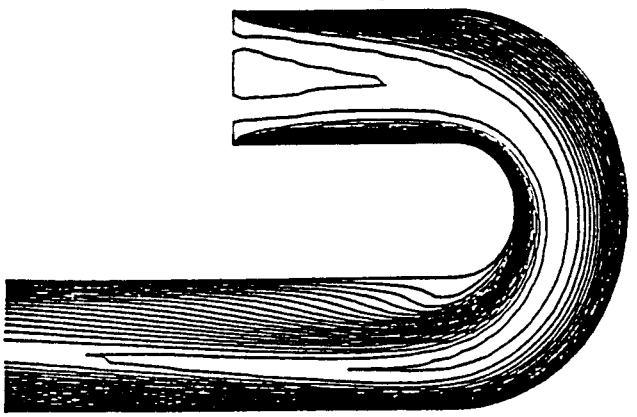
Existing iterative algorithms are based on evaluating a correction to each of the variables and then adding a certain fraction of the corrections to the present values of the variables thus forming the next iterative estimates.

$$q^{n+1} = q^n + \omega\, \delta q^n$$

The optimal values of the relaxation factor can be determined from the condition that the future residual is minimized [17]. Instead, corrections from several consecutive iterations could be saved and then added to the present value of the variable in a weighted fashion, where each of the consecutive iterative corrections is weighted by its own relaxation factor. Optimal values of the relaxation factors can be determined from the condition that the future residual is minimized. This is our GNLMR acceleration method [8,9], summarized as

$$q^{n+1} = q^n + \omega^n\, \delta q^n + \omega^{n-1}\, \delta q^{n-1} + \cdots + \omega^M\, \delta q^M$$

Now, let us consider an arbitrary system of L partial differential equations. If the GNLMR method is applied to each of the L equations so that each equation has its own sequence of M relaxation factors premultiplying its own M consecutive corrections, then this is our DMR acceleration method [14-16] (Figs. 1 and 2).



Isotherm contours

Fig.1 Heat conducting viscous flow inside U-shaped channel

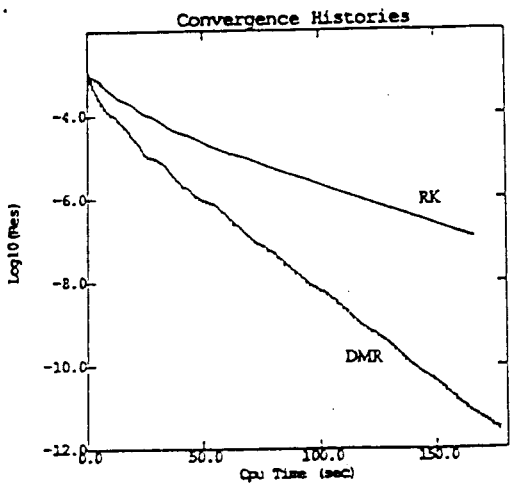


Fig.2 Heat conducting viscous flow inside U-shaped channel

Although offering between 15-75% reduction in total CPU time, the DMR method in its present form is not fully optimized. Specifically, the DMR is presently formulated as

$$q_1^{n+1} = q_1^{n+1} + \omega_1^n\, \delta q_1^n + \omega_1^{n-1}\, \delta q_1^{n-1} + \cdots + \omega_1^M\, \delta q_1^M$$

.....

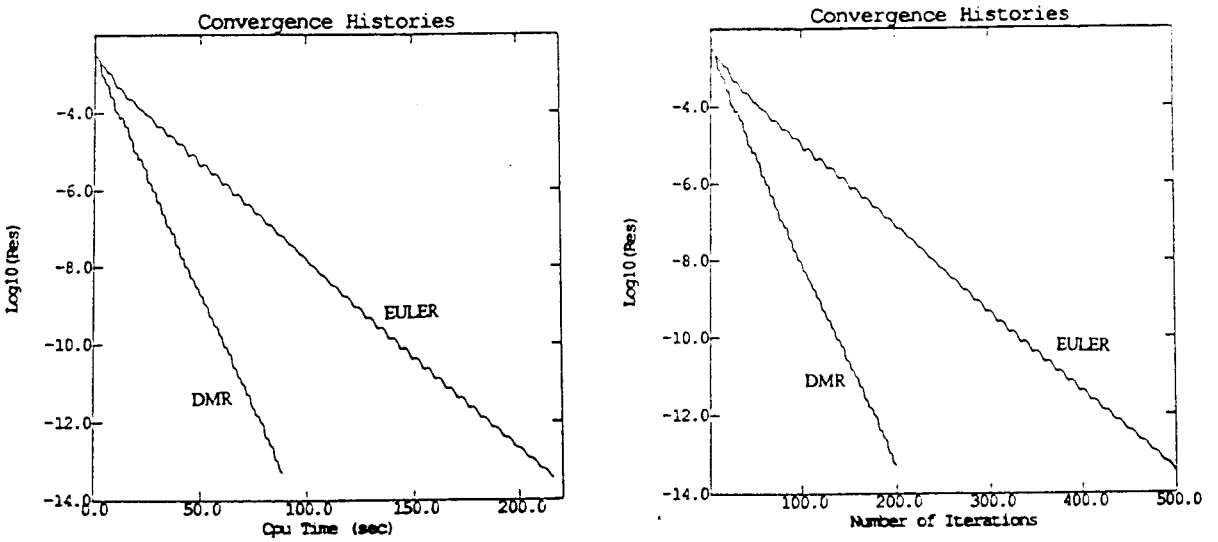
.....

$$q_L^{n+1} = q_L^{n+1} + \omega_L^n\, \delta q_L^n + \omega_L^{n-1}\, \delta q_L^{n-1} + \cdots + \omega_L^M\, \delta q_L^M$$

It is applied periodically where the number of iterations performed with the basic non-accelerated algorithm between two consecutive applications of the DMR is an input parameter. Furthermore, the present formulation of the DMR uses the same values of the L x M relaxation parameters at every grid point although different parts of the flow field converge at different rates. Consequently, we propose to apply the DMR to each distinctive flow subdomain separately. This approach will be especially suitable for parallel processing. Also, frequency of the application of DMR and the number of consecutive solutions combined will be optimized to achieve further gains in convergence rate and to enhance reliability of the DMR [2].

POINTWISE APPLICATION OF THE DMR

In the present formulation of the DMR, the optimal relaxation factors are constant over the entire domain. Nevertheless, if the the relaxation factors are computed using local values of residuals, they vary orders of magnitude. When the relaxation factors are allowed to vary not only from equation to equation in a PDE system, but also from grid point to grid point, and when we use one set of relaxation factors for each iteration level, i.e. M=1 in the DMR formulation, it can be shown that the DMR method is equivalent to the Newton iterations with the improvement that the relaxation factors are optimized. As a result, the DMR no longer needs other scheme to provide the corrections. This is [2] potentially the right answer to the acceleration on extremely clustered grids and we propose to investigate this approach in detail.



Convergence histories of the Euler implicit method for Hiemenz flow

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