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ACCELERATION OF ITERATIVE ALGORITHMS USING DISTRIBUTED MINIMAL RESIDUAL (DMR) METHOD

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ABSTRACT

Our new general method for acceleration of iterative algorithms for arbitrary systems of partial differential equations has been formulated and applied to explicit and implicit algorithms. The method, termed Distributed Minimal Residual (DMR) method, was recently successfully demonstrated with an explicit finite volume algorithm for compressible Euler equations. Here we present general formulation and numerical results obtained with an artificial compressibility algorithm for incompressible Navier-Stokes equations. The DMR is shown to perform equally well with an explicit Runge-Kutta scheme and with an implicit Euler algorithm. Reductions in total CPU time requirement range from 20% - 75% when using the DMR, while computer memory requirements increase approximately two to four times for two-dimensional problems.

BACKGROUND

A new Distributed Minimal Residual (DMR) method for the acceleration of explicit iterative algorithms for the numerical solution of systems of partial differential equations has been recently developed by Lee et al [1-7]. The DMR method belongs to a general class of time extrapolation techniques in which the solution is updated using information from a number of consecutive time steps in such a way that the L2 norm of future global residual is minimized. Unlike in other similar methods, each component of the solution vector is updated using a separate sequence of acceleration factors. The idea of using different sequence of acceleration factors for each component of a solution vector is similar to the concept of dynamic preconditioning. This allows each equation to evolve at its own optimal convergence rate. The acceleration factors are determined from the governing equations so that only a few consecutive solutions are required for each successful application of the DMR method. This acceleration scheme was applied to the system of time-dependent Euler equations of inviscid gasdynamics [2] in conjunction with the finite volume Runge-Kutta explicit time-stepping algorithm. Using DMR without multigridding, between 30% and 70% of the total computational time was saved in the low subsonic compressible flow calculations. The DMR method seems to be especially suitable for stiff systems of equations and can be applied to other systems of differential equations and other numerical algorithms. Specifically, the DMR method was applied to an artificial compressibility, explicit, Runge-Kutta time-stepping algorithm for steady, incompressible, Navier-Stokes equations [3]. A two-dimensional analysis computer code in a generalized curvilinear coordinate system was developed and its accuracy has been compared to known numerical solutions. The algorithm has been successfully accelerated using the DMR method, resulting in 25%-75% reduction in computing time, while requiring only 400% extra storage. Similar results were obtained when applying the DMR method to an Euler implicit algorithm [5,7] when solving the incompressible flow Navier-Stokes equations without and with heat transfer [6,7] and buoyancy forces. The DMR proves [7] to decrease the sensitivity of the basic non-accelerated algorithms to the variations in Reynolds number, grid skewness, grid clustering, and choice of CFL number in implicit algorithms.

ANALYSIS

The local residual, R, of a two-dimensional system of Navier-Stokes equations at iteration level t+1 is given by

$$R^{t+1} = \frac{\partial E^{t+1}}{\partial \xi} + \frac{\partial F^{t+1}}{\partial \eta} - D^2(JQ^{t+1}) + D(JQ^{t+1})$$

Here, Q^{t+1} is the solution vector, J is the Jacobian of geometric transformation $(\partial(\xi,\eta)/\partial(x,y))$, E^{t+1} and F^{t+1} are the components of the inviscid flux vector [3], $D^2(JQ^{t+1})$ is the viscous dissipation and $D(JQ^{t+1})$ is the artificial dissipation. Assume that the solution at the next iteration level t+1 is extrapolated from the previous M consecutive iteration levels. Then, we can say that

$$Q^{t+1} = Q^t + \sum_{m=1}^{M} \Theta^m$$

where $\Theta^m = \begin{bmatrix} \omega_1^m \Delta_1^m \\ \omega_2^m \Delta_2^m \\ \vdots \\ \omega_L^m \Delta_L^m \end{bmatrix}$

Here, ω's are the acceleration factors to be calculated, Δ's are the corrections computed with the original non-accelerated scheme after each of the consecutive iteration levels, M denotes the total number of consecutive iteration levels combined, and L denotes the total number of equations in the system.

Using Taylor series expansion in time and neglecting all terms that are higher than first order in δt, one gets approximately that

$$R^{t+1} = R^t + \sum_{m=1}^{M} [\frac{\partial}{\partial \xi}A^t. + \frac{\partial}{\partial \eta}B^t. - D^2J. + DJ.]\Theta^m$$

The global residual for the entire domain can be defined as

$$\bar{R}^{t+1} = \sum_{D} (R^{t+1}) \cdot (R^{t+1})^T$$

so that minimization of $\bar{R}^{t+1}$ leads to $\frac{\partial \bar{R}^{t+1}}{\partial \omega_r^m} = 0$, that is,

$$\sum_{n}^{M} \sum_{q}^{L} \omega_q^n \, c_{qr}^{nm} = b_r^m$$

representing the system of MxL linear algebraic equations for the L sets of M optimum acceleration factors ω. Here, coefficients c and b are functions of the corrections from M previous iterations [17] that is, they are known. For example, if we are to combine M = 2 consecutive time steps to extrapolate the solution and to solve the two-dimensional incompressible Navier-Stokes equations without heat transfer (L = 3), we need to solve 6 equations for 6 values of ω after every, say, 10 iterations of the basic non-accelerated algorithm.

RESULTS

When applying DMR method to an explicit algorithm for incompressible Navier-Stokes equations using artificial compressibliity concept [8], significant reductions in total number of iterations (Fig. 1) can be achieved with similar percentage reduction in the CPU time. When the DMR is applied to Euler equations of inviscid gasdynamics, the reduction in number of iterations and CPU time becomes even more impressive as the flow becomes effectively incompressible (Fig. 2). Driven cavity problem is a challenging test case for any Navier-Stokes algorithm, because of the corner singularities. Nevertheless, the DMR method applied to both explicit and implicit type algorithms for incompressible Navier-Stokes equations performs equally well (Figs. 3-4) on this test case. Furthermore, Fig. 5 demonstrate that DMR (with CFL = 2.8) is more cost effective than implicit residual smoothing (with CFL = 6) for a test case of an incompressible jet perpendicular to a wall (Hiemenz flow with Re = 400).

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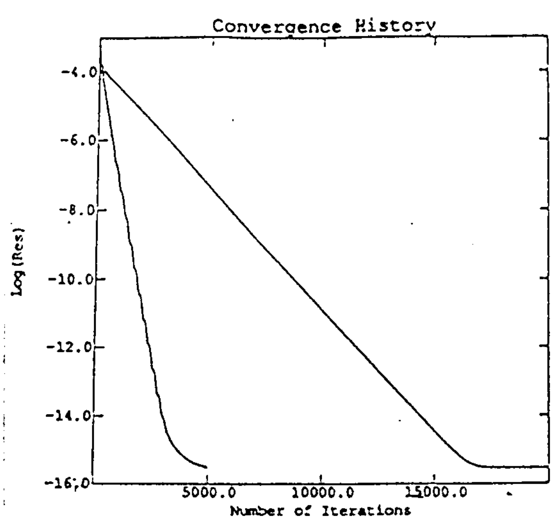

Fig. 1 Convergence histories with and without the DMR method; incompressible Navier-Stokes laminar flow through a ca-cade

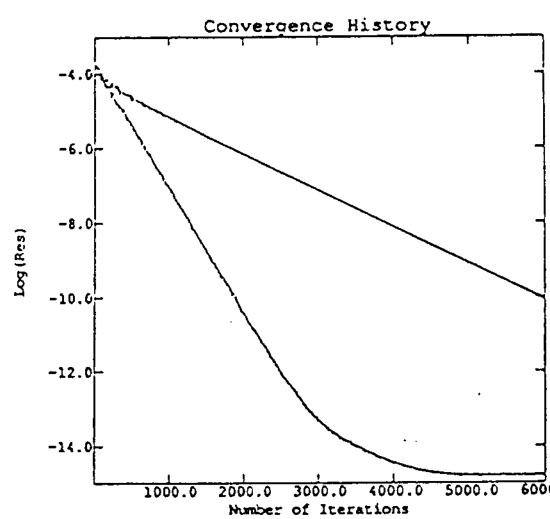

Fig. 2 Convergence histories with and without the DMR method; compressible inviscid flow at M =0.3 around a circle

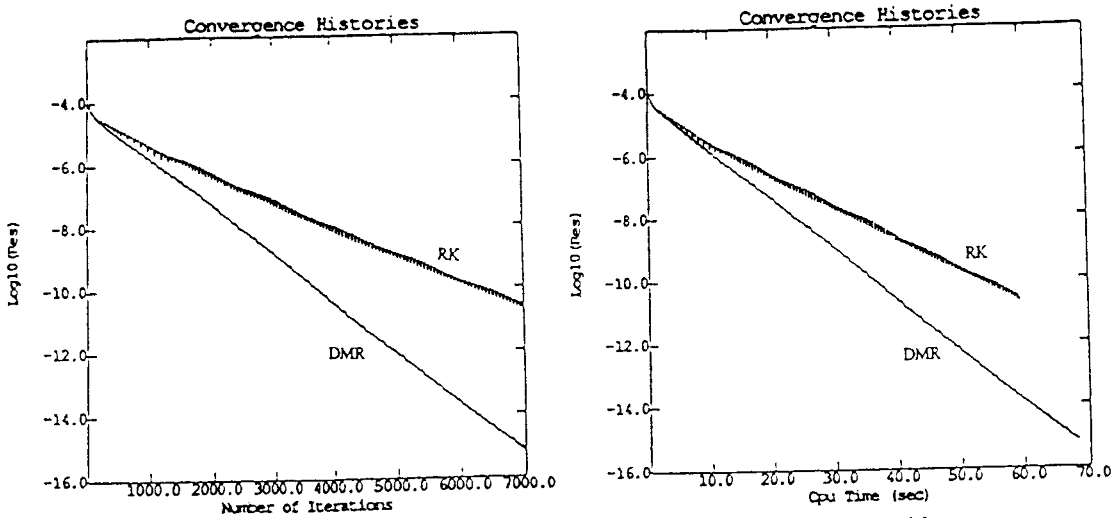

Fig. 3 Convergence histories for the driven cavity problem
Runge-Kutta code; Reynolds number 400; 44 x 44 grid

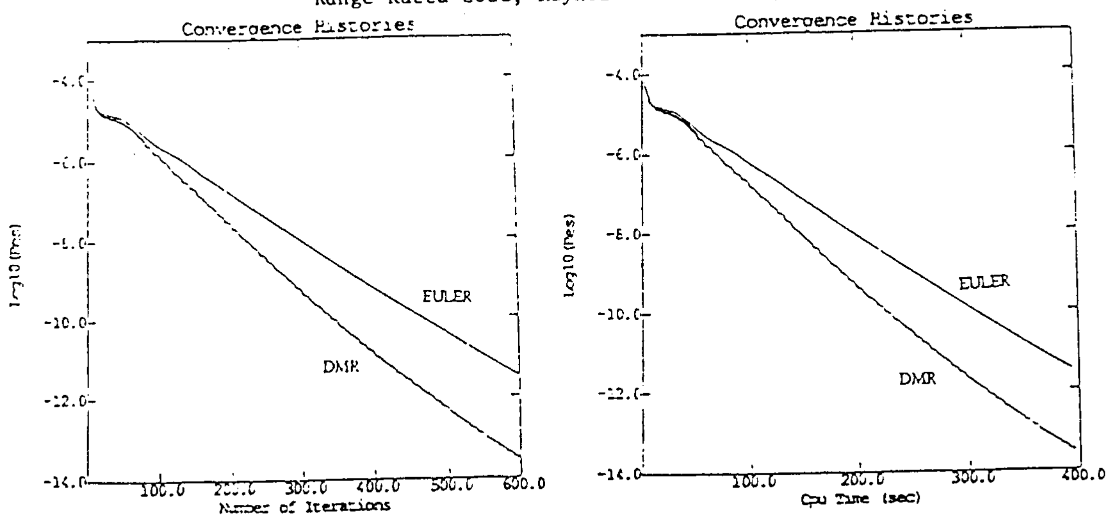

Fig. 4 Convergence histories for the driven cavity problem
Euler implicit code; Reynolds number 400; 44 x 44 grid

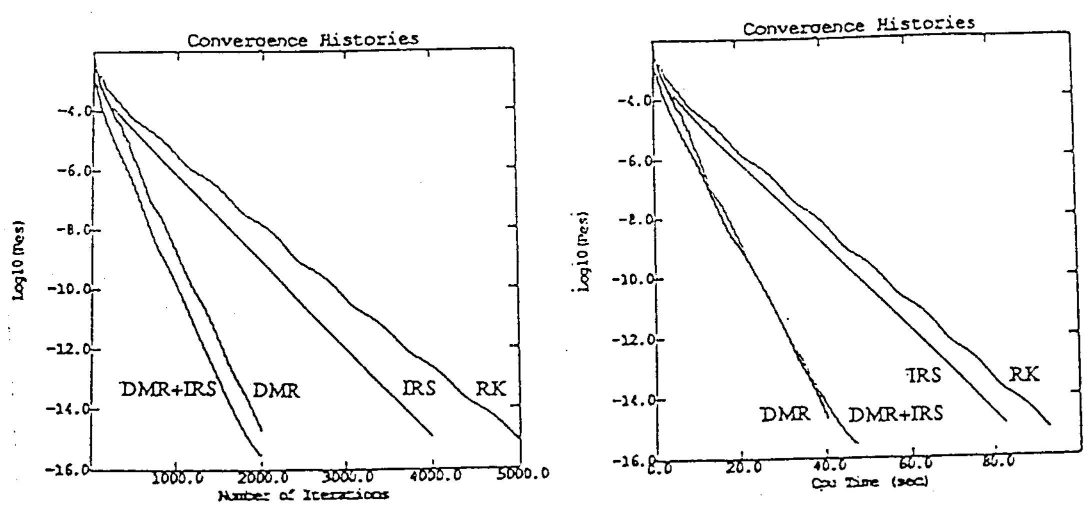

Fig. 5 Convergence histories for the Hiemenz flow with explicit Runge-Kutta scheme (Reynolds number 400; 60 x 30 grid; IRS = implicit residual smoothing; CFL=9)