Finding Unknown Surface Temperatures and Heat Fluxes in Steady Heat Conduction

Thomas J. Martin and George S. Dulikravich

Abstract— **We have developed a new direct noniterative methodology for determining unknown temperatures and heat fluxes on surfaces of arbitrarily-shaped solid objects where the thermal boundary conditions cannot be measured or evaluated otherwise. The method belongs to a general class of algorithms for the solution of steady inverse boundary value problems with the objective of determining the unknown boundary conditions. A requirement for this technique to work is that both temperatures and heat fluxes must be available and specified together creating an over-specified problem on at least a part of the object's surface. Our two-dimensional steady-state inverse boundary value problem—boundary element method (IBVP–BEM) algorithm computes the temperature field within the entire object and simultaneously calculates temperatures and heat fluxes on surfaces where thermal boundary values are unavailable. Our code has been tested on several simple geometries where the boundary conditions and the analytic solutions were known everywhere. The algorithm is highly flexible in treating complex geometries mixed thermal boundary conditions and temperature-dependent material properties. Results were in excellent agreement with the analytic values. The accuracy and reliability of this technique deteriorate when the known boundary conditions are only slightly over-specified and far from the inaccessible boundaries.**

Index Terms—**Heat transfer, inverse problems, boundary conditions, boundary element method.**

I. Introduction

It is often difficult and even impossible to place temperature and heat flux sensors and take measurements on a particular boundary of a conducting solid due to its small size or geometric inaccessibility or because of the severity of the environment. For example, in electronic-component cooling, prediction of detailed surface temperature and heat flux distributions is valuable. These data are very difficult to obtain because of the relatively large sizes of thermal probes as compared to the sizes of the electronic components and because of the electromagnetic field interference between the component and the electrically-operated probes. With our inverse method these steady-state unknown thermal boundary values are deduced from additional temperature or heat flux measurements made at a finite number of points within the solid or on some other accessible boundaries of the solid. Most of the existing methods for solving such steady-state inverse boundary value problems are iterative and based on an asymptotic time limit of one-dimensional transient heat conduction problem [1], [2]. In most of these inverse techniques the random noise due to round-off errors tends to magnify as the solution proceeds in time and quickly produces a useless solution, especially as the distance between the boundary and the over-specified information increases [3]. Various smoothing techniques for reducing this error growth could be used, but the effect of these operations on the accuracy of the solution is not easy to evaluate [4].

Our IBVP–BEM is noniterative and it does not utilize any artificial smoothing technique since it is not limited to transient or one-dimensional problems. In general, a matrix relating the over-specified data to the unknown thermal values on inaccessible boundaries is generated numerically, and then inverted using one of the existing specialized algorithms [5]. Our direct approach has been shown [6], [7] to compute meaningful and accurate thermal fields in a single analysis using a straightforward modification to the boundary element method (BEM). When analyzing steady-state heat conduction using the BEM [8], [9] either temperatures, T, or heat fluxes Q, are specified everywhere on the boundary of the solid where one of these quantities is known while the other is unknown. When performing IBVP–BEM of the steady-state heat conduction, both T and Q must be specified on a part of the solid's boundary, while both T and Q are unknown on another part of the boundary. Elsewhere on the solid's boundary, either T or Q should be applied. The boundary sections where both T and Q are specified simultaneously are called the over-specified boundaries. If temperatures or heat fluxes are also known at isolated points within the solid, they can be directly added to the IBVP–BEM equation set in order to enhance its accuracy.

II. Analytical and Numerical Formulations

Steady-state heat conduction in a homogeneous solid with a temperature-dependent coefficient of thermal conductivity, $k(T)$, is given by an elliptic partial differential equation

$$\nabla \cdot (k(T)\nabla T) = 0. \tag{1}$$

Using Kirchoff's transformation,

$$u = \int_0^T \frac{k(T)}{k_o}\, dT \tag{2}$$

where k_o is the reference coefficient of thermal conductivity, this equation can be converted to a Laplace's equation for the heat function, u. The boundary integral equation (BIE)

Manuscript received May 1994; revised September 12, 1994. This paper was presented at the Intersociety Conference on Thermal Phenomena, Washington, DC, May 5–7, 1994.

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IEEE Log Number 9411511.

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for Laplace's equation is obtained from the weighted residual statement or Green's Theorem

$$c(\mathbf{x}_i)u(\mathbf{x}_i) + \int_\Gamma u^* q \, d\Gamma = \int_\Gamma q^* u \, d\Gamma \tag{3}$$

where the integration is over the solid's boundary, Γ. Here $q = \partial u/\partial n, u^*$ is the fundamental solution [9], $q^* = \partial u^*/\partial n, n$ is the direction of the outward normal to the boundary, Γ, and $c(\mathbf{x}_i)$ is a free term arising from the integration over the singularity in the sense of the Cauchy principal value at the point $\mathbf{x}_i$ so that $c(\mathbf{x}_i) = 0.0$ when $\mathbf{x}_i$ is outside the domain $c(\mathbf{x}_i) = 1.0$ when $\mathbf{x}_i$ is inside the domain, and $c(\mathbf{x}_i) = \theta(\mathbf{x}_i)/2\pi$ when $\mathbf{x}_i$ is on the boundary, and $\theta(\mathbf{x}_i)$ is the internal angle between two neighboring boundary elements). The fundamental solution is a general Green's function solution for a point-source subject to the homogeneous boundary conditions. For the two-dimensional Laplace's equation it is given as

$$u^* = \frac{1}{2\pi}\ln\left(\frac{1}{|\mathbf{x}_j - \mathbf{x}_i|}\right) \tag{4}$$

where $\mathbf{x}_j$ is the position vector of the point of integration and $\mathbf{x}_i$ is the control point. A set of N boundary integral equations exists for each of the N control points on the boundary. The boundary integrals over the body's boundary may be discretized into a number of boundary panels connecting the N boundary points. The functions u and q are either constant over each panel or they may be linearly, quadratically, etc. distributed over each panel. After adding the contributions from each boundary integral the whole set of boundary integral, equations can be written in matrix form [9] as

$$[\mathbf{H}]\{\mathbf{U}\} = [\mathbf{G}]\{\mathbf{Q}\}. \tag{5}$$

For example, for constant elements the entries to the $[\mathbf{H}]$ and $[\mathbf{G}]$ matrices are

$$h_{ij} = \int_{\Gamma_j} q^* \, d\Gamma \quad g_{ij} = \int_{\Gamma_j} u^* \, d\Gamma. \tag{6}$$

If the domain's boundary is discretized with N points, initially there will be a total of $2N$ unknowns in the equation set. For a well-posed boundary value problem, at least one of the functions, u or q, (either Dirichlet or von Neumann boundary condition) will be known at each boundary point resulting in an equation set composed of N unknowns and N equations. For example, if we have a four-point quadrilateral domain with Dirichlet boundary conditions ($u = U$) specified everywhere on its boundary, the left-hand side of the discretized BIE may be multiplied out to form a vector of knowns, $\{\mathbf{F}\}$, while the right-hand side remains in the form $[\mathbf{A}]\{\mathbf{X}\}$. In this example, the equation set becomes a system of four linear algebraic equations that can be solved for the unknowns, q.

But, if the boundary conditions in the above example are partially not known, the problem becomes ill-posed although a solution may still be obtained. For example, if at each of the two boundary points both $u = U$ and $q = Q$ are known, while at the other two points neither is known, the BIE equation set before any rearrangement appears as

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{bmatrix} \begin{Bmatrix} U_1 \\ u_2 \\ U_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \begin{Bmatrix} Q_1 \\ q_2 \\ Q_3 \\ q_4 \end{Bmatrix}. \tag{7}$$

In order to solve this set, all of the unknowns will be collected on the left-hand side while all of the knowns are assembled on the right yielding

$$\begin{bmatrix} h_{12} & g_{12} & h_{14} & g_{14} \\ h_{22} & g_{22} & h_{24} & g_{24} \\ h_{32} & g_{32} & h_{34} & g_{34} \\ h_{42} & g_{42} & h_{44} & g_{44} \end{bmatrix} \begin{Bmatrix} u_2 \\ q_2 \\ u_4 \\ q_4 \end{Bmatrix} = \begin{bmatrix} h_{11} & g_{11} & h_{13} & g_{13} \\ h_{21} & g_{21} & h_{23} & g_{23} \\ h_{31} & g_{31} & h_{33} & g_{33} \\ h_{41} & g_{41} & h_{43} & g_{43} \end{bmatrix} \begin{Bmatrix} U_1 \\ Q_1 \\ U_3 \\ Q_3 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{Bmatrix}. \tag{8}$$

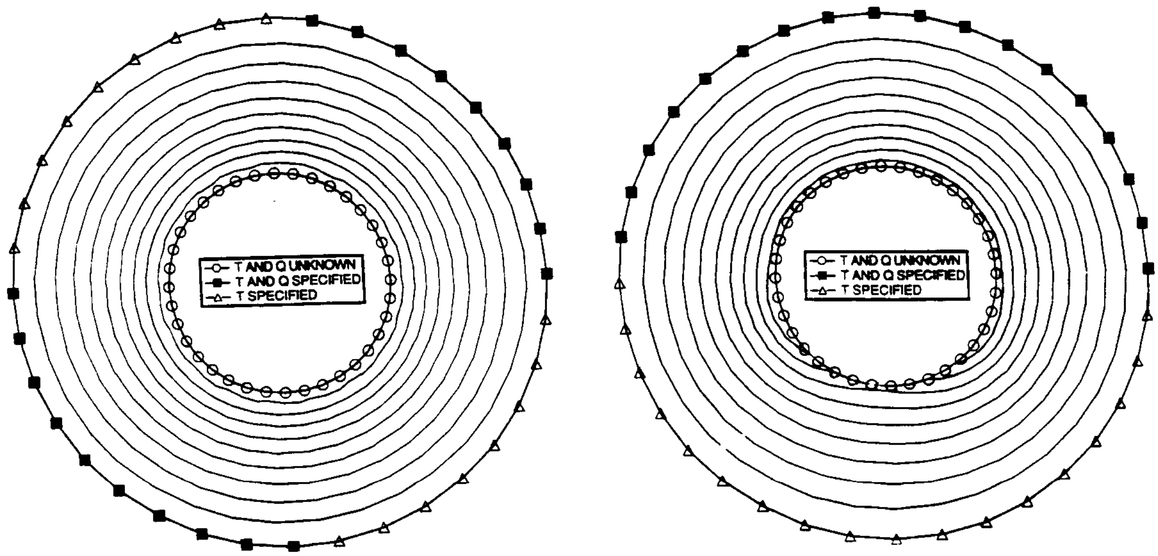
Since the vector on the right-hand side is known, it may be multiplied by its coefficient matrix to form a vector of knowns, $\{\mathbf{F}\}$. The left-hand side remains in the form $[\mathbf{A}]\{\mathbf{X}\}$. This equation set is highly singular and most standard matrix solvers will not work well enough to produce a correct solution. There are several techniques [8] for dealing with sets of equations that are either singular or very close to singular. We have decided to use singular value decomposition (SVD) method [5], [10] to solve the equation set and, most often, a solution to the highly singular BIE formulation was obtained. Since the SVD algorithm is capable of solving nonsquare matrices, the number of unknowns in the equation set need not be the same as the number of equations. Thus, virtually any combination of boundary conditions will yield at least some solution. Also, additional equations may be added to the equation set if, for example, temperature or heat flux measurements are known at locations within the solid domain.

III. Verification of the Nonlinear BEM Analysis Code

The accuracy of the BEM code for analysis of nonlinear heat conduction was tested on an example consisting of a 1×10^{-2} m by 1×10^{-3} m rectangular plate. The plate circumference was discretised with 22 equal-length linear boundary panels. The two vertical sides of the plate were kept adiabatic ($Q = 0$) and the two horizontal sides were subject to different uniform temperatures ($T_{\text{hot}} = 100$ K and $T_{\text{cold}} = 0$ K). The temperature-dependent thermal conductivity was given as

$$k(T) = k_o(B + CT) \tag{9}$$

where $k_o = 100 \text{ Wm}^{-1}\text{K}^{-1}$ and $B = 1.0$. Computed temperatures were collected for various degrees of thermal nonlinearity



(a) (b)

Fig. 1. Circular disk: geometry of the BEM points on the outer and inner boundaries, boundary condition types and isotherms computed with the BEM for each of the test cases.

given by the parameter, C. The results were compared with the one-dimensional analytic solution [11] given as

$$\frac{C}{2}T^2 + T = \left(T_{\text{hot}} + \frac{C}{2}T_{\text{hot}}^2\right) - \left(1 + \frac{C}{2}(T_{\text{hot}} + T_{\text{cold}})\right) \times \frac{(z - z_{\text{hot}})}{(z_{\text{cold}} - z_{\text{hot}})}(T_{\text{hot}} - T_{\text{cold}}). \quad (10)$$

The BEM results compared well with the analytic solution, averaging an error of less than 0.5%.

The behavior of this analysis algorithm was also examined for steady-state heat conduction in an annular solid disk. The outer radius of the disk was 1.2 m and the centrally-located circular hole had a radius of 0.5 m. The analytic solution for this problem was developed by applying temperature boundary conditions of $T_{\text{out}} = 100$ K on the outer boundary and $T_{\text{in}} = 50$ K on the inner boundary. The thermal conductivity of the solid was considered to be constant. The analytic solution for the temperature field within the disk is easily found as

$$T(r) = A + B \ln r \quad (11)$$

where $A = 89.59$ K and $B = 57.11$ K·m^{-1}. The radial temperature gradient is then

$$Q(r)/k = -\nabla T = -dT(r)/dr = B/r \quad (12)$$

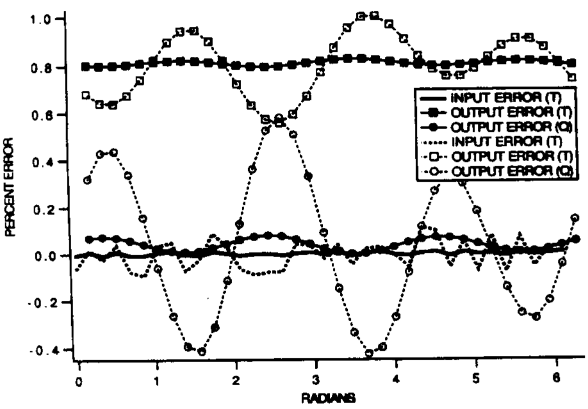
which yields $(dT/dr)_{\text{out}} = -47.59$ K·m^{-1} and $(dT/dr)_{\text{in}} = 114.22$ K·m^{-1} as normal temperature derivatives through the outer and inner boundaries respectively. The BEM analysis algorithm was run on the same problem. The geometry was discretized with 36 flat panels on outer and 36 flat panels on inner boundary. The BEM analysis program predicted the temperature field in the annular solid region which averaged only a 0.3% error versus the analytic solution.

IV. Testing IBVP–BEM

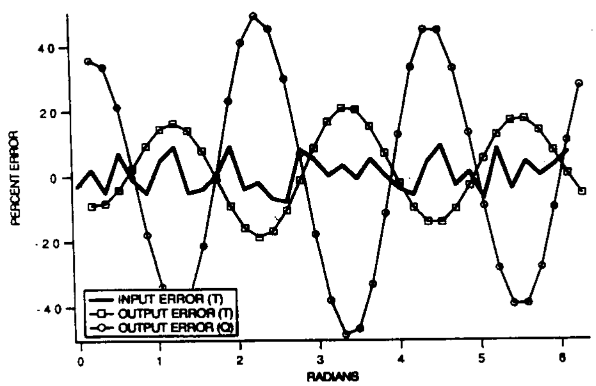
Next, a BEM computer program was developed using the inverse boundary condition theory discussed in this paper. The accuracy of this IBVP–BEM code was first verified for a solid square plate. The plate measured 6 × 10^{-2} m on each side having constant thermal conductivity. The top and bottom boundaries were specified to be adiabatic ($dT/dy = 0$), while the left side of the plate was over-specified with a temperature boundary condition ($T = 300$ K) and a normal derivative boundary condition ($dT/dx = -50$ K·m^{-1}). The right side of the plate was considered to be inaccessible and as such, both temperature and heat flux were treated as unknown on this boundary. The plate circumference was discretized with 32 equal-length panels. The IBVP–BEM solution was compared to the analytic solution and was found to be highly accurate with an error in temperature of less than 0.05% and an error in heat flux of less than 0.1%.

In order to further study the feasibility and accuracy of the steady-state IBVP–BEM solutions, several tests were performed [9] utilizing the same annular geometry and outer boundary thermal data as in the previous BEM analysis mode testing. We will demonstrate only a few cases here where the circular disk was discretized with 36 flat panels on both the inner and the outer boundaries. The objective was to find the unknown temperatures and heat fluxes on the inner circular boundary.

1) Disk-a: Constant temperature boundary conditions were specified on the entire outer boundary, and the additional constant heat flux boundary conditions were over-specified in the first and third quadrants of the outer boundary only. The IBVP–BEM solution set had 54 knowns, 90 unknowns, and 72 equations. The predicted temperature on the inner boundary was oscillatory, but had an rms error of only 0.85%. The predicted heat flux on the inner boundary was also oscillatory and averaged an rms error of about −2.0%. The predicted

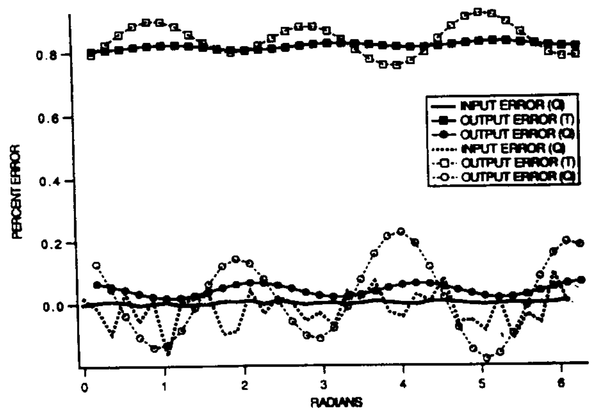


(a)

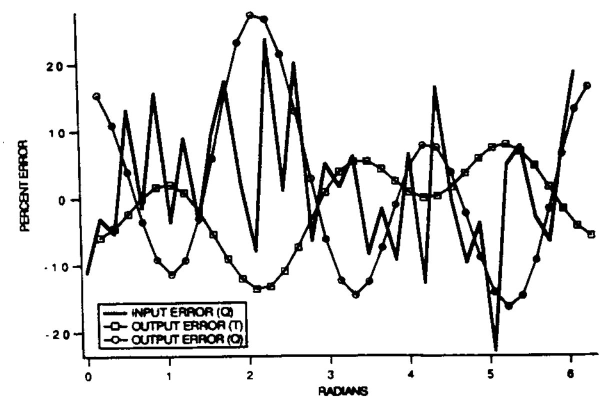


(b)

Fig. 2. Circular disk: relative percentage errors (IBVP–BEM versus analytic solution) of the inner boundary temperatures: (a) two cases of small outside temperatures errors; (b) one case of large outside temperature errors.



(a)



(b)

Fig. 3. Circular disk: relative percentage errors (IBVP–BEM versus analytic solution) of the inner boundary heat fluxes: (a) two cases of small outside heat flux errors; (b) one case of large outside heat flux errors.

temperature field (Fig. 1(a)) compared well with the analytic solution with nearly axisymmetric isotherms.

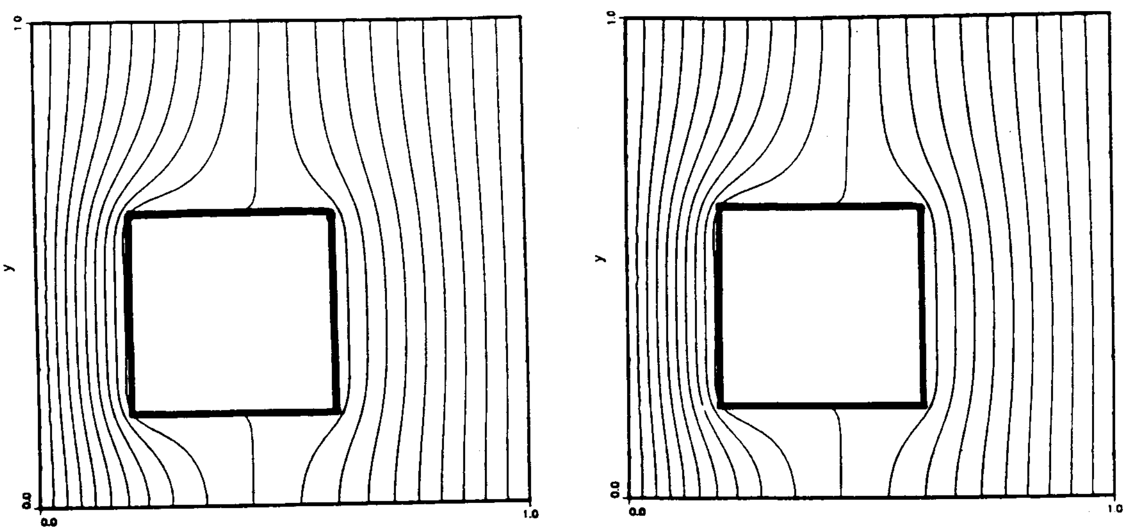
2) Disk-b: Constant temperature boundary condition was specified on the entire outer boundary, while constant heat flux boundary condition was over-specified only on the upper half of the outer boundary. As in the previous test the IBVP–BEM solution set contained 54 unknowns, 90 unknowns, and 72 equations. The predicted temperature field (Fig. 1(b)) was slightly asymmetric about the x-axis, but was very symmetric about the y-axis. The greatest error in the predicted temperature field occurred in the bottom half of the annular solid region. The errors in the predicted temperatures and heat fluxes are quite oscillatory in nature and noticeably peak at about 20% at the very bottom of the solid disk which is the point farthest from the over-specified data.

V. Sensitivity of the IBVP–BEM to Boundary Condition Errors

Next, truly random errors were intentionally introduced by hand into the outer boundary temperature and heat flux over-specified data. The output temperatures and heat fluxes computed by the IBVP–BEM on the hole boundary were then observed in terms of their accuracy with respect to the analytic solution. The standard deviation of the output (inaccessible boundary) errors stayed at about the same order of magnitude until a standard deviation of 0.1 was reached. Further increase in the input (overspecified outer boundary) errors caused a linear increase in the output (inner boundary) errors. In a response to the very small input errors in the outer boundary temperature data (Fig. 2(a)) the error in the output data is shown to be about two to four times larger than the input error. In addition, the output errors in both temperatures and heat fluxes were almost sinusoidal in shape for not only small input error magnitudes, but also for larger input errors (Fig. 2(b)). Sensitivity of the output errors to various levels of input errors in heat flux data is shown in Fig. 3(a) and (b). When comparing these figures to those of the previous two, the output data is observed to be more sensitive to errors in the input temperature data than to the input heat flux data. In addition, the amplitude of the sinusoidal error in the heat flux output was of the same order of magnitude as the amplitude of the input heat flux error.

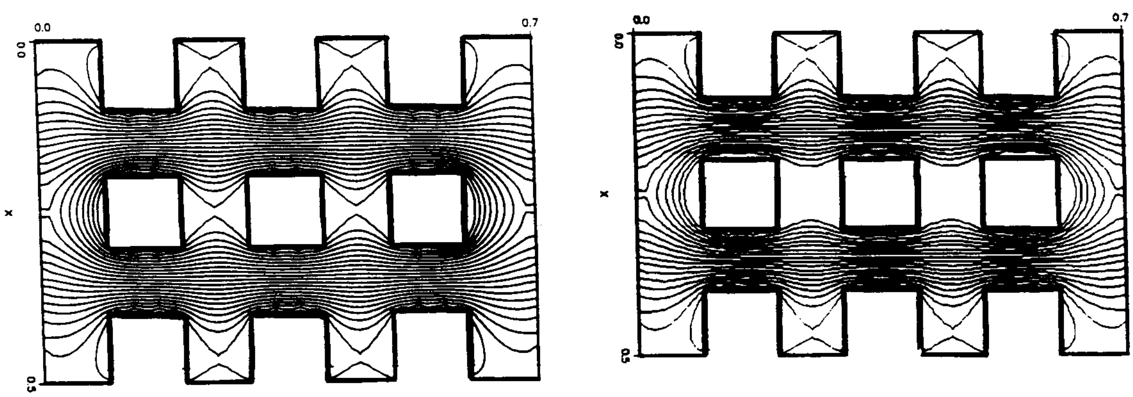
VI. Testing IBVP–BEM for Multiply Connected Domains

The feasibility of the IBVP–BEM technique in computing unknown temperature and heat flux data on inaccessible boundaries was next demonstrated on a two-dimensional square plate with a single noncentrally located square hole



(a) (b)

Fig. 4. Square plate with a square hole: isotherms obtained: (a) with analysis BEM code; (b) with IBVP–BEM code assuming that nothing is known on the walls of the square hole.



(a) (b)

Fig. 5. Square plate with three square holes and six legs: isotherms obtained: (a) with analysis BEM code; (b) with IBVP–BEM code assuming that nothing is known on the walls of the three square holds.

(Fig. 3(a)). The plate measured 1×10^{-2} m on each side and the square hole was 0.4×10^{-2} m on each side. In order to determine if the IBVP–BEM produced correct results, the temperature field within the solid was first determined using our BEM analysis code with well-posed boundary conditions. The left side of the plate was specified with $T = 1$ K, the right side was specified with $T = 0$ K, the top and bottom sides were specified to be adiabatic ($Q = 0$) and everywhere on the square hole boundary $T = 0.5$ K was specified. As a by-product of this BEM analysis (Fig. 4(a)), the temperatures on the top and the bottom outer walls and heat fluxes on the outer side walls of the plate were obtained. Then, both the temperatures and the heat fluxes were specified on all four outer walls of the plate and nothing was specified on the four walls of the square hole. Our IBVP–BEM solved this problem and the resulting temperature field is practically indistinguishable (Fig. 4(b)) from the one obtained in the preceding BEM analysis (Fig. 4(a)).

A more complex configuration was tested next that simulated cooling of a simplified electronic chip geometry which consisted of a rectangular plate with three square holes and six rectangular legs (Fig. 5(a)). First, we analyzed the thermal field in this configuration using our BEM analysis code. Temperatures on the top and bottom boundaries were specified as $T = 0$ K and on the walls of the three square holes as $T = 100$ K. Temperature derivatives normal to the outer side walls were specified as $dT/dn = -1\ \mathrm{K{\cdot}m^{-1}}$. The BEM solution of this well-posed boundary value problem is shown in Fig. 5(a). As a by-product of this BEM analysis, we obtained temperatures on the outer side walls and heat fluxes on the outer top and bottom walls (boundaries of the six rectangular legs). The inverse boundary value problem was then formulated by specifying both temperatures and the temperature normal derivatives on the outer side walls and on the outer top and bottom walls (boundaries of the six rectangular legs). It was assumed that nothing is known on the walls of the three square holes. The resulting thermal field (Fig. 5(b)) obtained by our IBVP–BEM is for all practical purposes identical to the thermal field obtained from the well-posed BEM analysis (Fig. 5(a)).

VII. Conclusion

We have developed a simple numerical algorithm capable of determining unknown steady temperatures and heat fluxes on boundaries of conducting solids where such quantities are unavailable. This means that given any over-specified thermal boundary conditions (such as temperatures and heat fluxes on boundaries where such data are readily available) the algorithm computes any unknown thermal boundary conditions on boundaries where the thermal boundary values are unavailable. A two-dimensional steady-state IBVP–BEM program has been developed to perform automatic noniterative determination of both temperatures and heat fluxes on parts of the interior and exterior boundaries. Our method is very fast since it uses BEM allowing it to be a noniterative direct approach. A typical two-dimensional BEM analysis or IBVP run consumes about one second on a personal computer. Our method is therefore an excellent candidate for thermal design optimization since it is easily applicable in three-dimensional arbitrarily shaped cooled configurations. Accuracy of our IBVP–BEM computer code tested on several simple geometries where the analytic solution for steady heat conduction is known, was found to be in excellent agreement with the analytic values in the regions relatively close to the over-specified data. The accuracy of the method deteriorated with the increased distance from the over-specified boundaries and the decrease of the amount of the available over-specified data. The method's accuracy and reliability strongly depend on the accuracy of the singular value decomposition matrix inversion routine. Furthermore, the method can be used economically and noniteratively for arbitrarily shaped objects with a reasonable number of domains made of different materials, but would have to become an iterative method for applications to objects having inhomogeneous thermal properties.

References

[1] J. V. Beck, B. Blackwell, and C. R. St. Clair, Jr., *Inverse Heat Conduction: Ill-Posed Problems.* New York: Wiley, 1985.

[2] E. C. Hensel, Jr., *Inverse Theory and Applications for Engineers.* Englewood Cliffs, NJ: Prentice-Hall, 1991.

[3] R. G. Hills and E. C. Hensel, Jr., "One-dimensional nonlinear inverse heat conduction technique," *Numerical Heat Trans.*, vol. 10, pp. 369–393, 1986.

[4] D. A. Murio, *The Mollification Method and the Numerical Solution of Ill-Posed Problems.* New York: Wiley, 1993.

[5] R. D. Throne and L. G. Olson, "A generalized eigensystem approach to the inverse problem of electrocardiography," *IEEE Trans. Biomedical Eng.*, vol. 41, no. 6, pp. 1-9, June 1994.

[6] T. J. Martin and G. S. Dulikravich, "A direct approach to finding unknown boundary conditions in steady heat conduction," in *Proc. 5th Ann. Thermal Fluids Workshop*, 1993, pp. 137–149.

[7] T. J. Martin and G. S. Dulikravich, "Inverse determination of temperatures and heat fluxes on inaccessible surfaces," in *Proc. BETECH*, 1994, pp. 69–76.

[8] C. A. Brebbia and J. Dominguez, *Boundary Elements an Introductory Course.* New York: McGraw-Hill, 1989.

[9] T. J. Martin, "Inverse design and optimization of two- and three-dimensional coolant flow passages," M.S. thesis, Department of Aerospace Engineering, The Pennsylvania State University, 1993.

[10] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in FORTRAN*, 2nd. ed. Cambridge, UK: Cambridge University Press, 1992.

[11] A. J. Chapman, *Heat Transfer.* New York: Macmillan, 1960.

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