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AERODYNAMIC SHAPE DESIGN

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1. SUMMARY

Design of aerodynamic shapes can be accomplished by using the methodologies from computational fluid dynamics and optimization. Two basic categories of the inverse (design) formulations are surface flow design and flow field design. A number of methods in both categories have been discussed and critically evaluated. Open questions remain to be specification of a more appropriate surface pressure, acceleration of iterative algorithms, increased versatility of the design methods, direct use of the existing and future flow field analysis software.

2. PREFACE

The field of aerodynamic shape design involves the ability to determine the geometry of an aerodynamic object that will satisfy the governing flow field equations and specified boundary conditions. For example, it is possible to determine the coordinates of an airfoil if a surface pressure distribution is specified. The resulting designs can be subject to certain specified constraints. Examples include finding aerodynamic configurations compatible with entirely shock-free transonic flow fields, obtaining shapes of objects that produce flow fields with minimum entropy generation, minimum noise generation, etc.

One of the first meetings on the general topic of shape design was the International Conferences on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES). The first ICIDES was organized and held October 17-18, 1984, at the University of Texas at Austin, while ICIDES-II was held October 26-28, 1988, at the Pennsylvania State University. They were followed by an AGARD Specialists' Meeting on Computational Methods for Aerodynamic Design (Inverse) and Optimization held in Loen, Norway, on May 22-23, 1989.

In the general field of aerodynamics as well as in any other field theory, we are basically faced with two problems: analysis and design. In the case of an analysis (direct problem) we are asked to predict the details of a flow field if the geometry of the flying object is given. In the case of a design (inverse problem) we are asked to predict the detailed geometry of the flying object so that it is compatible with specified features of the flow field.

Depending on the prescribed desired features of the flow field, the design (inverse) can be divided into two general categories: surface flow design and the flow field design [5,6]. Surface flow design is achieved by specifying a certain flow parameter (pressure, Mach number, etc.) on the surface of the flying object and finding the shape that will generate these surface conditions without regard for the rest of the flow field. The flow field design, on the other hand, enforces certain global flow field features (shock-free flow, minimal entropy generation, etc.) at every point of the flow field by finding the shape that will satisfy these global constraints at every point of the flow field. A large number of methods for performing the surface flow design have been developed, while only a few methods for the flow field design are known to exist.

Mathematical models used in the design are based on partial differential equations, integral equations, and algebraic equations. For example, Zhukovski conformal mapping is actually a technique for designing a class of airfoil shapes having specified surface distribution of pressure that corresponds to a flow around a rotating circle. Although we are dealing here with a simple algebraic expression, it is based on an integral equation formulation (a point-dipole and a point vortex) which resulted from the Laplace operator (a partial differential equation) governing the flow field. Thus, any global conformal mapping can be viewed as a very special method for designing certain simple shapes in a steady, planar, irrotational, inviscid flow field. Moreover, global conformal mapping is the only example that comes to mind as a method which combines the surface flow design concept and the flow field design concept by guaranteeing that the resulting shapes will have the specified surface distribution of the flow parameters while the flow field will be irrotational.

In a more general situation, arbitrary distribution of the surface flow parameters or the flow field distribution of the flow parameters could result in shapes that do not have to be physically meaningful and manufacturable. For example, the lower surface and upper surface of an airfoil could either cross over ("fish tail shapes") or never meet (open trailing edge shapes) although these solutions are mathematically acceptable (Fig. 1). Obviously, the problem is in choosing an appropriate surface distribution of the flow parameters. On the other hand, when performing the flow field design by minimizing the entropy generation at every point in the field, the resulting shape will most likely have zero thickness and no stagnation points, that is, the optimal shape will most likely be a flat plate. Certain constraints on the acceptable final geometry are needed especially since the final aerodynamic design is often incompatible with the minimum acceptability criteria posed by heat transfer, structural dynamics and vibrations, acoustics, and manufacturing, just to name a few [5,6].

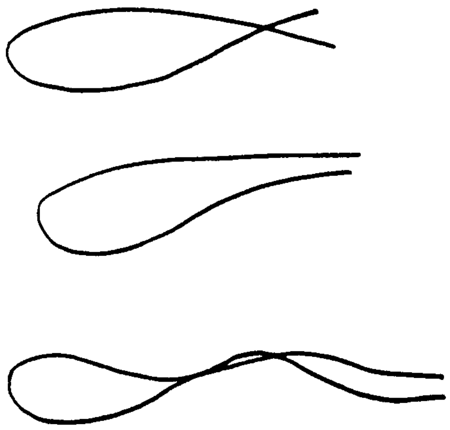

Fig. 1 Different configurations resulting from the unconstrained surface flow design

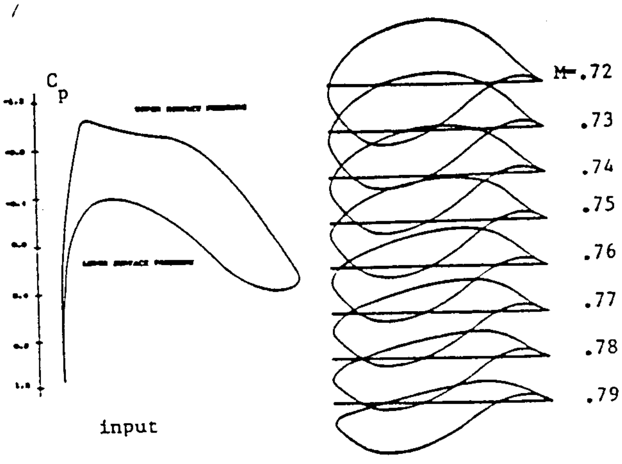

Fig. 2 Shock-free airfoil shapes having same surface pressure distribution; [7] vertical axis magnified five times

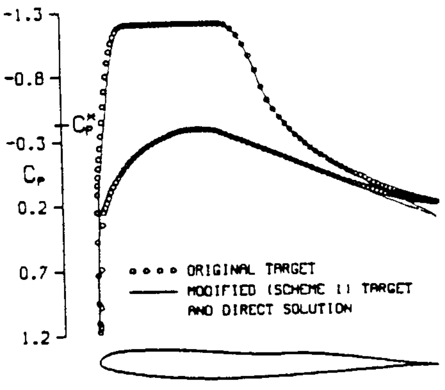

Fig. 3 An example of a "shock-free" surface pressure distribution with a concave suction surface [8]

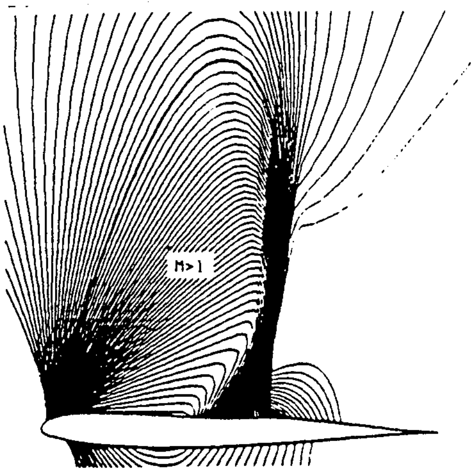

Fig. 4 Iso-Mach distribution for the shock-free surface designed airfoil; notice the hanging shock [8]

The main objection raised by the designers when discussing the inverse (design) methodologies is that these methods create strictly point-designs rather than range-designs. In other words, an aerodynamic shape (Fig. 2) designed by using a surface flow design method will have the desired [7] characteristics only at the design conditions. If the operating conditions (angle of attack, free stream Mach number, etc). are changed, the performance of the designed configuration can deteriorate rapidly. Moreover, when designing transonic shock-free shapes with any of the surface flow design methods, the resulting configuration could have a mildly concave surface (Fig. 3) that is covered by the supersonic flow. As a result, a "hanging shock" or a "loose-foot shock" will form (Fig. 4) even at the design conditions [8]. The aerodynamic efficiency of such a configuration will not be satisfactory even at the design operating point. At off-design values for the Mach number or the angle of attack, the hanging shock will violently re-attach itself to the airfoil surface thus causing rapid increase in drag due to the boundary layer separation. Consequently, it is more appropriate to design an almost shock-free shape even at the design conditions. Such shapes would have a weak family [9] of shocks that would not increase in strength appreciably at the off-design.

3. SURFACE DATA SPECIFICATION

This brings us to the question of what is the appropriate surface pressure distribution. The most desired feature of an aerodynamic design is to prevent flow separation over a wider range of angles of attack, Mach numbers, and Reynolds numbers. The answer to the question as to what is the optimal surface pressure distribution is not known. It might be an altogether wrong question to ask in light of the fact that the surface pressure distribution alone is not indicative of potentially hazardous flow field features as is the case of an unexpected hanging shock. Nevertheless, a number of researchers [10-14] have entertained this problem by using a classical approach based on the information from the boundary layer. A somewhat speculative approach using a concept of minimal kinetic energy rate [15] has been reported recently. A fast method capable of detecting laminar and turbulent flow separation from the prescribed surface pressure distribution would certainly be very useful. These relatively simple methods can help eliminate those surface pressure distributions that would separate the flow. Besides, these methods leave the designer with a psychologically important feeling that he is still in command, although knowing that all of his experience is still inadequate when compared to a true mathematical optimization.

Among a large number of publications using various surface flow designs, applications have been reported to single airfoils [16-24], multi-component airfoils [25], cascades of airfoils [26-32], ducts [34], rotors [35-46], isolated wings [47-48], wing-body combinations [49-50], nozzles and inlets [51-52], and axisymmetric bodies [53]. Some of the methods have received wider acceptance than the others. The general conclusion is that these methods which are more versatile, easy to comprehend and implement, are the more widely used. Since a number of flow field analysis codes are quite reliable, versatile, and efficient, most designers would like to make use of this software directly in the design process.

4. MODIFIED GARABEDIAN'S METHOD

Methods like Garabedian [19] and the modified Garabedian [20] are becoming quite popular since they require an extremely simple master code which can call any available flow field analysis code simply as a subroutine. Thus, as more sophisticated analysis codes become available, they can be directly substituted in the master code that computes corrections (Fig. 5) to the input geometry. The main drawback of the method is that it converges relatively slowly. The iterative motion of the surface which is undergoing design can become irregular very quickly if some sort of control over the motion of surface points is not enforced. The concept of treating such a surface as an elastic membrane which moves according to a simple linear time dependent damped model [19] is quite effective in enforcing a relatively smooth convergence of the surface geometry. A more thorough study on the stability of the surface motion model is necessary, since the choice of coefficients in the model [19,20] can seriously affect the convergence rate and the stability of the entire iterative process.

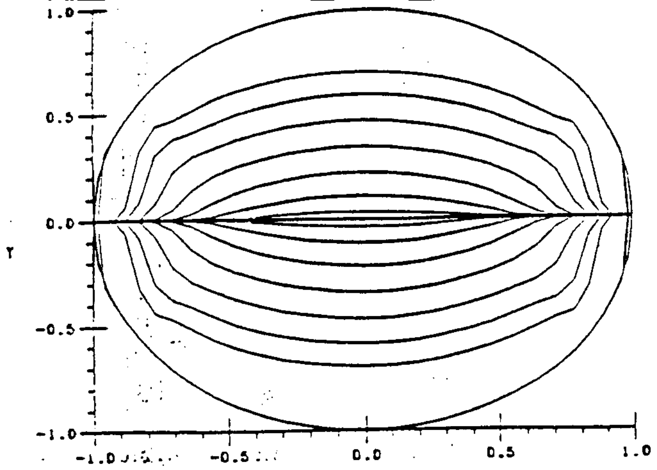

Fig. 5 Convergence history from a slit to a circle using panel code

5. STREAM FUNCTION BASED METHODS

A very interesting concept, termed Stream-Function-as-a-Coordinate (SFC), is based on a transformed flow field governing equations where the vertical coordinate of each stream line is treated as an unknown. Thus, the SFC formulation [32-33] solves directly for the unknown geometric coordinate which is the coordinate of a stream line (Fig. 6). A similar concept derived from the boundary element integral method [18] gives a fully converged solution in 10-20 iterations.

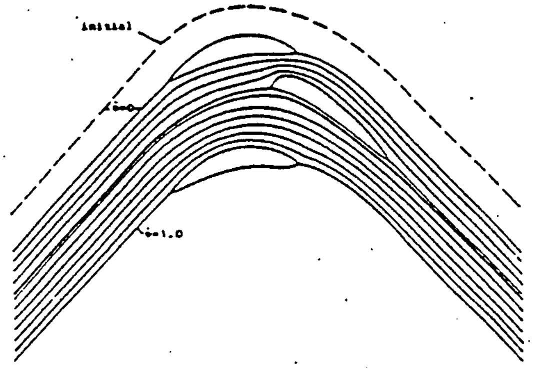

Fig. 6 Design of a tandem cascade using SFC formulation [32]

Another method that is based on the interplay of stream function and potential function in irrotational subsonic inviscid flows is due to Stanitz [34]. He has obtained fascinating configurations of channels and ducts subject to specified surface pressure along the duct walls (Fig. 7).

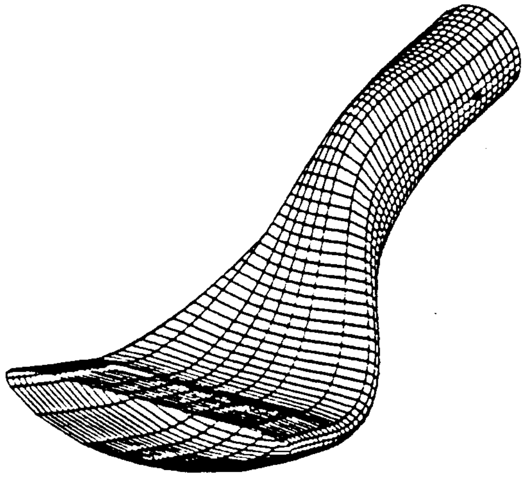

Fig. 7 Air intake scoop designed using Stanitz method [34]

6. TAYLOR SERIES EXPANSION METHOD

An extremely efficient and simple, although approximate method has been developed in China [37-39] and can be reportedly used on a pocket programmable calculator. The method is based on prescribing , say, Mach number distribution along the mid-passage streamline and then deducing values of the Mach number on the top and the bottom of the passage by expanding the prescribed data in the vertical direction using Taylor series. With more terms in the Taylor series, the larger gap-to-chord cascade can be designed. Since the analyticity is carried to an extreme, very little work needs to be performed iteratively. As a consequence, the method converges very fast. Errors in this method will be rapidly increasing towards the stagnation points especially if they are blunt. The method is applicable to radial turbomachinery as well (Fig. 8).

7. NEW THREE-DIMENSIONAL FORMULATIONS

Highly sophisticated and computationally complex computer codes have been developed and successfully applied in the design of three-dimensional coaxial nozzles [40] and turbomachinery blading [41]. The governing model is a complete set of three-dimensional Euler equations of gas dynamics.

Analytically novel and interesting are several new formulations [42-46] for quasi three-dimensional and fully three-dimensional turbomachinery inviscid flow field design. The main drawback of these approaches is the absence of viscosity and turbulence in the basic model.

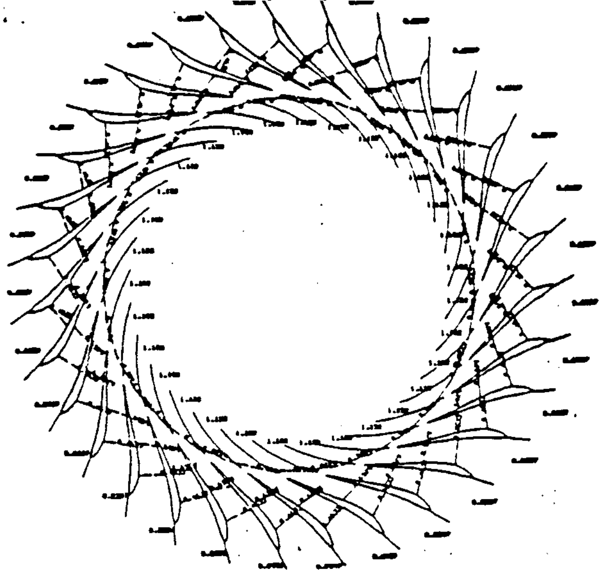

Fig. 8 Radial diffusor vanes designed using Taylor series expansion

The general concept of having a small master code and being able to utilize any available analysis code as a subroutine in the process of surface flow design has been successfully applied by Takanashi [47] in transonic wing design. The method converges extremely fast since he used a small perturbation integral formulation to evaluate geometry corrections of the wing surface.

Surface flow inverse designs of wings [48,50] and a wing-body combination [49] have been successfully accomplished recently using full potential transonic equation solvers [48,49] or higher order surface panel method [50] and fictitious surface transpiration concept.

Inverse designs of supersonic nozzles [51], supercritical jet engine inlets [52], and axisymmetric bodies in incompressible potential flow [53] have been accomplished. The approach of Ives [52] is especially innovative and unique.

8. TRANSONIC SHOCK-FREE DESIGN

Probably the best known method for the flow field design is a hodograph based method [54-57] for designing transonic shock-free shapes . Actually, the method is a unique combination of both surface flow design (surface Mach number can be specified on a point-by point basis) and flow field design formulations (no shocks are guaranteed to occur in the flow field). Consequently, the method suffers from the known problems (open trailing edges and fish-tail shapes) associated with both general approaches to design. The method has been well publicized in the seventies and the resulting software [18] found its use in industry. Nevertheless, any method based on the hodograph transformation is inapplicable to three dimensions. Since Garabedian's method is based on elliptic continuation approach [56] it requires two real and two imaginary characteristics. Needless to say, it is a highly complicated method and the resulting software is not easy to modify. The entire method is well described in a textbook by Schrier [57].

An alternative method is known in the West as Sobieczky's [58,59] fictitious gas or as Nakamura's gas [60] in Japan, since both researchers have developed and published the method independently. The concept is based on the basic fact that the shocks can form only if there is a supersonic flow, that is, if the governing partial differential equation is locally of a hyperbolic type. Consequently, if the conditions for possible shock formations are to be eliminated, the governing partial differential equation should never be allowed to become hyperbolic. Sobieczky and Nakamura accomplished this by switching from an isentropic expression for density to an appropriate analytical fictitious density relation at every point in the field and on the boundary where the flow would like to become supersonic. The resulting computations are acceptable in the subsonic regions (where the isentropic relations were used), but are not acceptable in the supersonic regions (where the fictitious gas relations were used). Nevertheless, the resulting sonic line which now separates the two regions is acceptable by both the isentropic and by the fictitious gas relations (Fig. 9). If we now decide to use the isentropic relations in the previously fictitious gas domain, the governing equations will be locally strictly hyperbolic. Hence, the sonic line values of the stream function can be used as initial data for a straight forward integration of the locally hyperbolic system. Moreover, the system becomes linear if transformed to a rheograph plane characterized by the Prandtl-Meyer function and the local velocity vector angle. The new shape coordinates will be determined from the condition that the stream function should maintain its constant value at every point of the airfoil surface. This method is fairly simple to comprehend and implement in the existing full potential codes. Nevertheless, the fictitious gas method does not give us freedom to specify surface values of flow parameters. It only guarantees that if our choice for the fictitious gas density - Mach number relation is not too restrictive, the supersonic bubble will become shallow and stretched along the surface (Fig. 9) resulting in an entirely shock-free flow field. The method is suitable for redesigning of the existing airfoils [58-62], cascades (Fig. 10) of airfoils [63-65], quasi three-dimensional rotors [66], wings [67-69] without having to worry about surface cross-over, fish-tail shapes, and hanging shocks.

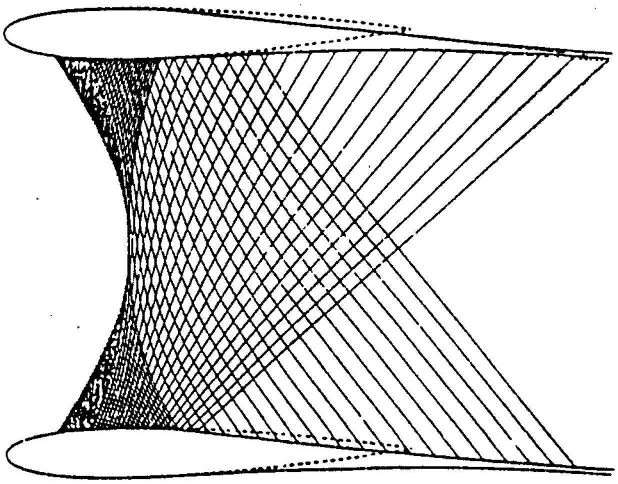

Fig. 10 Lifting choked shock-free cascade designed using fictitious gas [64]

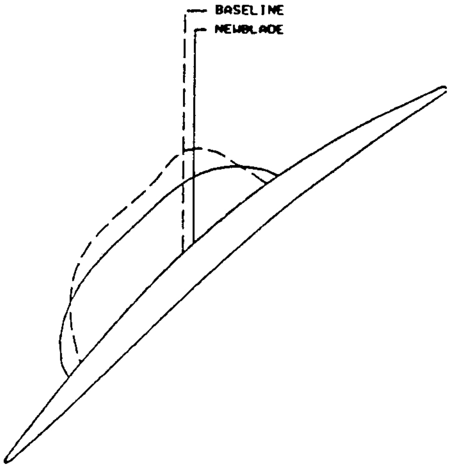

Fig. 9 Sonic line shape before and after the use of fictitious gas [66]

9. OPTIMIZATION

Due to the fact that aerodynamic shape design represents only a part of the overall design of a flying vehicle, the need for an interdisciplinary optimization is arising [4-6]. Simultaneously, the optimization algorithms are finding a rapidly growing applicability in the pure aerodynamic design [70-84]. The optimization algorithms are presently used mainly to minimize a difference between the specified and the computed surface flow data . This is obviously not a very imaginative use of the computational resources, since optimization codes are known to require a large number of flow field analysis solutions. Since the present use of the optimizers is largely not to minimize certain global measure of aerodynamic inefficiency but to enforce the surface flow data, such use of an optimizer has nothing to do with actually optimizing the shape. The noteworthy exceptions involve maximizing lift-to-drag ratio for a multicomponent airfoil [74], minimization of the total pressure loss across the shock waves in a supersonic inlet [77], minimization of the total pressure loss in an S-shaped duct [78], and optimization over a range of operating conditions [79]. Recent publication [80] exposes an interesting and potentially promising new formulation for the fast evaluation and optimization of off-design conditions. The approach of Rizk [81-83] is especially welcome since it allows for a stable iterative algorithm where an optimizer is used on each updated configuration even before the flow field has converged on the new geometry. As a consequence, a typical airfoil design involves the equivalent of 5-10 fully converged solutions (Fig. 11).

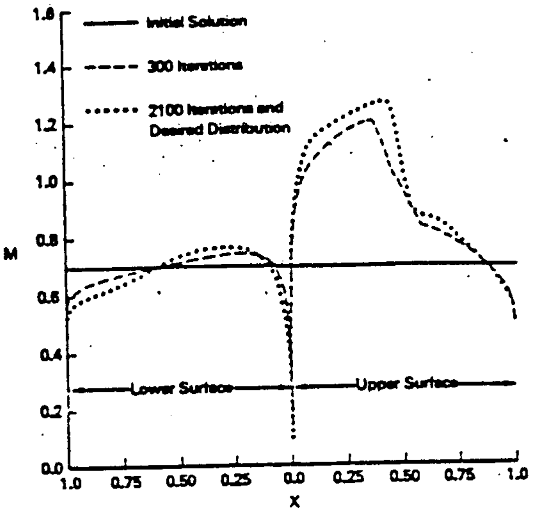

Fig. 11 Convergence history of a surface design using Rizk optimization [81]

10. EMERGING CONCEPTS

Recently, several researchers have looked into using control theory concepts [84-87] in order to achieve an inverse (design) algorithm. The approach is certainly novel and mathematically challenging since most of the fluid flow theory is based on partial differential equations, while the control theory is usually formulated via ordinary differential equations. Preliminary formulations [87] reconfirm earlier observations [84] that this type of formulation might not be efficient.

11. CONCLUSIONS

A survey of a vast number of different inverse (design) concepts and algorithms has been performed with an attempt to classify them. Positive and known negative characteristics of each of the more prominent methods have been outlined. Future research should concentrate on the use of Navier-Stokes equations and three-dimensionality of the problem. Optimization and especially interdisciplinary optimization should play a more prominent role in the near future.

12. ACKNOWLEDGEMENTS

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13. REFERENCES

1. Proceedings of the International Conference on Inverse Design Concepts in Engineering Sciences (ICIDES-I), ed. G.S. Dulikravich, Aero. Eng. Dept., Univ. of Texas, Austin, TX, Oct. 17-18, 1984.

2. Proceedings of the International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-II), ed. G. S. Dulikravich, Aero. Eng. Dept., Penn State Univ., University Park, PA, Oct. 26-28, 1987.

3. Proceedings of the AGARD Specialist's Meeting on Computational Methods for Aerodynamic Design (Inverse) and Optimization, ed. J.W. Sloof, Loen, Norway, May 24-25, 1989.

4. Voigt, R. G., "Requirements for Multidisciplinary Design of Aerospace Vehicles on High Performance Computers," ICASE Report No. 89-70, Sept. 1989.

5. Slooff, J. W., "Computational Methods for Subsonic and Transonic Aerodynamic Design," Proceedings of ICIDES-I, ed. G.S. Dulikravich, Univ. of Texas, Austin, TX, Oct. 17-18, 1984, pp. 1-68.

6. Sobieczky, H.,"Research on Inverse Design and Optimization in Germany", Appl. Mech. Rev., vol. 41, no. 6, June 1988, pp. 217-238.

7. Chin. W. C.,"Class of Shock-Free Airfoils Producing the Same Surface Pressure", AIAA J. of Aircraft, Vol.17, No. 4, April 1980, pp. 286-288.

8. Volpe, G. and Melnik, R.E.,"A Method for Designing Closed Airfoils", AIAA paper 85-5023 presented at the 3rd Applied Aerodynamics Conference, Colorado Springs, CO, Oct. 14-16, 1985.

9. Zhu, Z. and Sobieczky, H.,"An Engineering Approach for Nearly Shock-Free Wing Design", Proceedings of the Internat. Conf. on Fluid Mech., Beijing, China, July 1987.

10. Ives, D. C.," Inverse and Hybrid Compressor Cascade Design Methods", Proceedings of ICIDES-I, ed. G.S. Dulikravich, Univ. of Texas, Austin, TX, Oct. 17-18, 1984, pp. 555-572.

11. Papailiou, K., "An Investigation on Le Foll's Method used for Blade Optimization Based on Boundary Layer Concepts," paper presented at the XXth International Astronautical Congress, August 1969.

12. Citavy, J., "Two-Dimensional Compressor Cascades With Optimum Velocity Distribution Over the Blade," Journal of Engineering for Power, January 1975, pp. 101-110.

13. Lekoudis, S.G., Sankar, N.L., and Radwan, S.F.,"A Method for Designing Three-Dimensional Configurations With Prescribed Skin Friction," Journal of Aircraft, Vol.21, No. 11, Nov. 1984, pp.924-926.

14. Lekoudis, S.G., Sankar, N.L., and Malone, J.B.,"The Application of Inverse Boundary Layer Methods to the Three-Dimensional Viscous Design Problem", Commun. in Appl. Num. Meth., Vol.2, No.1, Jan.-Feb. 1986, pp.57-61.

15. Dulikravich, G. S., "A Criteria for Surface Pressure Specification in Aerodynamic Shape Design," AIAA paper 90-0124 presented at the 28th Aerospace Sciences Meeting, Reno, Nevada, January 8-11, 1990.

16. Taverna, F., "Advanced Airfoil Design for General Aviation Propellers," AAIA paper 83-1791 presented at the AIAA Applied Aerodynamics Conference, Danvers, MA, July 13-15, 1983.

17. Tranen, T., "A Rapid Computer Aided Transonic Airfoil Design Method," McDonnell Douglas Corporation Report No. MDC A2584, Dec. 31, 1973.

18. Soinne, E., and Laine, S., "An Inverse Boundary Element Method for Single Component Airfoil Design," Journal of Aircraft, Vol. 22, No. 6, June 1985, pp. 541-543.

19. Garabedian, P., and McFadden, G., "Design of Supercritical Swept Wings," AIAA Journal, Vol. 20, No. 3, March 1982, pp. 289-291.

20. Malone, J. B., Narramore, J. C., and Sankar, L. N., "An Efficient Airfoil Design Method Using the Navier-Stokes Equations," AGARD Specialists' Meeting on Computational Methods for Aerodynamic Design (Inverse) and Optimization, Loen, Norway, May 22-23, 1989.

21. Fray, J. M. J., Slooff, J. W., Boerstoel, J. W., and Kassies, A., "Design of Transonic Airfoils with Given Pressure, Subject to Geometric Constraints," National Aerospace Laboratory NLR, Amsterdam, The Netherlands, July 13, 1984.

22. Giles, M. B., and Drela, M., "Two-Dimensional Transonic Aerodynamic Design Method," AIAA Journal, Vol. 25, No. 9, September 1987, pp. 1199-1206.

23. Hassan, A. A., "A Viscous-Inviscid Coupling Method for the Design of Low Reynolds Number Airfoil Sections," Proceedings of the Conference on Low Reynolds Number Airfoil Aerodynamics, June 1985, pp. 53-63.

24. Hirose, N., Takanashi, S., and Kawai, N., "Transonic Airfoil Design Procedure Utilizing a Navier-Stokes Analysis Code," AIAA Journal, Vol. 25, No. 3, March 1987, pp. 353-359.

25. Kennedy, J. L., and Marsden, D. J., "A Potential Flow Design Method for Multicomponent Airfoil Sections," Journal of Aircraft, Vol. 15, No. 1, January 1978, pp. 47-52.

26. Jacquotte, O.-P., "A Finite Element Inverse Method for the Design of Turbomachinery Blades," presented at the 7th International Conference "Finite Element Methods in Flow Problems," Huntsville, Alabama, April 3-7, 1988.

27. Gelder, T. F., Moore, R. D., Sanz, J. M., and McFarland, E. R., "Wind Tunnel Turning Vanes of Modern Design," AIAA paper 86-0044 prepared for the Twenty-fourth Aerospace Sciences Meeting, Reno, Nevada, January 6-8, 1986.

28. Tong, S. S., and Thompkins, Jr., W. T., "A Design Calculation Procedure for Shock-free or Strong Passage Shock Turbomachinery Cascades," ASME 27th Gas Turbine Conference, London, England, April 18-22, 1982.

29. Miton, H. H., "Properties of a Space-wise Periodic Flow Application to Flow Computation and Design for Blade Cascades," Communications in Applied Numerical Methods, Vol. 2, No. 1, Jan.-Feb. 1986, pp. 101-112.

30. Meauze, G., "An Inverse Time Marching Method for the Definition of Cascade Geometry," ASME paper 81-GT-167 presented at the International Gas Turbine Conference and Products Show, Houston, Texas, March 9-12, 1981.

31. Meauze, G., "On the Use of Inverse Modes of Calculation in Two-dimensional Cascades and Ducts," Communications in Applied Numerical Methods, Vol.2, no. 1, Jan.-Feb. 1986, pp. 73-81.

32. Chen, N.-X. and Zhang, F.-X.,"A Generalized Numerical Method for Solving Direct, Inverse and Hybrid Problems of Blade Cascade Flow by Using Streamline-Coordinate Equation", ASME paper 87-GT-29, Anaheim, CA, June 1987.

33. Huang, C.-Y. and Dulikravich, G.S.,"Stream Function and Stream-Function-Coordinate (SFC) Formulation for Inviscid Flow Field Calculations",

Computer Meth. in Appl. Mech. and Eng., Vol. 59, 1986, pp.155-177.

34. Stanitz, J.D.,"A Review of Certain Inverse Methods for the Design of Ducts With 2- or 3-Dimensional Potential Flow", Appl. Mech. Rev., Vol. 41, no. 6, June 1988, pp. 217-238.

35. Cedar, R. D., and Stow, P., "The Role of a Mixed Design and Analysis Method in a Turbomachinery Blade Design System," Communications in Applied Numerical Methods, Vol. 2, No. 1, Jan.-Feb. 1986, pp. 91-99.

36. Ribaut, M., and Martin, D., "A Quasi Three-dimensional Inverse Design Method Using Source and Vortex Integral Equations," Communications in Applied Numerical Methods, Vol. 2, No. 1, Jan.-Feb. 1986, pp. 63-72.

37. Zhengming, W., "A Method of Aerodynamic Design of Blades in Quasi-Three-Dimensional Calculation of Turbomachine," ASME paper 86-GT-192 presented at the International Gas Turbine Conference and Exhibit, Dusseldorf, West Germany, June 8-12, 1986.

38. Xiaolu, Zhao, and Lisen, Qin, "An Approximate 3-D Aerodynamic Design Method for Centrifugal Impeller Blades," ASME paper 89-GT-73 presented at the Gas Turbine and Aeroengine Congress and Exposition, Toronto, Canada, June 4-8, 1989.

39. Gong, Yifang, and Cai, Ruixian, "3D Mean-Stream-Line Method--A New Engineering Approach to the Inverse Problem of 3D Cascade," ASME paper 89-GT-48 presented at the Gas Turbine and Aeroengine Congress and Exposition, Toronto, Canada, June 4-8, 1989.

40. Couaillier, V. and Veuillot, J.P.,"Inverse Method for the Euler Equations", Proceedings of ICIDES-II, ed. G.S. Dulikravich, Penn State Univ., University Park, PA, Oct. 26-28, 1987, pp. 131-146.

41. Zannetti, L., di Torino, P., and Ayele, T. T., "Time Dependent Computation of the Euler Equations for Designing Fully 3D Turbomachinery Blade Rows, Including the Case of Transonic Shock Free Design," AIAA paper 87-0007 presented at the 25th Aerospace Sciences Meeting, Reno, Nevada, January 12-15, 1987.

42. Miyazaki, T., and Hirayama, N., "A Theoretical Solution of Three-Dimensional Flows in Subsonic, Transonic and Supersonic Turbomachines: An Exact Solution and Its Numerical Method," ASME paper 86-GT-111 presented at the 31st Internat. Gas Turbine Conf.and Exhibit, Dusseldorf, Germany, June 1986.

43. Borges, J. E., "A Three Dimensional Inverse Method in Turbomachinery; Part 1--Theory," ASME paper 89-GT-136 presented at the Gas Turbine and Aeroengine Congress and Exposition, Toronto, Canada, June 4-8, 1989.

44. Borges, J. E., "A Three Dimensional Inverse Method in Turbomachinery; Part 2--Experimental Verification," ASME paper 89-GT-137 presented at the Gas Turbine and Aeroengine Congress and Exposition, Toronto, Canada, June 4-8, 1989.

45. Hawthorne, W.R. and Tan, C.S.,"Design of Turbomachinery Blading in Three-Dimensional Flow by the Circulation Method; A Progress Report", Proceedings of ICIDES-II, ed. G.S. Dulikravich, Penn State Univ., University Park, PA, Oct. 26-28, 1987, pp. 207-226.

46. Ghaly, W. S., "A Design Method for Turbomachinery Blading in Three-Dimensional Flow," International Journal for Numerical Methods in Fluids, Vol. 10, 1990, pp. 179-197.

47. Takanashi, S., "Iterative Three-Dimensional Transonic Wing Design Using Integral Equations," Journal of Aircraft, Vol. 22, No. 8, August 1985, pp. 655-660.

48. Gally, T. A., and Carlson, L. A., "Transonic Wing Design Using Inverse Methods in Curvilinear Coordinates," Journal of Aircraft, Vol. 25, No. 11, November 1988, pp. 1009-1017.

49. Hazarika, N., "An Efficient Inverse Method for the Design of Blended Wing-Body Configurations," Ph.D. thesis, Aero. Eng. Dept., Georgia Institute of Technology, June 1988.

50. Fornasier, L.,"An Iterative Procedure for the Design of Pressure-Specified Three-Dimensional Configurations at Subsonic and Supersonic Speeds by Means of a Higher-Order Panel Method",AGARD Specialists' Meeting on Computational Methods for Aerodynamic Design (Inverse) and Optimization, Loen, Norway, May 22-23, 1989.

51. Meauze, G., and Fourmaux, A., "A Coupled Inverse-Inverse Method for Over-Expanded Supersonic Nozzles," paper 109 presented at the 7th ISABE International Symposium on Air Breathing Engines, Beijing, People's Republic of China, September 2-6, 1985.

52. Ives, D. C., "Supercritical Inlet Design," AIAA paper 83-1866 presented at the AIAA Applied Aerodynamics Conference, Danvers, Massachusetts, July 13-15, 1983.

53. Zedan, M. F., and Dalton, C., "The Inverse Method Applied to a Body of Revolution with an

Extended Favourable Pressure Gradient Forebody," Communications in Applied Numerical Methods, Vol. 2, No. 1, Jan.-Feb. 1986, pp. 113-119.

54. Boerstoel, J.W. and Huizing, G.H.,"Transonic Airfoil Design by an Analytic Hodograph Method", AIAA paper 74-539, 1974.

55. Hobson, D.E.,"Shock-Free Transonic Flow in Turbomachinery Cascades", Univ. of Cambridge, UK, CUED/A Turbo/TR 65, 1974.

56. Bauer,F., Garabedian, P., Korn, D. and Jameson, A.,"Supercritical Wing Sections 1,2,3", Lecture Notes in Economics, Math. Syst., No. 66, 108, 150, Springer Verlag, Berlin, 1972, 1975, 1977.

57. Schrier, S.,"Compressible Flow", John Wiley & Sons, New York, 1982.

58. Sobieczky, H., Fung, K.-Y., Seebass, R.A., Yu, N.J.,"New Method for Designing Shock-Free Transonic Configurations", AIAA Journal, Vol.17, No. 7, 1979.

59. Sobieczky, H.," Related Analytical, Analog and Numerical Methods in Transonic Airfoil Design", AIAA paper 79-1980, Williamsburg, VA, June 1979.

60. Nakamura, M., "A Method for Obtaining Shockless Transonic Flows past Two-Dimensional Airfoils whose Profiles are Partially Modified from a Given Arbitrary Profile," Trans. Japan Soc. Aero. Space Sci., Vol. 23, No. 62, Feb. 1981, pp. 195-213.

61. Hassan, A., Sobieczky, H., and Seebass, A. R., "Subsonic Airfoils with a Given Pressure Distribution," AIAA Journal, Vol. 22, No. 9, September 1984, pp. 1185-1191.

62. Dulikravich, D. S., and Sobieczky, H., "Shockless Design and Analysis of Transonic Cascade Shapes," AIAA Journal, Vol. 20, No. 11, November 1982, pp. 1572-1578.

63. Dulikravich, G.S. and Sobieczky, H.,"CAS22-FORTRAN Program for Fast Design and Analysis of Shock-Free Airfoil Cascades", NASA CR 165596, 1982.

64. Sobieczky, H. and Dulikravich, G.S.,"A Computational Design Method for Turbomachinery Cascades", ASME paper 82-GT-117, London, UK, April, 1982.

65. Dulikravich, G.S. and Sobieczky, H.,"Design of Shock-free Compressor Cascades Including Viscous Boundary Layer Effects",ASME paper 83-GT-134, Phoenix, AZ, March 27-31, 1983.

66. Prince, T. C., and Beauchamp, P. P., "Shock-free Transonic Rotor Design," Communications in Applied Numerical Methods, Vol. 2, No. 1, Jan.-Feb. 1986, pp. 21-27.

67. Fung, K-Y., Sobieczky, H., and Seebass, R., "Shock-Free Wing Design," AIAA Journal, Vol. 18, No. 10, October 1980, pp. 1153-1158.

68. Yu, N. J., "Efficient Transonic Shock-Free Wing Redesign Procedure Using a Fictitious Gas Method," AIAA Journal, Vol. 18, No. 2, February 1980, pp. 143-148.

69. Raj, P., and Miranda, L. R., "A Cost-Effective Method for Shock-Free Supercritical Wing Design," Journal of Aircraft, Vol. 19, No. 4, April 1982, pp. 283-289.

70. Silver, B., and Ashley, H., "Optimization Techniques in Aircraft Configuration Design," Department of Aeronautics and Astronautics, Stanford University, SUDAAR No. 406, June 1970.

71. Reneaux, J., and Thibert, J-J., "The Use of Numerical Optimization for Airfoil Design," AIAA paper 85-5026 presented at the AIAA 3rd Applied Aerodynamics Conference, Colorado Springs, CO, October 14-16, 1985.

72. Sanger, N. L., "The Use of Optimization Techniques to Design Controlled Diffusion Compressor Blading," NASA Technical Memorandum 82763, April 1982.

73. Ormsbee, A. I., and Chen, A. W., "Multiple Element Airfoils Optimized for Maximum Lift Coefficient," AIAA J., Vol. 10, No. 12, Dec. 1972, pp. 1620-1624.

74. Misegades, K. P., "Optimization of Multi-Element Airfoils," Von Karman Institute for Fluid Dynamics, Belgium, Project Report 1980-5, June 1980.

75. Cosentino, G. B., "Numerical Optimization Design of Advanced Transonic Wing Configurations," AIAA paper 85-0424 presented at the AIAA 23rd Aerospace Sciences Meeting, Reno, NV, January 14-17, 1985.

76. Lock, R. C., "Comment on 'Numerical Optimization Design of Advanced Transonic Wing Configurations'," Journal of Aircraft, Vol. 24, No. 8, August 1987, pp. 575-576.

77. Huddleston, D. H., and Mastin, C. W., "Optimization of Aerodynamic Designs Using Computational Fluid Dynamics," Proceedings of the 7th Internat. Conference on Finite Element Methods in Flow Problems, ed. Chung, T.J. and Karr, G.R., Univ. of Alabama in Huntsville Press, April 3-7, 1989, pp. 899-907.

78. Madabhushi, R.K., Levy, R. and Pinkus, S.M.,"Design of Optimum Ducts Using an Efficient 3-D Viscous Computational Flow Analysis", Proceedings of ICIDES-II, ed. G.S. Dulikravich, Penn State Univ., University Park, PA, Oct. 26-28, 1987, pp.147-166.

79. Ghielmi, L., Marazzi, R. and Baron, A.,"A Tool for Automatic Design of Airfoils in Different Operating Conditions",AGARD Specialists' Meeting on Computational Methods for Aerodynamic Design (Inverse) and Optimization, Loen, Norway, May 22-23, 1989.

80. Sharp, H. T., and Sirovich, L., "Constructing a Continuous Parameter Range of Computational Flows," AIAA Journal, Vol. 27, No. 10, 1989, pp. 1326-1331.

81. Rizk, M. H., "The Single-Cycle Scheme: A New Approach to Numerical Optimization," AIAA Journal, Vol. 21, No. 12, Dec. 1983, pp. 1640-1647.

82. Rizk, M. H., "A New Approach to Optimization for Aerodynamic Applications," Journal of Aircraft, Vol. 20, No. 1, Jan. 1983, pp. 94-96.

83. Rizk, M. H., "Application of the Single-Cycle Optimization Approach to Aerodynamic Design," Jour. of Aircraft, Vol. 22, No. 6, June 1985, pp. 509-515.

84. Angrand, F., "Optimum Design for Potential Flows," International Journal for Numerical Methods in Fluids, Vol. 3, 1983, pp. 265-282.

85. Gu, Chuan-gang, and Miao, Yong-miao, "Blade Design of Axial-Flow Compressors by the Method of Optimal Control Theory--Physical Model and Mathematical Expression," ASME paper 86-GT-183 presented at the 31st Internat. Gas Turbine Conf. and Exhibit, Dusseldorf, Germany, June 1986.

86. Gu, Chuan-gang, and Miao, Yong-miao, "Blade Design of Axial-Flow Compressors by the Method of Optimal Control Theory--Application of Pontryagin's Maximum Principles, a Sample Calculation and Its Results," ASME paper 86-GT-182 presented at the 31st Internat. Gas Turbine Conf. and Exhibit, Dusseldorf, Germany, June 1986.

87. Jameson, A., "Aerodynamic Design Via Control Theory," ICASE Report No. 88-64, 1988.