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Aerodynamic Shape Design and Optimization: Status and Trends

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A limited number of aerodynamic shape design concepts have been surveyed and an attempt has been made to classify them. Characteristics, both positive and negative, of the more prominent methods were outlined. Future research is expected to concentrate on the use of Navier-Stokes equations and applications to three-dimensional configurations. Interdisciplinary constrained optimization is expected to play a more prominent role in the immediate future. Adjoint operator/control theory and its variations are the most promising concepts for interdisciplinary aerodynamic shape design which involves a large number of variables. This theory is expected to constitute the major development area in future research.

Introduction

IN the general field of aerodynamics, as with any field theory, we are faced with two problems, analysis and design. In the case of analysis (direct problem) we are asked to predict the details of a flowfield if the geometry of the object is specified. In the case of a design (inverse or indirect) problem we are asked to predict the geometry of the object, which must be compatible with the desired features of the flowfield. Although the words "design" and "inverse" have the same meaning in the present context, it has become customary to use the expression "inverse design" instead. Thus, aerodynamic shape design involves the ability to determine the geometry of an aerodynamic object that will satisfy the governing equations for the flowfield and the desired boundary conditions. For example, it is possible to determine the coordinates of an airfoil if a surface pressure distribution is specified. The resulting designs can be subject to certain specified constraints. Examples that include such constraints, may entail finding aerodynamic configurations that are compatible with entirely shock-free transonic flowfields, or obtaining shapes of objects that produce flowfields with minimum entropy generation, minimum noise generation, desired surface heat flux distribution, etc.

Depending on the prescribed features of the flowfield, design (inverse) methodologies can be grouped into two general categories, 1) surface flow design, and 2) flowfield design. Surface flow design involves specifying certain flow parameters (pressure, Mach number, etc.) on the surface of the flying object, then finding the shape that will generate these surface conditions without regard for the rest of the flowfield. Flowfield design approach, on the other hand, enforces certain global flowfield features (shock-free flow, minimal entropy generation, etc.) at every point of the flowfield by determining the shape that will satisfy these constraints locally. A large number of methods for performing surface flow design have been developed, whereas only a few methods for flowfield design are known to exist.

Although numerous methods[1-5] for performing inverse design and optimization of aerodynamic configurations have been devised, these methods are not routinely used by the engineering community. Since there are no comprehensive textbooks covering this field (except for a few publications in the form of survey articles), these methods have not been taught in engineering programs. Consequently, present and future engineers are inadequately trained in this rapidly growing field. In other words, future aerodynamicists are not taught how to achieve the mathematically optimal aerodynamic configurations most economically by means of inverse design and optimization. Instead, cut-and-try design based on repetitive analysis and intuition, is still widely practiced in the aerospace industry. Needless to say, this classical inefficient approach cannot survive the rigors of the competitive global world market. This fact has been recognized by a few leading companies where these methods are being developed and implemented by a few highly specialized individuals. These efforts, although resulting in actual hardware, have been mostly covered by a veil of undue secrecy. This concise survey article is an attempt to classify and briefly evaluate the most prominent aerodynamic design methods available in the open literature.

One of the reasons for the slow acceptance of inverse design and optimization methodologies in the field of aerodynamics has been the notion that these methods are hard to comprehend and difficult to mathematically formulate. The fact is that most of the design methods are as complex analytically and numerically as their direct (analysis) counterparts. A typical inverse design computer code needs a single run to generate a new shape that duplicates the desired surface flow parameters. An advanced constrained optimization code might require computer time that is equivalent to a few dozen analysis runs in order to arrive at a mathematically optimal configuration, rather than just an improved shape that the cut-and-try approach would be able to provide.

Most of the methods that will be mentioned in this article have been available for quite some time. It was the author's deliberate decision to write this survey article by quoting only the methods that have either found their acceptance in industry or that could be further perfected. Details of each methodology could be found by reading the original papers quoted in the concise list of references. The author would like to apologize for not being able to quote all the publications available because of the page limitations.

Mathematical models used in aerodynamic shape design are based on partial differential equations, integral equations, and algebraic equations. Detailed reviews have been presented at specialists meetings[1-5] in the form of survey articles.[6-13] For example, Zhukovskii conformal mapping is actually a technique for designing a class of airfoil shapes with a specified pressure distribution at the surface that corresponds to a flow around a rotating circular cylinder. Here we are dealing with a simple algebraic expression, but that expression is based on

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a general solution of an integral equation formulation (a point-dipole and a point vortex) or a Laplace operator (a partial differential equation) governing the flowfield. Thus, many global conformal mappings can be viewed as very special methods for designing certain simple shapes in a steady, planar, irrotational, inviscid flowfield. Moreover, global conformal mapping is the only example that comes to mind as a method that combines the surface flow design concept and the flowfield design concept. It guarantees that the resulting airfoil shapes have the specified surface distribution of the flow parameters while maintaining the irrotationality of the flowfield.

In a more general situation, the arbitrary distribution of the surface flow parameters, or an arbitrary field distribution of the flow parameters, could result in shapes that are not physically meaningful and cannot be manufactured. For example, the lower surface and the upper surface of an airfoil could either cross over (fish tail shapes) or never meet (open trailing edge shapes), although these solutions are mathematically acceptable. Obviously, the problem rests in choosing an appropriate surface distribution of the flow parameters that satisfies certain global flow field constraints.[14] Certain constraints on the geometry are needed since the final aerodynamic design is often incompatible with heat transfer, structural dynamics, acoustics, or manufacturing requirements.

The main objection raised by designers when discussing inverse (design) methodologies is that these methods create strictly point-designs rather than range-designs. In other words, an aerodynamic shape designed by using a surface flow design method will have the desired characteristics only at the design conditions.[15] If the operating conditions (angle of attack, free stream Mach number, etc.) vary from the design conditions, then the configuration will have to be changed (Fig. 1) in order to main the desired surface-flow parameters. For example, when designing transonic shock-free shapes with any of the surface-flow design methods, the resulting configuration could have a mildly concave surface, locally covered by the supersonic flow. As a result, a "hanging shock" or a "loose-foot" shock[14] will form even at the design conditions. At off-design, the hanging shock reattaches itself to the airfoil surface causing a rapid increase in drag due to boundary-layer separation. Consequently, it is more appropriate to design an almost shock-free shape, even at the design conditions. Such shapes could have a weak family of shocks[16] that do not increase appreciably in strength at off-design conditions.

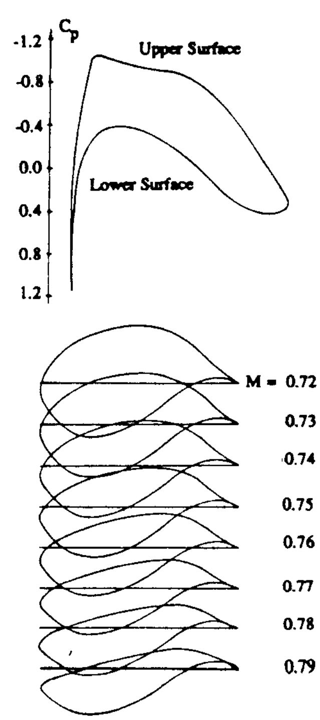

Fig. 1 Airfoil shapes[15] (y-axis magnified five times) having identical surface pressure distributions at different freestream Mach numbers.

Surface Data Specification

Prevention of uncontrolled flow separation over a wider range of angles-of-attack, Mach numbers, and Reynolds numbers, is the most important goal of an aerodynamic design. We must now face the question, what is the best surface pressure distribution? It might not be an appropriate question in light of the fact that the surface pressure distribution alone is not indicative of potentially hazardous flowfield features, such as an unexpected hanging shock. A number of researchers[17-19] have entertained this issue using an approach based on boundary-layer information. A different (although somewhat speculative) fast method, for detection of flow separation[20] from a given surface pressure distribution, utilizes a concept of minimal kinetic energy rate from fracture mechanics. All these methods leave the designer with an important feeling that he is still in command, although realizing that all of his experience is inadequate when compared to a true mathematically constrained optimization.

Among the large number of publications using various surface flow designs, applications to single airfoils,[21-28] multicomponent airfoil,[29] cascades of airfoils,[30-37] ducts,[38] rotors,[39-48] isolated wings,[49,50] wing-body combinations,[51,52] complete airplanes,[53] nozzles,[54-57] inlets,[58,59] and axisymmetric bodies[60] can be found. Some of these methods have received wider acceptance than others. The general conclusion is that methods that are more economical, versatile, robust, and easier to comprehend and implement are more widely used. There are even instances in which three-dimensional aircraft configuration[53] have been aerodynamically optimized on personal computers.

Inverse Design Using Analysis Codes

This general approach to aerodynamic shape design is becoming quite popular since it requires only a simple master code that can call any available flowfield analysis code as a subroutine. Thus, as more sophisticated analysis codes become available, they can be directly substituted in the master code that computes corrections to the input geometry. The surface motion is often modeled as an elastic membrane that moves according to an instantaneous difference between the specified and the computed local surface pressures.[22,24] This is quite effective in enforcing a relatively smooth convergence of the surface geometry (Fig. 2). A more thorough study on the stability of the surface motion model is necessary, since the choice of coefficients in the model[24] can affect the con-

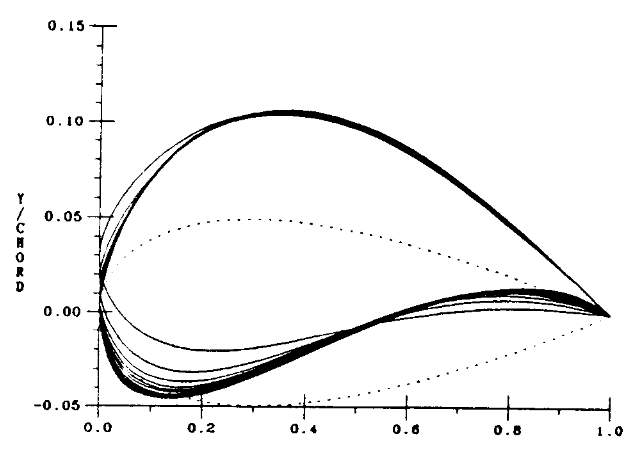

Fig. 2 Example of the convergence history using a master code method[24] and a surface panel analysis code; initial shape was a NACA0010 airfoil; and target pressure distribution was for a 15%-thick cambered Zhukovski airfoil.

An even more efficient method[30] is based on an equivalent surface transpiration concept.

Stream Function-Based Methods

A very interesting concept, termed stream-function-as-a-coordinate (SFC), is based on a transformed stream function formulation where the vertical coordinate of each streamline is treated as the unknown. Thus, the SFC formulation[36,61,62] directly solves the unknown geometric coordinates that are the coordinates of the streamlines (Fig. 3). A three-dimensional version of the SFC formulation remains to be developed. A similar concept derived from the boundary element integral method[23] gives a fully converged solution for an airfoil design on a personal computer in 10–20 iterations. Another method that is based on the interplay of two stream functions, and the potential function in irrotational subsonic inviscid flows,[37,38] is capable of generating fascinating configurations of channels and three-dimensional ducts subject to a specified surface pressure along the duct walls (Fig. 4).

Taylor Series Expansion Method

An extremely fast and simple (although approximate) method of preliminary design can reportedly be used on a pocket programmable calculator. The method is based on prescribing, e.g., a Mach number distribution along the midpassage streamline, and then deducing values of the Mach number on the top and bottom of the passage by expanding the prescribed data in the vertical direction, using the Taylor series. With more terms in the Taylor series, a larger gap-to-chord cascade can be designed. Errors in this method rapidly increase to-

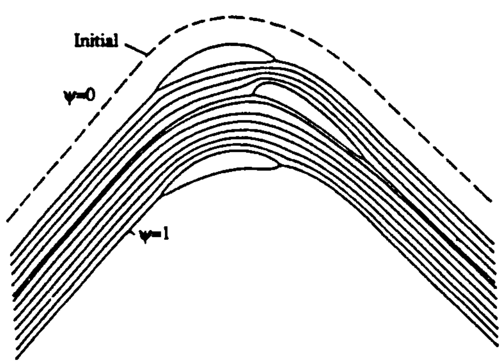

Fig. 3 SFC method[36] generates streamline shapes directly as its solution; example of a turbomachinery cascade designed simultaneously with a splitter blade in a single computer run.

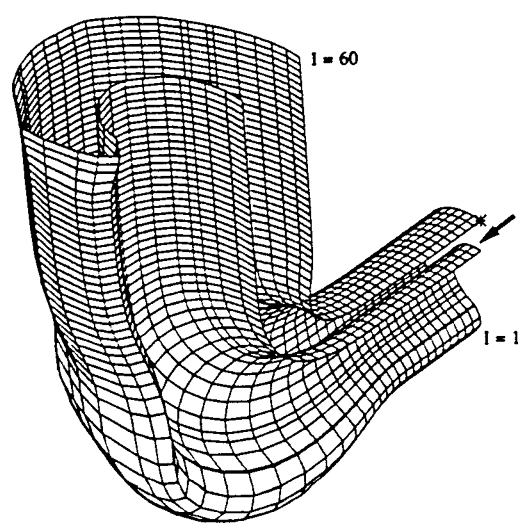

Fig. 4 Stream function—potential function inverse design method,[38] can generate complex realistic three-dimensional configurations.

method is equally applicable to a preliminary design of axial and radial turbomachinery.[43]

Three-Dimensional Formulations

Highly sophisticated computer codes have been developed and successfully applied in the design of three-dimensional coaxial nozzles[54] and turbomachinery blading.[44] The model includes a complete set of the three-dimensional Euler equations of gas dynamics. Although complex, the method converges quickly, since the geometry corrections are calculated using information that propagates along the characteristic lines. Several new methods[45–48] for quasi-three-dimensional and fully three-dimensional turbomachinery design, using inviscid flowfield formulations, are analytically novel and interesting. The main drawback of these approaches is that the basic model does not take into account either viscosity or turbulence.

The general concept of using a small master code to call any available flowfield analysis code, as a subroutine in the process of surface flow design, can be made to converge relatively quickly if a suitable formulation is used to evaluate and distribute the local geometry (Fig. 5). Recently, inverse designs of isolated shock-free transonic wings have been reported by using a full potential analysis code and a geometry correction redistribution algorithm based on an integro-differential formulation.[49] A typical design process would require less than 10 flowfield analysis runs. Inverse designs of wing-body combinations were successfully performed using the surface transpiration concept and a small perturbation transonic potential flow equation,[50] full potential transonic equation,[51] and higher-order surface panel methods,[52,53] together with a gradient search optimization code.

Transonic Shock-Free Design

Probably the best known method for flowfield design is a hodograph-based method[63–65] for designing transonic, shock-free shapes. Actually, the method is a combination of both

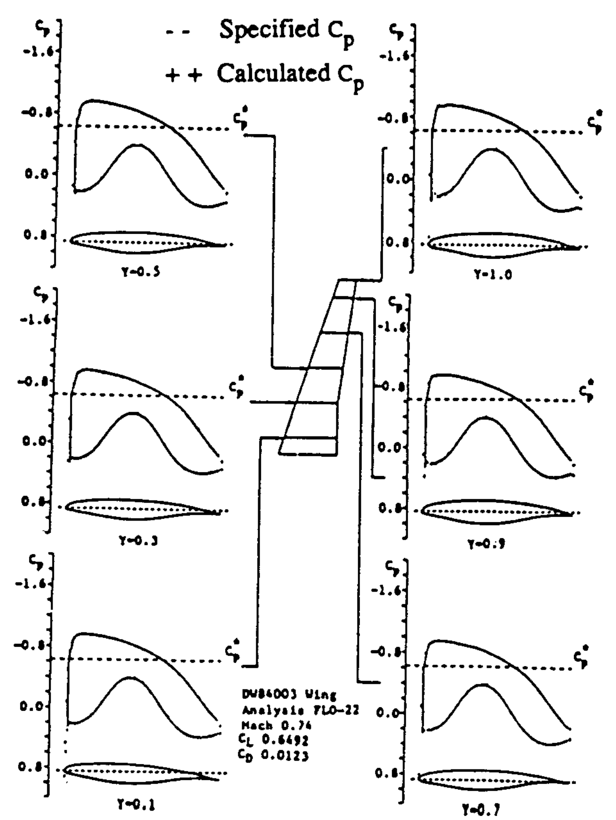

Fig. 5 Inverse design of an entirely shock-free transonic transport wing using a master code based on an integral method[49] requires only 10 flowfield analysis runs.

surface flow design (the surface Mach number can be specified on a point-by-point basis) and flowfield design formulations (it can be guaranteed that no shocks occur in the flowfield). Consequently, the method suffers from the problems previously mentioned (open trailing edges and fish-tail shapes) that are associated with both general approaches to design. The method was well publicized in the seventies and the resulting software[64] found its way into industry. Nevertheless, methods based on the hodograph transformation are not applicable to general three-dimensional configurations. Since the hodograph method is based on an elliptic continuation approach,[65-67] it requires two real and two imaginary characteristics. Needless to say, it is a highly complicated method and the resulting software is not easy to modify.

An alternative design method is known as the fictitious gas[66,67] method. This concept is based on the fact that shocks can form only in supersonic flow, i.e., if the governing partial differential equation is locally hyperbolic. Consequently, if the conditions for possible shock formations are to be eliminated, the governing partial differential equations should never be allowed to become hyperbolic. This can be ensured by switching from an isentropic expression for density and local speed of sound, to an appropriate analytical fictitious density relation at every point in the field, and on the boundary where the flow is likely to become supersonic. Computational results are acceptable in the subsonic regions (where the isentropic relations are used), but are not acceptable in the supersonic regions (where the fictitious gas relations are used). The resulting sonic line that separates the two regions, is compatible with both the isentropic and fictitious gas relations. Hence, the sonic line values of the stream function and the potential function, can be used as initial data for integration of the purely supersonic domain underneath the sonic line. We can now use the isentropic relations in this region so that the governing equations will be locally strictly hyperbolic. Moreover, the system becomes linear if transformed to a rheograph plane[66] characterized by the Prandtl-Meyer function and the local velocity vector angle. The new shape coordinates will be determined from the condition that the stream function should maintain a constant value at every point of the airfoil surface. This method is fairly simple to comprehend and implement in existing full potential codes. Nevertheless, the fictitious gas method does not allow us the freedom to specify surface values of flow parameters. It only guarantees that if our choice for the fictitious gas density—Mach number relation is not too restrictive, the supersonic bubble will become shallow and stretched along the surface, that results in an entirely shock-free flowfield. The method is suitable for redesigning existing airfoils,[66,67] cascades of airfoils,[68-70] quasi three-dimensional rotors,[71] and transonic wings[72,73] without having to worry about surface crossover, fish-tail shapes and hanging shocks.

Optimization Attempts

Due to the fact that aerodynamic shape design represents only a part of the overall design of a flying vehicle, the need for interdisciplinary optimization arises. Simultaneously, optimization algorithms are rapidly finding applicability in pure aerodynamic design.[52,53,57,73-91] Presently, optimization algorithms are often used to minimize the difference between the specified and the computed surface flow data. It should be pointed out that such use of an optimizer has nothing to do with optimizing the aerodynamic shape. Noteworthy exceptions involve maximizing lift-to-drag ratio for isolated helicopter blade airfoils[76] and multicomponent airfoils,[78] minimizing the total pressure loss across the shock waves in supersonic inlets[82] and around nonaxisymmetric hypersonic vehicles,[83] minimizing the total pressure loss in an incompressible viscous flow inside an S-shaped duct,[84] optimizing airfoils over a range of operating conditions,[85] and minimizing induced drag[52,53,86] of an entire three-dimensional airplane (Fig. 6). Recent publications[87-89] expose interesting and potentially promising sensitivity analysis formulations for evaluation and optimization at off-design conditions. Nevertheless, sensitivity analysis is a very costly process requiring a large number of analysis runs. Single cycle optimization[90] is therefore, welcome, since it allows for a stable iterative algorithm where an optimizer is used on each updated configuration even before the flowfield has fully converged to the new geometry. As a consequence, a typical airfoil design involves an equivalent of 5–10 fully converged flowfield solutions. A very readable and thorough comparative analysis of optimization-based approaches[91] confirms that more economical approaches are possible with the gradient-based algorithms. Besides a wide variety of the gradient-based optimization algorithms, it should be pointed out that truly remarkable results were obtained using an evolution type algorithm[79] that seems to be less sensitive to local minimums. Initial applications of neural networks to aerodynamic design are also encouraging.[57]

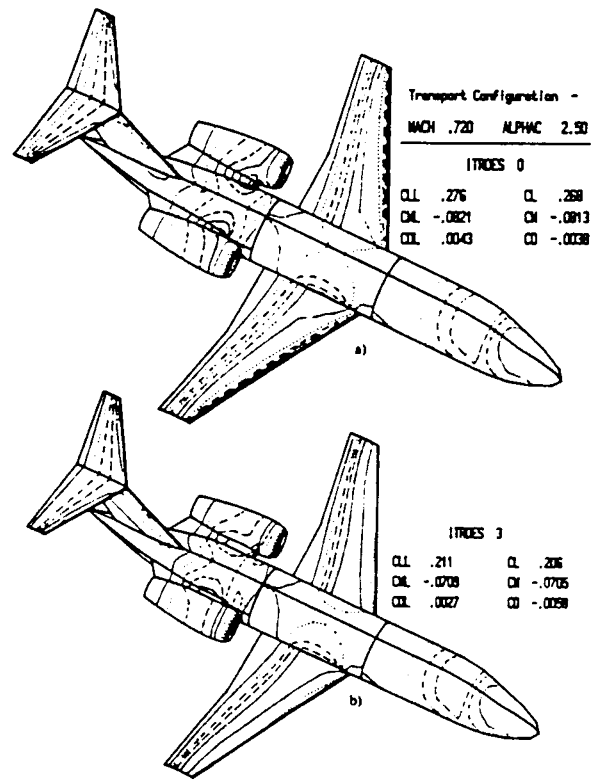

Fig. 6 Entire business jet configuration can be optimized[53] on a personal computer using surface transpiration concept and a panel method a) before and b) after three optimization cycles. Notice improvements in aerodynamic coefficients.

Adjoint Operator/Control Theory

Control theory has recently been applied to systems of partial differential equations governing fluid flow.[92-99] In this context, control theory can be thought of as a minimization process performed in a continuous function space, that is beneficial when optimizing a large number of variables. This approach requires derivation of adjoint systems of partial differential equations and their appropriate boundary conditions. The adjoint operators must be separately derived for each new system of flow governing equations. The methodology is explained in the articles mentioned in the list of references which appear to be the most complete[95,96] and readable[98,99] texts on this subject to date. They also provide convincing results (Figs. 7 and 8) for the design of nozzles and ducts with maximum pressure rise and minimum total pressure drop. These preliminary results dispel earlier reservations that these formulations might not be computationally efficient since they involve the solution of an additional

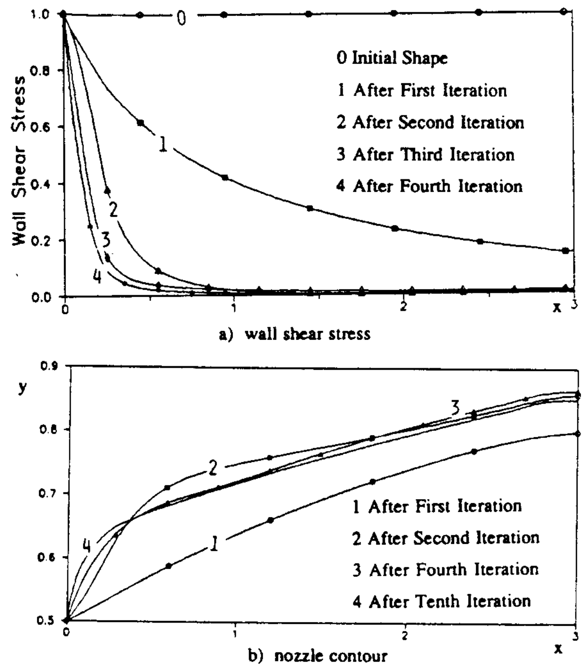

Fig. 7 Two-dimensional nozzle designed using laminar incompressible Navier-Stokes equations and adjoint operator/control theory[97-99]: iteration histories for a) the surface shear stress and b) the nozzle geometry. Inlet-to-exit pressure rise was maximized while minimizing total pressure loss.

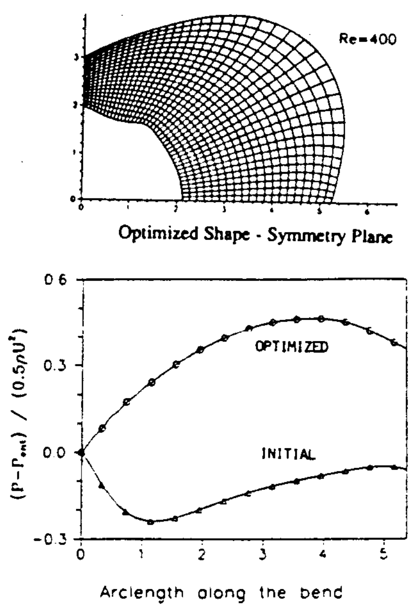

Fig. 8 Three-dimensional curved duct designed using Navier-Stokes equations and adjoint operator/control theory[98]; initial duct had constant radius; and inlet-to-exit pressure rise was maximized while minimizing total pressure loss.

set of adjoint equations and several more interface equations. Typically, the adjoint operator approach requires 5–15 analysis runs.

Conclusions

Realistic, aerodynamic shapes, can be designed using methodologies from computational fluid dynamics and optimization. Two basic categories of the inverse (design) formulation are 1) surface flow design, and 2) flowfield design. A number of methods, in both categories, including novel methods based on flow control theory, have been discussed and critically evaluated. Many issues remain unresolved. They include: specification of a more appropriate set of design constraints, acceleration of iterative algorithms, minimization of artificial dissipation, increased versatility and robustness of the design methods, and direct use of existing and future flowfield analysis software.

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