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Minimization of The Number of Cooling Holes in Internally Cooled Turbine Blades

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Abstract

Our inverse design methodology for the determination of the proper locations, shapes and sizes of a given number of coolant flow passages (holes) subject to specific surface temperatures and heat fluxes has been extended to allow the designer the freedom to guess the required number of holes and to specify the minimal allowable diameter of a hole. A constrained optimization algorithm is then used to minimize the total number of cooling holes, while satisfying user-specified hot surface temperatures and heat fluxes. Premature termination of the optimization process due to the existence of local minimas has been satisfactorily resolved by automatic switching of the objective function formulation whenever the local minima is detected. The convergence criteria of the iterative process, which can be specified by the user, was found to have a strong influence on the accuracy of the entire inverse design optimization algorithm.

Nomenclature

D_N	diameter of a hole N
F	objective function
s_N, y_N	coordinates of the center of a hole N
T	temperature
q	heat flux
ϵ	a small number ($\epsilon < 10^{-6}$)

Subscripts

hot	outer surface of the blade
cold	surface of a coolant passage (hole)

Superscripts

d	desired or specified
c	computed

Introduction

During the past several years a new method has been developed (Dulikravich, 1988) and has been constantly improved, that allows a thermal cooling system designer to determine proper sizes, shapes and locations of coolant passages (holes) in, say, an internally cooled turbine blade. Using this method the designer can iteratively enforce desired heat flux distribution on the hot outer surface of the blade, while simultaneously enforcing desired temperature distributions on the hot surface and on the surfaces of each of the holes. This constitutes an overspecified problem which is solved by allowing the sizes, locations and shapes of the holes to adjust iteratively until the final multiply connected configuration satisfies the overspecified thermal boundary conditions and the governing Laplace's equation for the steady temperature field. The problem is solved by minimizing an error function expressing the difference between the specified and the computed hot surface heat flux. Constraints on minimal allowable distance between any two holes and between any hole and the outer boundary are specified by the user.

Theoretical Formulation

This method deals with heat transfer by conduction inside the solid material of the blade.

We will assume that the temperature field is already steady and that the solid material of the internally cooled configuration is thermally isotropic and incompressible. Phase change of the material and its thermal expansion are neglected although, in principle, they could be included. The blade can be ceramically coated or otherwise made of regions having different thermal diffusivities (Chiang and Dulikravich, 1986). Consequently, the governing energy equation is simple Laplace's equation for the temperature, T, that is

$$\nabla^2 T = 0$$

Mathematically, boundary conditions defining the solution of this equation can be of three basic types: specified surface temperature (Dirichlet), specified surface heat flux (Neumann) and a mixed boundary condition (linear combination of the temperature and the heat flux).

Dirichlet conditions mean that the designer can specify desired values of the temperatures T_{hot}^d on a point-by-point basis along the entire outer (hot) surface of the object of a known and fixed geometry. At the same time the desired temperature T_{cold}^d should be specified along the surfaces (walls) of the yet unknown cooling passages. This would be a rudimentary example of a well-posed elliptic problem of a Dirichlet type except for the fact that the configuration of the holes is yet unknown.

In addition to the desired specified hot surface temperature distribution, T_{hot}^d, we allow the designer to specify also the desired hot surface heat flux, q_{hot}^d, on a point-by-point basis. This combination of T_{hot}^d and q_{hot}^d represents an overspecified elliptic problem meaning that in general it has no solution, except, possibly, for a very special configuration of holes.

This can be explained as follows. Since hot gases are flowing around the blade, the heat flux q_{hot}^d on the hot surface will be determined by the properties of the hot fluid and its flow regime. In other words, when analyzing the hot flowfield we will use the specified hot surface temperature, T_{hot}^d, as one of the solid wall boundary conditions in order to account for the convective and radiative heat transfer between the blade and the fluid. As a by-product of the flowfield computation we will then obtain the corresponding heat flux, q_{hot}^c, at the hot surface. This calculated heat flux we will use as an additional thermal boundary condition for the inverse heat conduction problem inside the solid material of the blade. Notice that instead of obtaining q_{hot}^c from a hot fluid flow analysis code, we could have used an experimentally obtained distribution of hot surface heat fluxes.

After the desired surface temperatures T_{hot}^d and T_{cold}^c have been specified, the designer will have to guess the number of coolant flow passages, N, their diameters, D_N, and their locations characterized by the coordinates x_N, y_N of their centers. For simplicity we have decided to work with circular cross section passages only (Chiang and Dulikravich, 1986), although arbitrary shapes for the passage cross sections could be used (Kennon and Dulikravich, 1985; 1986). The Laplace's equation can now be integrated numerically in this multiply connected domain subject to desired specified surface temperatures T_{hot}^d and T_{cold}^d only. The solution of the Laplace's equation for heat conduction in the solid material will be a detailed map of temperatures and corresponding local heat conduction fluxes including the values of the computed heat fluxes on the hot surface, q_{hot}^c. Since we have integrated the Laplace's equation for the guessed values of N, D_N and x_N, y_N, the desired hot surface heat flux q_{hot}^d will not be the same as the computed hot surface heat conduction flux, q_{hot}^c. This difference in the locally specified hot surface heat flux and the locally computed hot surface heat flux can be either positive or negative. The objective is to minimize the squared and properly normalized difference of the fluxes by iteratively changing the sizes of the holes, D_N, and the locations of their centers, x_N, y_N. That is, the objective function that needs to be minimized is the integrated and normalized global L-2 norm of the differences in the two hot surface heat fluxes.

$$F = \frac{\sum_i (q_{hot}^d - q_{hot}^c)_i^2}{\sum_i (q_{hot}^d)_i^2 + \epsilon}$$

Here, ϵ is a very small number which should eliminate the singular behavior of F when

$$\sum_i (q^d_{hot})^2_i = 0$$

An alternative objective function could be understood as the sum of the local normalized L-2 norms of the differences in the hot surface heat fluxes, that is

$$F = \sum_i \left\{ \frac{(q^d_{hot} - q^c_{hot})^2}{(q^d_{hot})^2 + \epsilon} \right\}_i$$

The minimization of F can be performed using any of a number of standard optimization algorithms. We chose to use standard Davidon-Fletcher-Powell minimization (Vanderplaats, 1984) which requires on average between 10 and 20 complete solutions of the Laplace's equation per each of the N x (D_N + x_N + y_N) parameters to be optimized. Any version of a finite difference or a finite element temperature analysis code would require considerable amounts of computer time even in the case of a single hole. Obviously, only an accurate form of the boundary element method will be fast enough (Kishimoto, Miyasaka and Aoki, 1989; Tanaka and Jamagiwa 1989; Zabaras, Morellas and Schnur, 1989) to provide an economically feasible design procedure.

Results

To illustrate this method, we have considered a test case consisting of a disk shaped domain with a thick coating of another solid material. The disk had a large centrally located hole (Fig. 1). There is a known analytical solution for a steady state temperature field in such a doubly-connected circular domain when constant temperatures T_{hot}^d and T_{cold}^d are specified on the outer boundary and the inner (hole) boundary, respectively. We have decided to use the corresponding analytical value of the heat flux q_{hot}^d = const. on the outer surface and try to satisfy these overspecified thermal boundary conditions by drilling three holes (Fig. 1) instead of one centrally located hole. Using our inverse design methodology, the sizes of two holes have been iteratively reduced to practically zero (dots marked by arrows in Fig. 1). At the same time, the third hole was enlarged and became practically indistinguishable from the analytically correct centrally located one-hole configuration. The convergence history of the optimization process (Fig. 2) indicates that the total count of thermal field analysis calls was 636 in order to reduce the global L-2 norm of the surface error in heat flux to 0.1%.

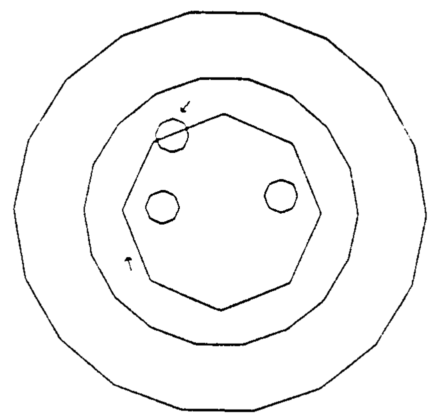

Fig. 1. Initial configuration (three holes of equal radii) and final configuration (one large centrally located hole and two dots marked with arrows) corresponding to a solution with 0.1% integrated flux error.

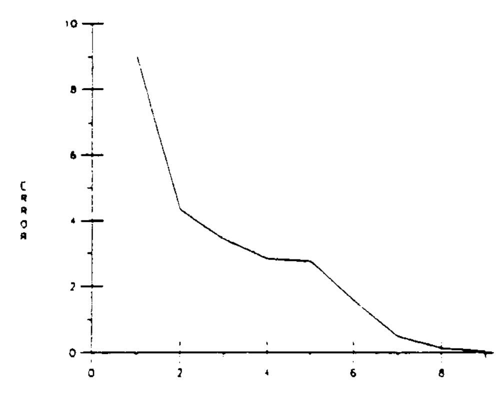

Fig. 2. Integrated heat flux error (L-2 norm) convergence history during the optimization.

Obviously, the thermal field analysis routine should be as efficient as possible. For this reason we have used our version of a Boundary Element Method (BEM) with linearly varying temperature distribution along each straight surface panel. In addition, there is a problem of underiteration or insufficient accuracy which has been recognized in the field of numerical inverse methodologies (Chiang and Dulikravich, 1986; Das and Mitra, 1989). To illustrate this point, notice (Fig. 3) that if we would have terminated the iterative process after only six iterations when the error was seemingly acceptably low (1.868%), the sizes and locations of the holes would have been unacceptably incorrect. In conclusion, only a highly efficient and accurate analysis algorithm should be used in this design procedure until a convergence level of less than 0.1% error is achieved.

In another test case where the initial sizes of the three holes were identical and they were initially located at the same radial distance and spaced 120 degrees apart (Fig. 4), each of the three holes changed its size and location equally since there was no preference for treating any of the holes differently. The apparently low error of the circumferentially integrated heat flux corresponding to the final converged solution (Fig. 5) was possible for two reasons. The thermal conductivities of the disk and the outer ring coating were considerably different with the coating acting as a temperature gradient smoothing device. Also, we did not minimize the local L-2 norm of the error at each point on the outer surface. Instead, we have minimized the circumferentially integrated L-2 norm error, thus allowing for oscillations of the final outer surface heat flux. In conclusion, any new software for multiply-connected configurations should be first tested on multi-symmetric problems where they should maintain the multi-symmetry. Only then can such software be applied with full confidence in the inverse shape design of arbitrary multiply-connected configurations.

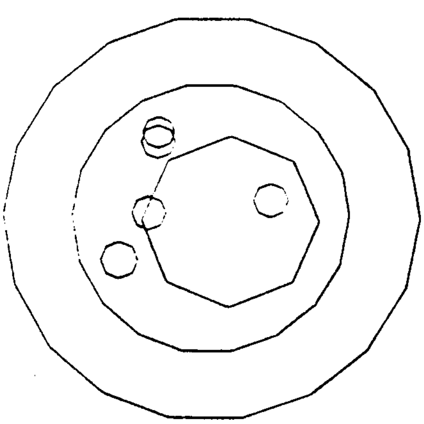

Fig. 3. Underiterated result with the initial three hole configuration. Final integrated heat flux error is 1.868%.

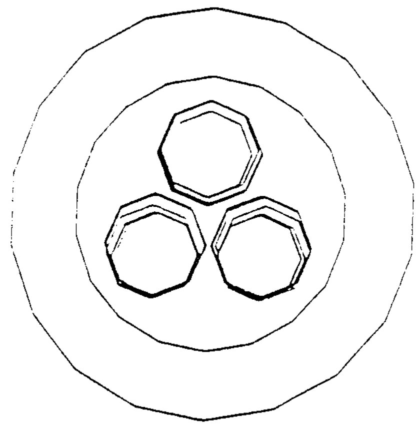

Fig. 4. Initially symmetrically located holes of identical size maintain a symmetric configuration throughout the iterative process.

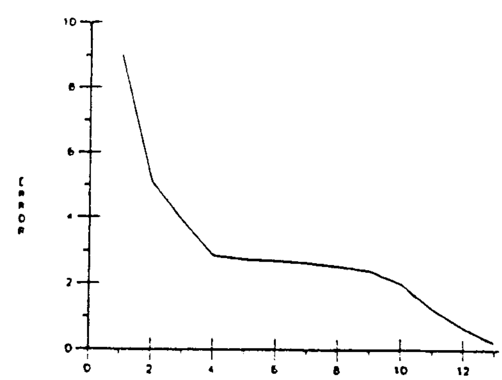

Fig. 5. Convergence history of the three-hole symmetrical configuration.

Hole Number Minimization

In this example, we should be allowed to initially guess an arbitrarily large number of holes whereby all holes except one should shrink to zero size. This procedure is extremely time consuming and often terminates in a local minima (Fig. 6) thus preventing the minimization process from fully converging (Fig. 7). Obviously, when the size of a hole becomes very small, its influence on the solution becomes also negligible. Thus, instead of wasting the computational time on recomputing the entire solution including these very small holes and further optimizing their sizes and locations, we can eliminate any hole that reduced beyond a prespecified size. In Fig. 8 we see an optimization history involving automatic hole elimination whenever a hole diameter became smaller than 1% of the average domain diameter. The solution is considerably better converged (Fig. 9) as compared to the solution without elimination of the holes (Fig. 6). The hot surface heat flux error convergence history (Fig. 9) shows clearly that the convergence is rapid, although occurrence of local minima again seems to terminate the iterative optimization process. Therefore, we have decided to automatically switch from the integral L-2 norm error (Eq. 2) to local L-2 norm error (Eq. 3) each time when the convergence becomes too slow. This sudden change in the way that the objective function is evaluated causes the

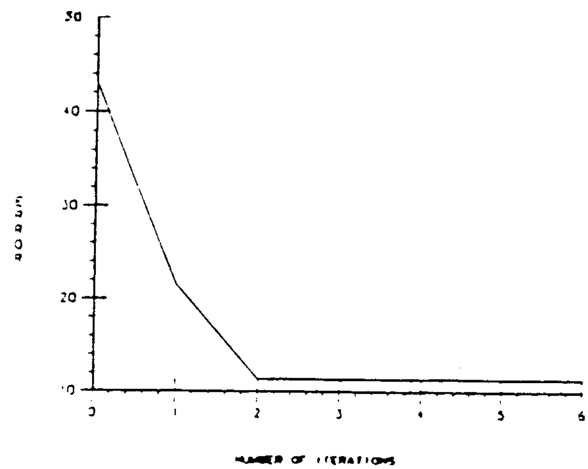

Fig. 7. An L-2 norm of the hot surface heat flux error: convergence history for a circular domain with initially ten holes. Hole elimination method not used. Minimization process terminates in a local minima.

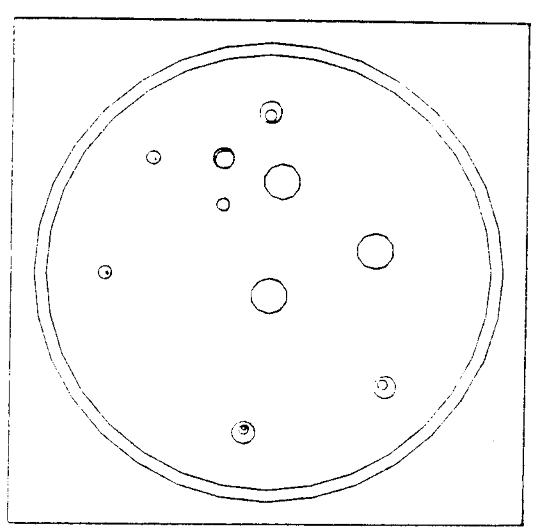

Fig. 6. Coated disk problem with initially ten holes. Convergence history shows that five holes are not reducing to zero. Hole elimination method not used.

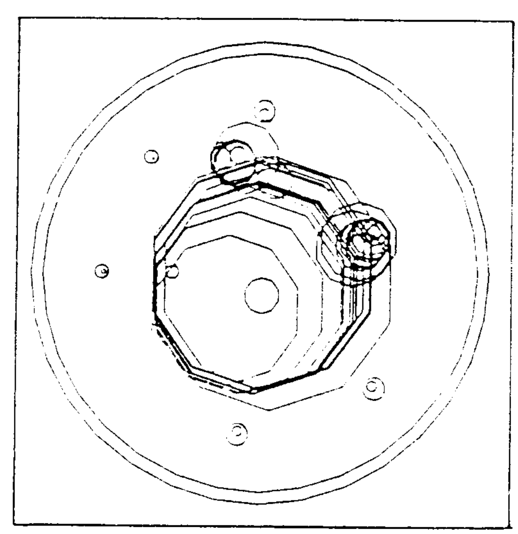

Fig. 8. Geometric convergence history for a circular domain with initially ten holes. Hole elimination method used together with L-2 norm switching.

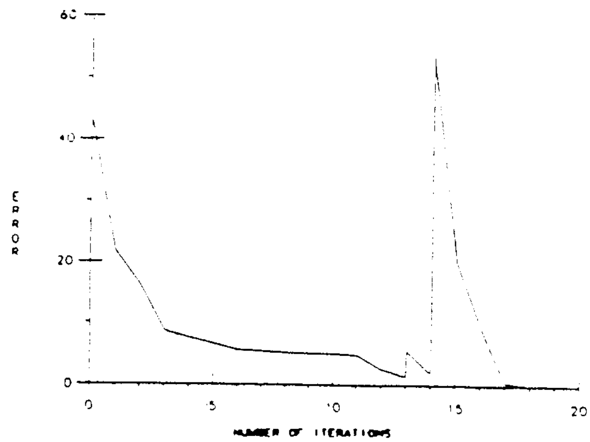

Fig. 9. An L-2 norm error history of the hot surface heat flux for a circular domain with initially ten holes. Hole elimination method and L-2 norm switching used.

solution to depart from a local minima and continue on its search for a global minima. Large jumps in the values of L-2 norm are evident (Fig. 9) whenever the switch from one formula (Eq. 2) to another formula (Eq. 3) was performed.

A second example involved an application of these techniques to a noncircular domain. The temperature field inside a coated turbine blade of a realistic shape (Fig. 10) and having five holes was obtained using our boundary element analysis code. The optimization process was then initiated by

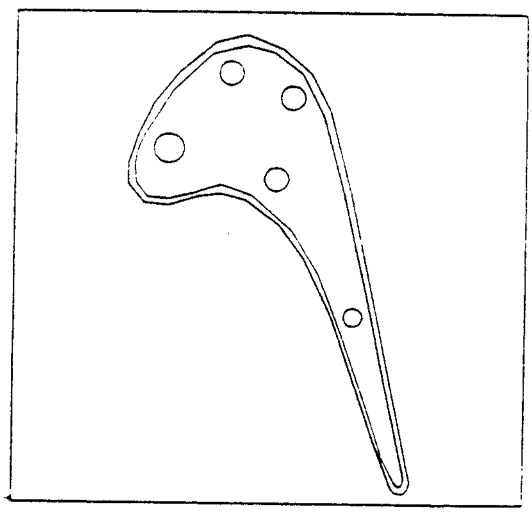

Fig. 10. Actual solution for example no. 2. A five-hole coated turbine blade from which thermal boundary conditions were used.

guessing that there should be ten holes (Fig. 11) with each of them having the same T_{cold}^{c} temperature as the actual five holes. Also, desired hot surface temperature T_{hot}^{c} and heat flux q_{hot}^{c} corresponding to the five-hole configuration were iteratively enforced. After 30 iterations requiring 3775 calls to the Laplace's equation integration routine, the error reduced below 0.25%. The optimized solution (Fig. 12) had six holes with five of them having almost the

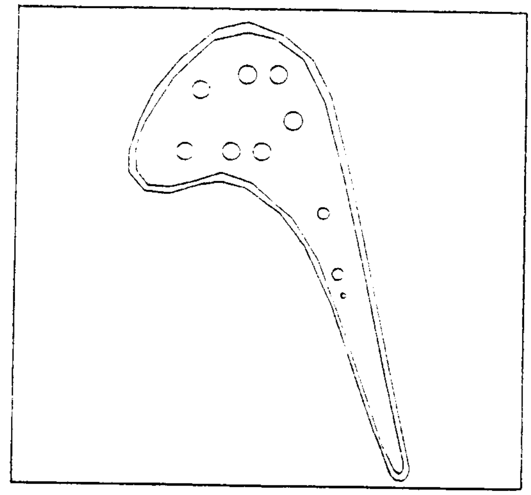

Fig. 11. Initial guess for example no. 2, using ten holes and the thermal boundary conditions from the five-hole configuration (Fig. 10).

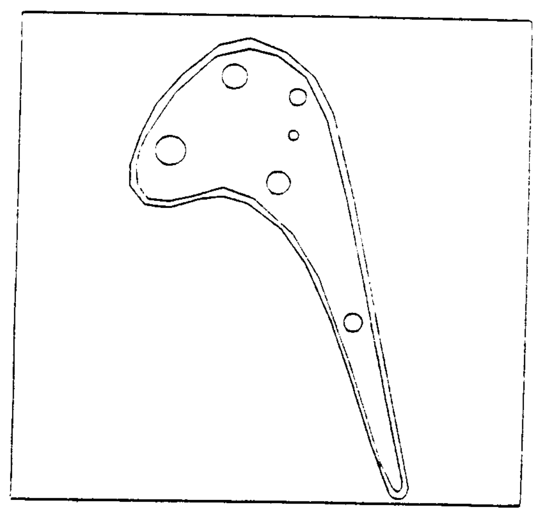

Fig. 12. Optimized solution that started with ten holes and resulted in six holes, where the sixth hole continued to shrink. Hole elimination method and L-2 norm switching used.

same sizes and locations as the correct solution, while the sixth hole was still reducing to zero. Iterative convergence history (Fig. 13) indicates that automatic hole elimination and objective function switching was used. The turbine blade example consumed about 5000 seconds on an IBM 3090 computer.

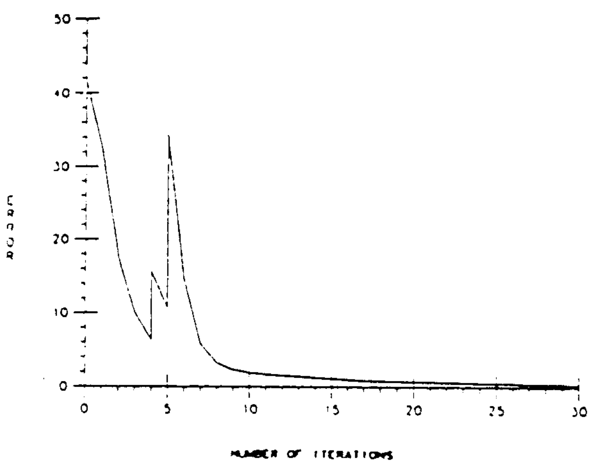

Fig. 13. Convergence history for the turbine blade solution that started with ten holes and resulted in six holes. Hole elimination method and L-2 norm switching used.

Summary

A technique for performing an automatic determination of proper sizes and locations of multiple coolant flow passages has been augmented by allowing the designer to also freely guess the number of holes needed. The algorithm automatically eliminates any guessed hole if it reduces in diameter below a user-specified minimum value. This feature allows even unexperienced designers to create a good thermal design in a single computer run. Local minima in the minimization process are automatically avoided by switching from an integrated L-2 norm to a local L-2 norm of the hot surface heat flux error.

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