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CONTROL OF INTERIOR COOLING RATES IN HETERGENEOUS MATERIALS BY VARYING SURFACE THERMAL BOUNDARY CONDITIONS

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ABSTRACT

Recent experimental research has confirmed that the cooling rate is one of the most crucial factors in determining the survival of frozen tissues and organs which are destined for transplantation. Optimal local cooling rates are difficult to obtain due to the nonuniform geometries of organs and the variation of thermal properties in the various tissue layers. It is shown that an optimal time-space variation of the container wall temperature can be determined using an inverse formulation which will produce optimal local cooling rates in the interior of the system. As a specific example, the cooling of a canine kidney is considered, and nearly optimal, constant cooling rates are obtained throughout the entire kidney.

INTRODUCTION

When preserving living human tissues (kidney, heart, liver, embryo, bone, spleen, semen, etc.) for the purpose of performing transplant surgery, the organ is cooled in a cryoprotective agent (CPA) to a prescribed low temperature and stored at this temperature until ready for use. During the cooling process there is an optimal cooling rate [1] for each particular type of tissue of an organ. Figure 1 shows typical cell survivability which is maximized at a specific cooling rate and falls off at cooling rates above and below the optimum. It has been suggested [2] and computationally demonstrated that the proper surface thermal conditions of the container in which the CPA and the organ are located can be determined so that the optimal local cooling rates are achieved at each instant of time at every point in the organ. These surface conditions are very difficult to determine because of the irregular shape of human organs and the fact that different cell types have different optimal cooling rates. At the present time, freezing protocols use a fixed cooling rate at every point on the outside surface of the organ [1,3]. However, this procedure results in considerably different values of local cooling rates inside the organ [4], because thermal boundary conditions are not propagated uniformly into the interior of a heterogeneous tissue. We have decided to use an inverse design procedure to determine the transient conditions that should be applied on the surface of a container in order to produce near optimal cooling rates inside an organ. A specific example of cooling a canine kidney is considered. It is shown that it is feasible to determine a priori surface conditions which can produce local prescribed cooling rates throughout irregular geometries.

MATHEMATICAL FORMULATION

A two-dimensional heat conduction equation is used:

$$\partial T/\partial t = \alpha \nabla^2 T \tag{1}$$

Equation (1) is solved for a model of a kidney shown in Figure 2. Here, α is the heat diffusivity coefficient ($m^2/°Csec$), T is the absolute temperature (°K), and t is the time (sec). In this example the organ is cooled and does not undergo phase change. In practice, phase change is an important physical phenomena [4] and the temperature dependent thermal properties and latent heat release could be included in the thermal model. The boundary element method [5] was used since it allows easy treatment of arbitrary geometrical shapes.

While the size and geometry of the transplant organ and the container remain fixed, the wall temperature of the cooling container can be continuously adjusted in time in order to maintain the prescribed local cooling rates throughout the organ. The circumferential temperature distribution on the container wall was approximated with a Chebyshev polynomial. The initial values of the coefficients of the polynomial are specified and the transient temperature distribution is computed at every point of the organ. From this, the actual local cooling rates are determined at each point of the organ. A normalized error function can then be formed as a sum of least squares of deviations of the computed and the specified optimal local cooling rates. A nonlinear constrained function minimization method [6] was used to update the polynomial coefficients. The new temperature distribution on the container walls is determined in order to reduce the error function at the next time step during the cooling process. Thus, the desired optimal local cooling rates are achieved throughout the organ by determining the proper instantaneous values of the coefficients of the polynomials representing the temperature variation on the surface of the container.

NUMERICAL RESULTS

To demonstrate the practical application of the optimization design process, an actual canine kidney was approximated by three multiply connected regions shown in Figure 2. This includes the outer annulus simulating the CPA fluid and the two inner regions simulating two distinct tissues of an actual kidney. The diffusivities used were $\alpha_1 = 0.00145$, $\alpha_2 = 0.0169$ and $\alpha_3 = 0.0255$ in the CPA, outer and inner kidney layers, respectively. Linear spatial interpolation and a constant time step of $\Delta t = 30$ seconds were used. The initial temperature of the entire system was 305°K and the prescribed optimal cooling rate in every triangle forming the kidney domain was -2.5°K. No cooling rate was prescribed in the CPA. A sixth order Chebyshev polynomial was used to represent the variable temperature on the surface of the container. The circumferential variation of optimized container wall temperatures is shown in Figure 3. The temperature fields are not axisymmetric due to the kidney geometry and placement in the container. When the initial guess for the surface temperature is used for the entire simulation, the relative average error of the cooling rate in the kidney was 11.7% at ten minutes. In contrast, when the container wall temperatures are determined using optimization techniques, the relative average error of the cooling rate in the kidney was reduced to less than 1% during the entire 20 minute cooling protocol which was simulated.

CONCLUSIONS

Inverse design techniques can be used to determine both spatial and transient surface temperature varations which will produce specific, local thermal conditions inside a heterogeneous material. This technique can be used to maximize cell survivability during the cryopreservation process.

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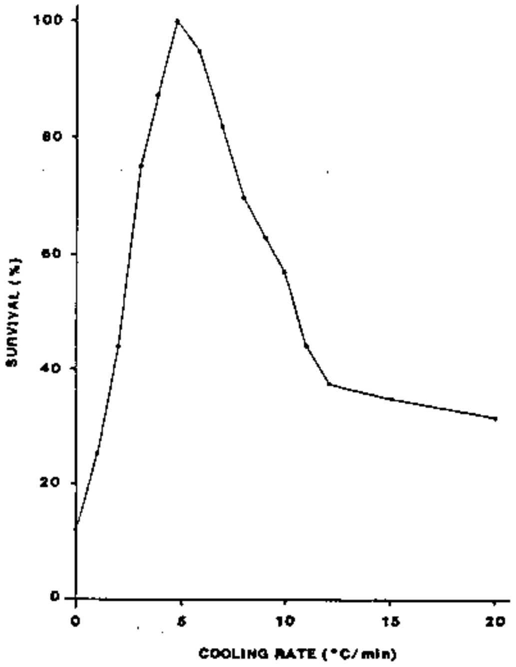

Figure 1. Representative Cell Survival Signature Normalized to 100% at 5 C/min Cooling Rate

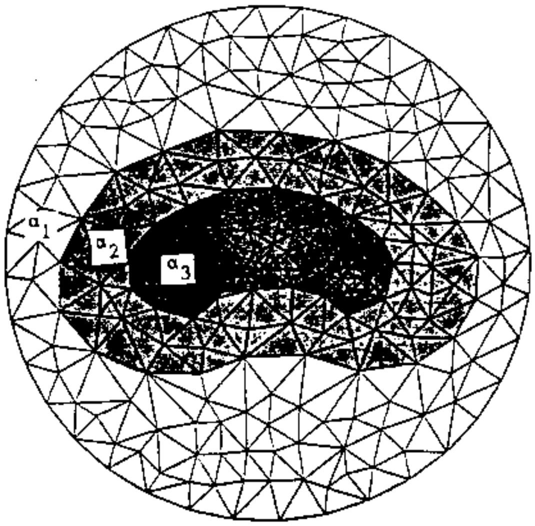

Figure 2. Computational Grid for a Kidney/CPA System

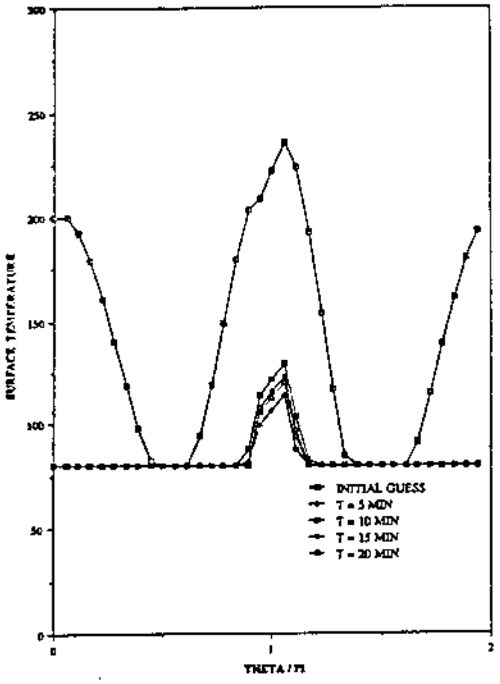

Figure 3. Circumferential distribution of Optimized Container Wall Temperatures.