PHYSICALLY CONSISTENT MODELS FOR ARTIFICIAL DISSIPATION IN TRANSONIC POTENTIAL FLOW COMPUTATIONS

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The effects that artificial dissipation has on numerical solutions of the transonic full potential equation (FPE) are investigated by comparing the artificially dissipative FPE to a physically dissipative potential (PDP) equation. Analytic expressions were derived for the variables C and M_c that are used in the artificial density formulation. It was shown that these new values generate artificial dissipation, which is equivalent to the physical dissipation which exists in the PDP equation. The new expressions for the variables C and M_c can easily be incorporated into the existing full potential codes, which are based either on the artificial density or on the artificial viscosity formulation. Comparisons of physically dissipative potential (PDP) with the artificial density or viscosity (ADV), artificial mass flux (AMF), and ADV with variable C and M_c formulation (MCC) illustrate the fact that the ADV dissipation creates excessively large amounts of equivalent entropy. As an alternative, AMF and MCC physically consistent formulations offer considerably lower equivalent entropy jumps across shocks.

1. Introduction

A mathematical model for nondissipative, irrotational, compressible, inviscid flows is known as the full potential equation (FPE). Numerical techniques for integrating the FPE in transonic shocked regions require addition of artificial dissipation in an attempt to stabilize these schemes.

The artificial dissipation must be added in a fully conservative form if shock strengths and locations are to be computed accurately. Although the common belief is that the shock jumps are independent of the details of the artificial dissipation, we have found that this is not true when computing equivalent entropy jumps. The artificial dissipation usually has the form of artificial viscosity [1], artificial density [2, 3] or artificial mass flux [4, 5]. In this work, the artificially dissipative FPE, that is, FPE with an artificial density or viscosity formulation (ADV) and the FPE with an artificial mass flux (AMF) formulation were compared to a recently derived physically dissipative potential (PDP) equation [6].

From these comparisons, a new form of numerical dissipation has been derived which has physical origins and an analytic formulation for the constants presently used in the ADV. This new formulation is termed variable M_c and C or (MCC) formulation. Our objective is to demonstrate that the ADV formulation generates large amounts of entropy and that AMF and MCC formulations generate considerably less entropy across a shock.

2. Physically dissipative potential (PDP) equation

Dulikravich and Kennon [6] have derived a new mathematical model which governs irrotational, non-isentropic, viscous flows of calorically perfect gases without body forces, surface tension, radiation heat transfer, internal heat generation and mass sources. This model includes the physical dissipation due to certain effects of shear viscosity, secondary viscosity and heat conductivity. The full three-dimensional version of their physically dissipative potential (PDP) equation can be expressed [6] in a canonical nondimensional form as

$$
\begin{aligned}
\rho\Big\{ & [(1 - M^2)\phi_{ss} + \phi_{mm} + \phi_{nn}] - \frac{1}{a^2}(\phi_{tt} + 2\phi_s\phi_{st})\Big\} \\
& = \frac{\mu''}{\mathrm{Re}}\Big\{ -\frac{\phi_s}{a^2}\Big(1 + \frac{\gamma - 1}{\mathrm{Pr}''}\Big)(\phi_{sss} + \phi_{smm} + \phi_{snn}) \\
& + \frac{\gamma - 1}{a^2}\Big(1 - \frac{1}{\mathrm{Pr}''}\Big)[(\phi_{ss})^2 + (\phi_{mm})^2 + (\phi_{nn})^2] \\
& + 2\frac{\gamma - 1}{a^2}\Big(1 - \frac{2}{\mu''}\Big)(\phi_{ss}\phi_{mm} + \phi_{ss}\phi_{nn} + \phi_{mm}\phi_{nn}) \\
& - 2\frac{\gamma - 1}{a^2}\Big(\frac{1}{\mathrm{Pr}''} - \frac{2}{\mu''}\Big)[(\phi_{sm})^2 + (\phi_{sn})^2 + (\phi_{mn})^2] \\
& - \Big(1 - \frac{\gamma - 1}{\mathrm{Pr}''}\Big)\frac{1}{a^2}(\phi_{sst} + \phi_{mmt} + \phi_{nnt})\Big\}.
\end{aligned}
\tag{1}
$$

Here, s is the locally streamline aligned coordinate direction and m and n are the mutually orthogonal remaining coordinates of the locally streamline-aligned Cartesian coordinate system. The left-hand side of this equation represents the nondissipative FPE and the right-hand side represents physical dissipation due to viscous effects and heat conduction. Since Reynolds number, Re, is the common denominator of the entire right-hand side, the PDP equation is applicable to flow fields with negligible vorticity and very high Re number, that is, outside the boundary layer. Notice also that the PDP equation does not use small perturbations and that it represents a more complete version of the well-known transonic-viscous (T–V) small perturbation equation model [7, 8] which contains only the essential nonlinearities. Here, ρ is the local fluid density, ϕ is the local velocity potential function, $(V = \nabla\phi)$, a is the local isentropic speed of sound, t is the time, μ is the coefficient of shear viscosity, λ is the coefficient of secondary viscosity, μ'' is the coefficient of longitudinal viscosity $\mu'' = 2\mu + \lambda$, γ is the ratio of specific heats, M is the local Mach number, Re is the

Reynolds number [7], Pr″ is defined [8] as the longitudinal Prandtl number $\mathrm{Pr}'' = \mathrm{Pr}\,\mu''/\mu$ where $\mathrm{Pr} = \bar{C}_p \bar{\mu}_\infty / \bar{k}_\infty$ is the Prandtl number and k is the coefficient of heat conductivity.

All quantities in (1) have been nondimensionalized, that is,

$$\rho = \frac{\bar{\rho}}{\bar{\rho}_*}, \quad T = \frac{\bar{T}}{\bar{T}_*} = a^2 = \frac{\bar{a}^2}{\bar{a}_*^2}, \quad \phi_s = M_*, \quad \mu'' = \frac{\mu''}{\bar{\mu}_\infty}, \quad k = \frac{\bar{k}}{\bar{k}_\infty}, \quad \lambda = \frac{\bar{\lambda}}{\bar{\mu}_\infty},$$

$$\mathrm{Re}^{-1} = \frac{\bar{\mu}_\infty}{\bar{\rho}_* \bar{a}_* \bar{L}_*}, \quad x = \frac{\bar{x}}{\bar{L}_*}, \tag{2}$$

quantities have an overbar. Coefficients λ, μ and k are treated as constants. The one-dimensional version of (1) for steady flows,

$$\rho(1 - M^2)\phi_{xx} + \frac{\mu''}{Re}\left(1 + \frac{\gamma - 1}{Pr''}\right)\frac{\phi_x}{a^2}\phi_{xxx} - \frac{\gamma - 1}{a^2}\frac{\mu''}{Re}\left(1 - \frac{1}{Pr''}\right)(\phi_{xx})^2 = 0 , \qquad (3)$$

was numerically integrated using a fourth-order Runge-Kutta integration scheme with $\Delta x = 10^{-7}$ and several values for $\bar{\lambda}/\bar{\mu}$. The results indicate (see Figs. 1 and 2) that the PDP can produce shocks of various total jumps depending on the specified value of the ratio of viscosities λ/μ. Specifically, Stokes hypothesis that $\lambda/\mu = -2/3$ leads to Rankine-Hugoniot shock jumps, and $\lambda/\mu \cong -2$ leads to isentropic shock jumps [9]. Other values used in all test cases were $Pr = \frac{3}{4}$, $\gamma = \frac{7}{5}$, $Re = 10^5$.
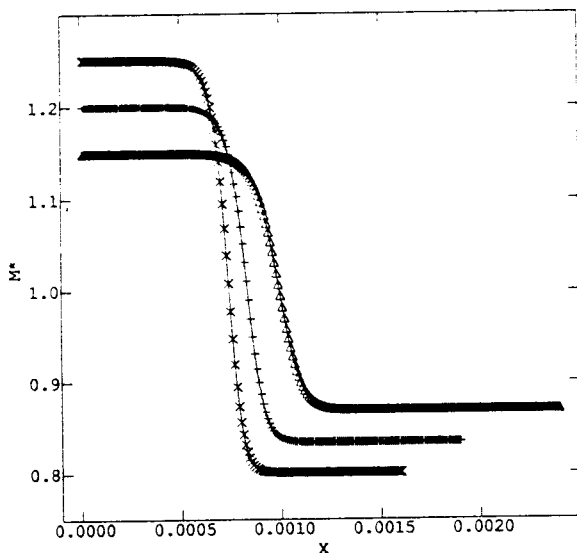

Fig. 1. PDP formulation: Variation of critical Mach number through normal shock using $\lambda/\mu = -0.666$ which produces Rankine-Hugoniot shock jumps: △△△△ $(\phi_x)_{-\infty} = 1.15$; ┼┼┼┼ $(\phi_x)_{-\infty} = 1.20$; ✕✕✕ $(\phi_x)_{-\infty} = 1.25$.
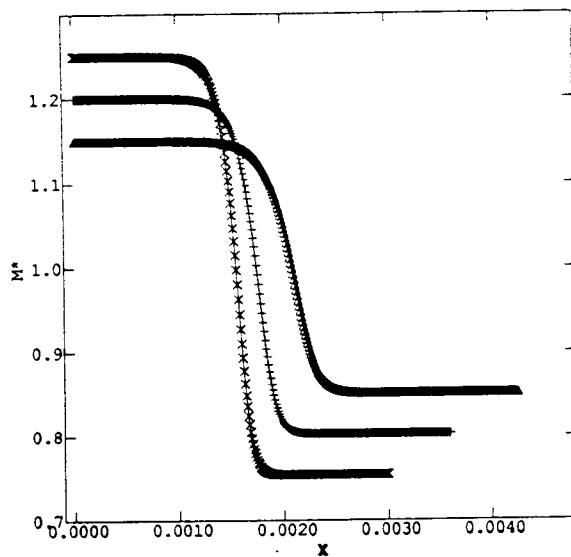

Fig. 2. PDP formulation: Variation of critical Mach number through a normal shock using $\lambda/\mu = -2.118$ which produces istentropic shock jumps: △△△ $(\phi_x)_{-\infty} = 1.15$, ┼┼┼┼ $(\phi_x)_{-\infty} = 1.20$; ✕✕✕ $(\phi_x)_{-\infty} = 1.25$.

3. Entropy generation

Dissipation effects in a flowfield can be most rigorously evaluated by computing the entropy change due to viscous effects and heat conduction. The entropy generation equation can be expressed as

$$\bar{\rho}\bar{T}\,\frac{\mathrm{d}\bar{S}}{\mathrm{d}\bar{t}} = \bar{\Phi} + \bar{k}\bar{\nabla}^2\bar{T}\,, \tag{4}$$

where Φ is the viscous dissipation function and S is the specific entropy. All the variables were consequently nondimensionalized with their critical values. Also, $\bar{\mu}''$ and λ were nondimensionalized with $\bar{\mu}_*$, $\bar{k}$ with $\bar{k}_*$ and $\bar{S}$ with $\bar{R}$, where the speed of sound is $\bar{a}^2 = \gamma\bar{R}\bar{T}$. Then

$$\nabla^2 T = \nabla^2\left[\frac{\gamma+1}{2} - \frac{\gamma-1}{2}\,(\nabla\phi\cdot\nabla\phi)\right] \tag{5}$$

and the nondimensionalized one-dimensional steady version of (4) becomes

$$\mathrm{Re}\,\rho T\phi_x\,\frac{\mathrm{d}S}{\mathrm{d}x} = \gamma\mu''(\phi_{xx})^2 - \frac{\bar{k}_*(\gamma-1)\gamma}{\gamma\bar{R}\bar{\mu}_*}\,k[\phi_x\phi_{xxx} + (\phi_{xx})^2]\,. \tag{6}$$

Notice that

$$\frac{(\gamma-1)}{\gamma\bar{R}} = \frac{1}{\bar{C}_p}\,, \qquad \mathrm{Pr}'' = \frac{\bar{C}_p\bar{\mu}_*''}{\bar{k}_*}\,, \qquad k = \frac{\bar{k}}{\bar{k}_*} = 1\,. \tag{7}$$

Then, the normalized entropy generation equation for one-dimensional steady flows without radiation and internal heat sources is

$$\frac{\mathrm{d}S}{\mathrm{d}x} = \frac{\gamma\mu''}{\rho T\,\mathrm{Re}}\left[\left(1 - \frac{1}{\mathrm{Pr}''}\right)\frac{(\phi_{xx})^2}{\phi_x} - \frac{\phi_{xxx}}{\mathrm{Pr}''}\right]\,. \tag{8}$$

If, for simplicity, we neglect the influence of entropy generation on the variation of density, the nondimensional entropy generation equation becomes

$$\frac{\mathrm{d}S}{\mathrm{d}x} \cong \frac{\gamma\mu''}{\mathrm{Re}\left[\dfrac{\gamma+1}{2} - \dfrac{\gamma-1}{2}\,(\phi_x)^2\right]^{\frac{\gamma}{\gamma-1}}}\left[\left(1 - \frac{1}{\mathrm{Pr}''}\right)\frac{(\phi_{xx})^2}{\phi_x} - \frac{\phi_{xxx}}{\mathrm{Pr}''}\right]\,. \tag{9}$$

Notice that S in this equation is actually a nondimensional quantity $S = \bar{S}/\bar{R}$. Equation (9) was integrated using the fourth order Runge–Kutta scheme and several values of $(\phi_x)_{-x}$ and λ/μ, while $\mathrm{Re} = 10^5$. For comparison, the exact values of the total entropy jump across a normal shock satisfying Rankine–Hugoniot conditions can be found from

$$\Delta S = \frac{\gamma}{\gamma-1}\ln\left[\frac{2}{(\gamma+1)M_1^2} + \frac{\gamma-1}{\gamma+1}\right] + \frac{1}{\gamma-1}\ln\left[\frac{2\gamma}{\gamma+1}\,M_1^2 - \frac{\gamma-1}{\gamma+1}\right]\,, \tag{10}$$
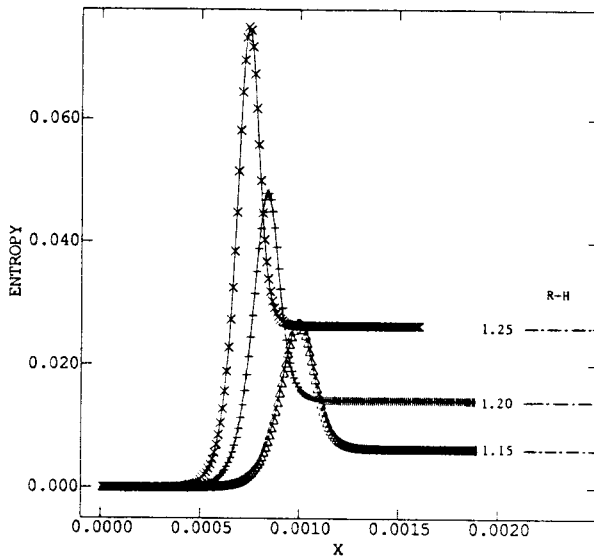
Fig. 3. PDP formulation: Variation of nondimensional entropy through a normal shock using $\lambda/\mu = -0.666$: $\triangle\!\triangle\!\triangle$ $(\phi_x)_{-\infty} = 1.15$; $+\!+\!+\!+$ $(\phi_x)_{-\infty} = 1.20$; $\times\!\!-\!\!\times\!\!-\!\!\times$ $(\phi_x)_{-\infty} = 1.25$.
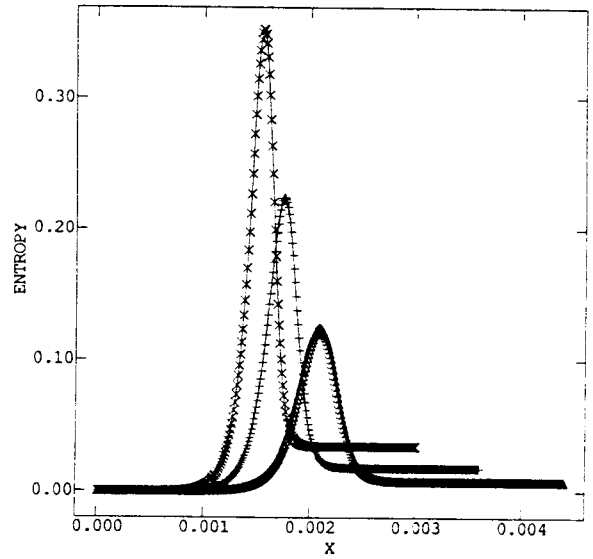
Fig. 4. PDP formulation: Variation of nondimensional entropy through normal shock using $\lambda/\mu = -2.118$: $\triangle\!\triangle\!\triangle$ $(\phi_x)_{-\infty} = 1.15$; $+\!+\!+\!+$ $(\phi_x)_{-\infty} = 1.20$; $\times\!\!-\!\!\times\!\!-\!\!\times$ $(\phi_x)_{-\infty} = 1.25$.

where M_1 is the Mach number ahead of the normal shock. Comparison of the numerically computed and the exact values of the total entropy jumps are good (Figs. 3 and 4), despite the fact that we have used an isentropic relation for the density, ρ, in (9). This result provides a detailed picture of entropy variation through the compression shock showing that it has a strong maximum in the middle of the shock [6]. Nevertheless, the objective of this work is not to study the structure of shock waves. Instead, our goal is to demonstrate the capability to compute and compare the total entropy jumps across the shocks by using different formulations for artificial dissipation.

4. Artificial density or viscosity (ADV) formulation

It was shown by Dulikravich [5] analytically that the conventional formulation for artificial density [3] generates (in the one-dimensional steady case) the following terms:

$$\rho\Bigg\{(1 - M^2)\phi_{xx} + C(M^2 - M_c^2)(M^2)^n\phi_{xxx}$$
$$+ C\bigg[(M^2 - M_c^2)(2 - (2 - \gamma)M^2)$$
$$+ \frac{(\gamma + 1)}{a^2}(M_c^2 + n(M^2 - M_c^2)\bigg](M^2)^n(\phi_{xx})^2 \frac{1}{\phi_x}\Bigg\} = 0,$$

$$(11)$$

where the constants C and n, and the constant cut-off [10] Mach number, M_c, are the user-specified input parameters. This equation is a result of using the following values of
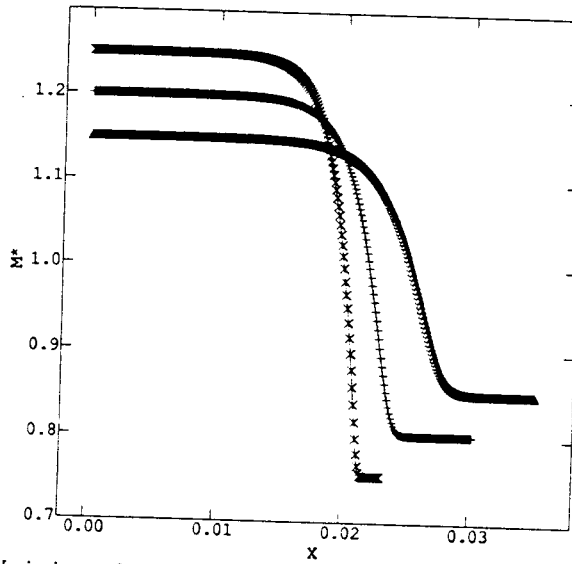
Fig. 5. ADV formulation: Variation of critical Mach number through a normal shock using $C = 0.001$ and $M_c = 0.70$: △△△ $(\phi_x)_{-\infty} = 1.15$; ┼┼┼ $(\phi_x)_{-\infty} = 1.20$; ✕✕✕ $(\phi_x)_{-\infty} = 1.25$.

artificial density, $\tilde{\rho}$, and the switching function, $\tilde{\mu}$, in the conventional form [3] of the artificial density or viscosity (ADV):

$$\tilde{\rho} = \rho - \tilde{\mu}\rho_s, \qquad \tilde{\mu} = C \max\left\{0, \quad 1 - \frac{M_c^2}{M^2}\right\}(M^2)^n. \tag{12}$$

Both C and M_c are arbitrary *constants* in the conventional ADV formulation [3]. The coefficient C is usually chosen to be of the order one [5]. The exponent n [5] is usually zero. The cut-off Mach number M_c is usually chosen [1] as having the constant value between 0.8 and 1.0. It should always be less than the post-shock Mach number [5].

Although M_c does not affect the total shock jump, it strongly affects the shock thickness. Since the artificial viscosity formulation is a truncated version of the artificial density formulation and since the directionally biased flux formulation [11] is equivalent [5] to the ADV, only ADV will be discussed. Critical Mach number variations through a normal shock resulting from the numerical integration of the ADV formulation (11) are shown in Fig. 5. The total Mach number jumps correspond to the isentropic shock strengths [9].

5. Effects of the numerical dissipation based on ADV

Equating the coefficients of like derivatives (ϕ_{xx}^2 and ϕ_{xxx}) in (3) and in (11) produces two simultaneous equations, namely

and

$$-\frac{\gamma - 1}{a^2} \frac{\mu''}{\text{Re}}\left(1 - \frac{1}{\text{Pr}''}\right) = C\rho\left\{(M^2 - M_c^2)(2 - (2 - \gamma)M^2) + \frac{\gamma + 1}{a^2} M_c^2\right\} \frac{1}{\phi_x} \tag{13}$$

$$\frac{\mu''}{\text{Re}}\left(1 + \frac{\gamma - 1}{\text{Pr}''}\right)\frac{\phi_x}{a^2} = C\rho(M^2 - M_c^2)\,.\tag{14}$$

These two equations can be solved for $(\mu''/\text{Re})_{\text{eq}}$ and for $(1/\text{Pr}'')_{\text{eq}}$. The result is

$$\left(\frac{\mu''}{\text{Re}}\right)_{\text{eq}} = \frac{C\rho a^2}{\gamma\phi_x}\left\{(M^2 - M_c^2)((2 - \gamma)M^2 - 1) - \frac{\gamma + 1}{a^2}M_c^2\right\}\tag{15}$$

and

$$\left(\frac{1}{\text{Pr}''}\right)_{\text{eq}} = 1 + \frac{C\rho a^2}{(\gamma - 1)}\frac{1}{(\mu''/\text{Re})_{\text{eq}}}\left\{(M^2 - M_c^2)(2 - (2 - \gamma)M^2) + \frac{\gamma + 1}{a^2}M_c^2\right\}\frac{1}{\phi_x}\,.\tag{16}$$

Thus

$$(\mu'')_{\text{eq}} = \frac{1/\text{Pr}}{(1/\text{Pr}'')_{\text{eq}}}\,.\tag{17}$$

These expressions provide physically equivalent values for $(\mu''/\text{Re})_{\text{eq}}$ and $(1/\text{Pr}'')_{\text{eq}}$ generated by the conventional formulations [3] of the artificial density where C and M_c are kept constant. Thus, we could now analyze the physically equivalent dissipative features of the ADV formulation. For this purpose, (15) and (16) were substituted in the entropy generation equation (9) and integrated. The results indicate (see Fig. 6) that the ADV formulation generates entropy which could even be larger than the entropy generated by Rankine–Hugoniot shocks. Moreover, when (17) is plotted (see Fig. 7) for three different constant values of M_c, it is noticeable that μ''_{eq} is not contant. Similar results are obtained (see Fig. 8) when the equivalent Prandtl number, $(\text{Pr}'')_{\text{eq}}$, is computed from (16).
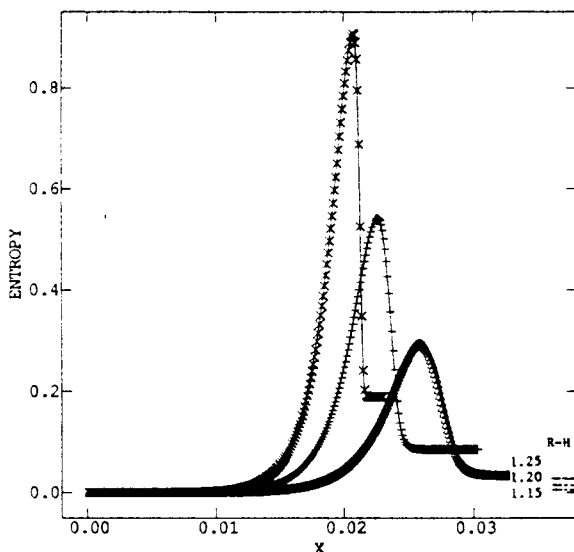

Fig. 6. ADV formulation: variation of nondimensional equivalent entropy through a normal shock using $C = 0.001$ and $M_c = 0.70$: ▵▵▵ $(\phi_x)_{-\infty} = 1.15$; ┼┼┼ $(\phi_x)_{-\infty} = 1.20$; ✕✕✕ $(\phi_x)_{-\infty} = 1.25$.
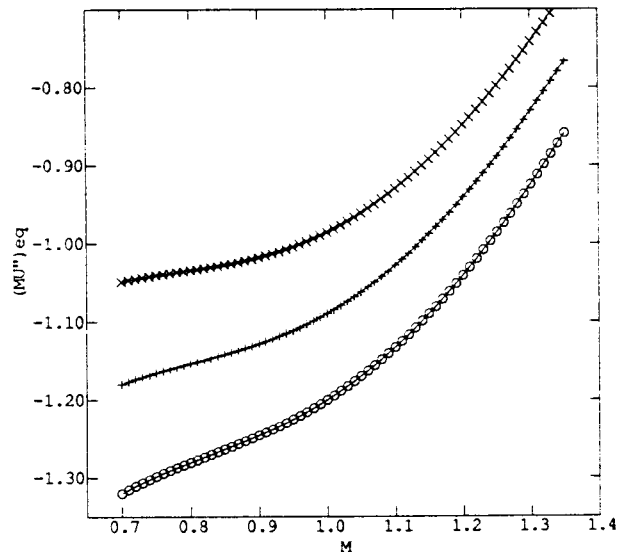
Fig. 7. ADV formulation: Variation of equivalent nondimensional longitudinal viscosity $(\mu'')_{\text{eq}}$ using $C = 1.0$: ⊖⊖⊖ $M_c = 0.80$; ┼┼┼ $M_c = 0.75$; ✕✕✕ $M_c = 0.70$.
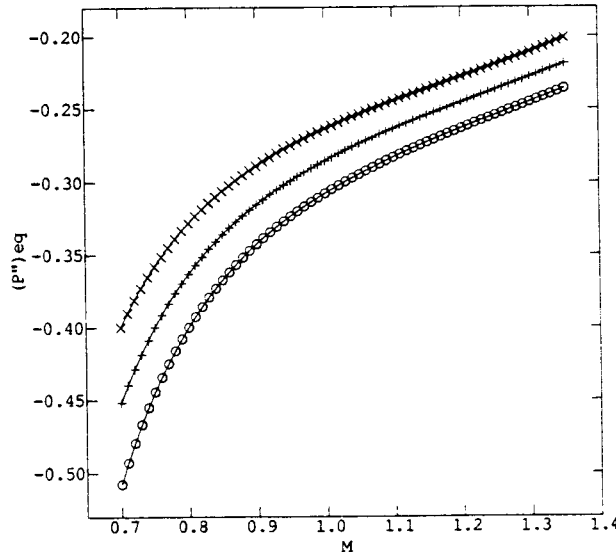
Fig. 8. ADV formulation: Variation of equivalent longitudinal Prandtl number $(Pr'')_{eq}$ using $C = 1.0$; ⊖⊖⊖ $M_c = 0.80$; | | | | $M_c = 0.75$; × × × $M_c = 0.70$.

6. Artificial mass flux (AMF) formulation

In addition to PDP and ADV formulations, it is possible to work with an artificial mass flux (AMF) dissipation [4, 5, 12]. Here, the entire mass flux $(\rho\phi_s)$, instead of just density, ρ, is differentiated in the locally upstream direction

$$\nabla \cdot (\rho V) = \left(\frac{\partial}{\partial s}\,\hat{e}_s + \frac{\partial}{\partial n}\,\hat{e}_n\right) \cdot \{[(\rho\phi_s) - (\rho\phi_s)_s]\hat{e}_s + (\rho\phi_n)\hat{e}_n\} = 0 , \qquad (18)$$

where $\hat{e}_s$ and $\hat{e}_n$ are the unit vectors in s and n direction, respectively.

In the two-dimensional case, the resulting artificially dissipative full potential equation contains non-physical dissipation [5] of the form

$$\rho[(1 - M^2)\phi_{ss} + \phi_{nn}] + A\left\{\frac{\phi_s}{a^2}\,\phi_{sss} + \frac{1}{a^2}\,\phi_{ss}\phi_{nn}\right.$$

$$\left. + \left[3 + 2(\gamma - 1)M^2 - \frac{\gamma + 1}{a^2}\right]\frac{(\phi_{ss})^2}{a^2}\right\} = 0 , \qquad (19)$$

where

$$A = -\frac{\mu''}{Re}\,(\gamma - 1)2\left(1 - \frac{2}{\mu''}\right) \quad \text{and} \quad \mu'' = \frac{(\gamma - 1)\left(4 - \frac{1}{Pr}\right)}{1 + 2(\gamma - 1)} . \qquad (20)$$

The switching function $\tilde{\mu}$ and the constant C in the AMF formulation were evaluated [5] by equating the coefficients multiplying ϕ_{sss} and $\phi_{ss}\phi_{nn}$ terms in the full two-dimensional AMF
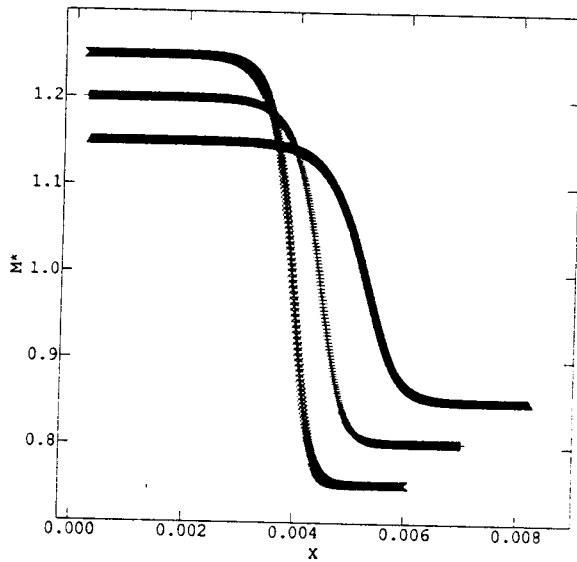
Fig. 9. AMF formulation: Variation of critical Mach number through a normal shock: $\triangle\!\!-\!\!\triangle\!\!-\!\!\triangle$ $(\phi_x)_{-\infty} = 1.15$; $+\!\!+\!\!+\!\!+\!\!+$ $(\phi_x)_{-\infty} = 1.20$; $\times\!\!-\!\!\times\!\!-\!\!\times$ $(\phi_x)_{-\infty} = 1.25$.
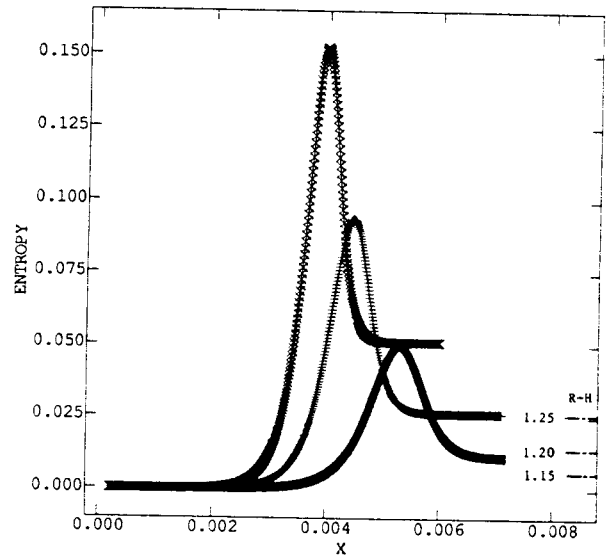
Fig. 10. AMF formulation: Variation of nondimensional equivalent entropy through a normal shock: $\triangle\!\!-\!\!\triangle\!\!-\!\!\triangle$ $(\phi_x)_{-\infty} = 1.15$; $+\!\!+\!\!+\!\!+\!\!+$ $(\phi_x)_{-\infty} = 1.20$; $\times\!\!-\!\!\times\!\!-\!\!\times$ $(\phi_x)_{-\infty} = 1.25$.

equation and in the PDP equation. Figure 9 demonstrates that the AMF formulation provides shock jumps [9] corresponding to isentropic shocks and that it produces positive entropy jumps (see Fig. 10) which are significantly smaller than the entropy jumps produced by the ADV formulation (see Fig. 6). AMF formulation requires the Reynolds number as the only input parameter [5] since $\gamma = \frac{7}{5}$ and $\mathrm{Pr} = \frac{3}{4}$ for diatomic gases.

7. Variable C and M_c (MCC) formulation

If (13) and (14) are solved simultaneously for C and M_c, the result is an analytic expression for variable values of C and M_c that are consistent with the physical dissipation generated by the PDP equation. Hence

$$M_c^2 = \frac{M^2\left\{\left(1 + \dfrac{\gamma - 1}{\mathrm{Pr}''}\right)[2 - (2 - \gamma)M^2] + (\gamma - 1)\left(1 - \dfrac{1}{\mathrm{Pr}''}\right)\right\}}{\left(1 + \dfrac{\gamma - 1}{\mathrm{Pr}''}\right)\left[2 - (2 - \gamma)M^2 - \dfrac{\gamma + 1}{a^2}\right] - (\gamma - 1)\left(1 - \dfrac{1}{\mathrm{Pr}''}\right)}, \tag{21}$$

$$C = \frac{\left(1 + \dfrac{\gamma - 1}{\mathrm{Pr}''}\right)\dfrac{\phi_s}{a^2}\,\mu''}{\rho(M^2 - M_c^2)\mathrm{Re}}. \tag{22}$$

The new variable values for C and M_c in the ADV formulation are analytically defined so that they duplicate the physical dissipation from the one-dimensional version of the PDP equation.
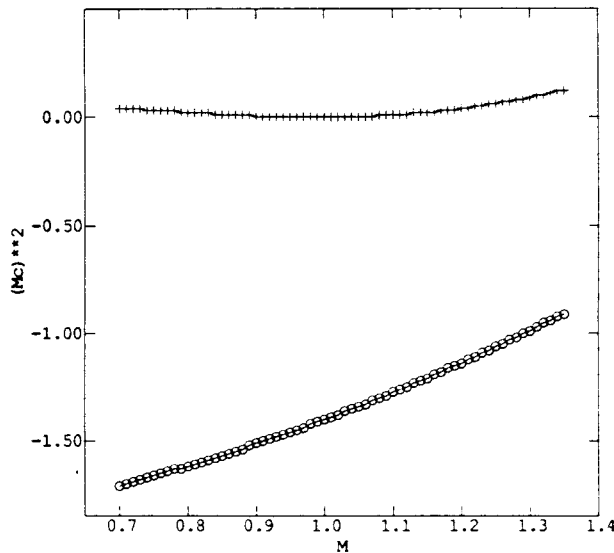
Fig. 11. MCC formulation: Variation of M_c^2 with M: ⊖⊖⊖ $\lambda/\mu = -0.666$ (producing Rankine–Hugoniot shock jumps); ┼┼┼┼ $\lambda/\mu = -2.118$ (producing isentropic shock jumps).
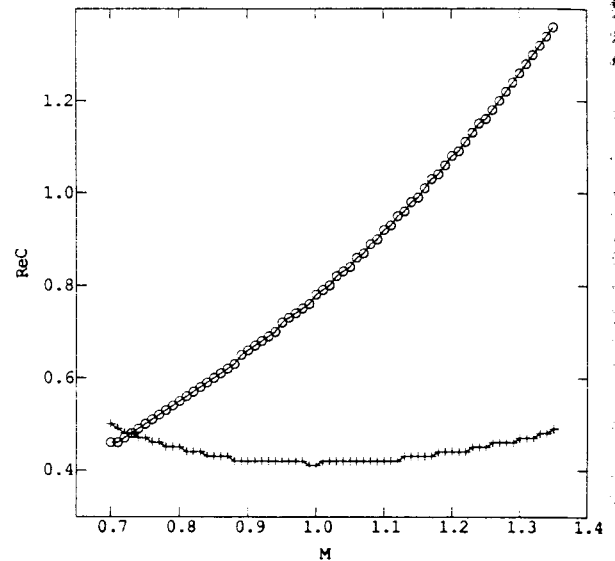
Fig. 12. MCC formulation: Variation of C (multiplied with Re) ⊖⊖⊖ $\lambda/\mu = 0.666$; producing Rankine–Hugoniot shock jumps; ┼┼┼$\lambda/\mu = -2.118$ producing isentropic shock jumps.

Notice that the new formulation gives vanishing value for C as the specified value of Re is increased, that is, C is not grid dependent. Substituting (21) and (22) in (11) and using the fourth order Runge–Kutta integration scheme produced the results shown in Figs. 11 and 12. Thus, the variable M_c and C formulation (MCC) is capable of producing Rankine–Hugoniot shock jumps if $\lambda/\mu = -\frac{2}{3}$ and isentropic shock jumps if $\lambda/\mu \cong -2$.

8. Summary

The artificial density or viscosity (ADV) formulation as used in the existing computer codes for the solution of the transonic full potential equation utilizes constant, user specified values for the cut-off Mach number M_c, and a constant C in the switching function $\tilde{\mu}$. Numerical results demonstrate that such an ADV formulation generates entropy jumps across the shocks. These jumps are not small; they are actually larger [13] than the entropy jumps generated by the physically dissipative potential (PDP) equation when using Stokes hypothesis, that is, Rankine–Hugoniot jumps. New analytic expressions for both M_c and C were derived that introduce the effects of physical dissipation and reduce the number of user specified parameters to only one Reynolds number. The existing full potential codes that use artificial density formulation can easily accommodate this physically consistent numerical dissipation by evaluating C and M_c analytically at every point in the flow field. Moreover, full potential codes could be used for computing flows with either isentropic or Rankine–Hugoniot shock jumps. Consequently, the strengths and thicknesses of the resulting shocks and the amount of entropy generated can be controlled with the physically known coefficients Pr″, μ'' and Re needed in the new (AMF and MCC) physically consistent artificially dissipative models.

Table 1
Comparison of coefficients multiplying corresponding derivatives of ϕ in four dissipation models for the numerical integration of the steady two-dimensional full potential equation

	ϕ_{ss}	ϕ_{nn}	ρ	ϕ_{sss}	ϕ_{snn}	ϕ_{ss^2}	ϕ_{nn^2}	$\phi_{ss}\phi_{nn}$	ϕ_{sn^2}
FPE-PDP	$\rho(1-M^2)$	ρ		$\frac{\mu''}{Re}\frac{\phi_s}{a^2}\left(1+\frac{\gamma-1}{Pr''}\right)$	$\frac{\mu''}{Re}\frac{\phi_s}{a^2}\left(1+\frac{\gamma-1}{Pr''}\right)$	$\frac{\mu''}{Re}\frac{\gamma-1}{a^2}\left(\frac{1}{Pr''}-1\right)$	$\frac{\mu''}{Re}\frac{\gamma-1}{a^2}\left(\frac{1}{Pr''}-1\right)$	$\frac{\mu''}{Re}\frac{\gamma-1}{a^2}2\left(\frac{2}{\mu''}-1\right)$	$\frac{\mu''}{Re}\frac{\gamma-1}{a^2}2\left(\frac{1}{Pr''}-\frac{2}{\mu''}\right)$
FPE-ADV	$\rho(1-M^2)$	ρ	$\rho C(M^2-M_c^2)$	0	0	$\rho C\left[(M^2-M_c^2)\cdot(2-(2-\gamma)M^2)+(\gamma+1)\frac{M_c^2}{a^2}\right]\frac{1}{\phi_s}$	0	$\rho C\left(\frac{M^2-M_c^2}{\phi_s}\right)$	0
FPE-AMF	$\rho(1-M^2)$	ρ		$\frac{\mu''}{Re}\frac{\phi_s}{a^2}\left(1+\frac{\gamma-1}{Pr''}\right)$	0	$\frac{\mu''}{Re}\frac{[1+(\gamma-1)/Pr'']}{a^2}\cdot\left[3+2(\gamma-1)M^2-\frac{\gamma+1}{a^2}\right]$	0	$\frac{\mu''}{Re}\frac{(1+1/Pr'')}{a^2}$	0
FPE-MCC	$\rho(1-M^2)$	ρ		$\frac{\mu''}{Re}\frac{\phi_s}{a^2}\left(1+\frac{\gamma-1}{Pr''}\right)$	0	$\frac{\mu''}{Re}\frac{\gamma-1}{a^2}\left(\frac{1}{Pr''}-1\right)$	0	$\frac{\mu''}{Re}\frac{1}{a^2}\left(1+\frac{\gamma-1}{Pr''}\right)$	0

Finally, Table 1 summarizes the analytic forms of all three artificial dissipation formulations (ADV, MCC and AMF) and compares them with the physically dissipative potential (PDP) flow formulation.

Acknowledgment

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