AIAA 91-0476

Aerodynamic Shape Design and Optimization

George S. Dulikravich
The Pennsylvania State University
University Park, PA 16802, USA

29th Aerospace Sciences Meeting

January 7-10, 1991/Reno, Nevada

AERODYNAMIC SHAPE DESIGN AND OPTIMIZATION

George S. Dulikravich
Department of Aerospace Engineering
The Pennsylvania State University
University Park, PA 16802, USA

ABSTRACT

Realistic aerodynamic shapes can be designed using methodologies from computational fluid dynamics and optimization. Two basic categories of the inverse (design) formulation are surface flow design and flow field design. Several methods, in both categories, including novel methods based on flow control theory, are being discussed and critically evaluated. Many issues remain unresolved. These issues include: specification of a more appropriate set of design constraints, acceleration of iterative algorithms, minimization of artificial dissipation, increased versatility of the design methods, and direct use of the existing and future flow field analysis software.

INTRODUCTION

In the general field of aerodynamics as with any field theory, we are faced with two problems: analysis and design. In the case of an analysis (direct problem) we are asked to predict the details of a flow field if the geometry of the object is specified. In the case of a design (inverse problem) we are asked to predict the geometry of the object, which must be compatible with the desired features of the flow field.

Thus, the field of aerodynamic shape design involves the ability to determine the geometry of an aerodynamic object that will satisfy the governing equations for the flow field and the associated boundary conditions. For example, it is possible to determine the coordinates of an airfoil if a surface pressure distribution is specified. The resulting designs can be subject to certain specified constraints. Examples which include such constraints may entail finding aerodynamic configurations that are compatible with entirely shock-free transonic flow fields or obtaining shapes of objects that produce flow fields with minimum entropy generation, minimum noise generation, uniform surface heat flux, etc.

One of the first meetings on the general topic of shape design was the International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES). The first ICIDES was organized and held[1] on October 17-18, 1984, the second at the University of Texas at Austin, while ICIDES-II was held[2] on October 26-28, 1988, at the Pennsylvania State University. These conferences were followed by an AGARD Specialists' Meeting on Computational Methods for Aerodynamic Design (Inverse) and Optimization held[3] in Loen, Norway, on May 22-23, 1989; and by the AGARD/FDP Lecture Series on Inverse Methods in Airfoil Design for Aeronautical and Turbomachinery Applications held[4] at VKI, Belgium on May 14-18, 1990. The next ICIDES is scheduled for October 23-25, 1991 in Washington, D. C.

Depending on the prescribed features of the flow field, the design (inverse) methodologies can be grouped into two general categories: surface flow design and flow field design. Surface flow design involves specifying certain flow parameters (pressure, Mach number, etc.) on the surface of the flying object, then finding the shape that will generate the surface conditions without regard for the rest of the flow field. Flow field design, on the other hand, enforces certain global flow field features (shock-free flow, minimal entropy generation, etc.) at every point of the flow field by determining the shape that will satisfy these constraints locally. A large number of methods for performing the surface flow design have been developed, while only a few methods for the flow field design are known to exist.

---

* Associate Professor. Senior Member AIAA

Mathematical models used in aerodynamic shape design are based on partial differential equations, integral equations, and algebraic equations. Detailed reviews were presented by Slooff[5] and Sobieczky.[6] For example, Zhukovski conformal mapping is actually a technique for designing a class of airfoil shapes with a specified pressure distribution at the surface that corresponds to a flow around a rotating cylinder. Here we are dealing with a simple algebraic expression, but that expression is based on a general solution of an integral equation formulation (a point-dipole and a point vortex) or a Laplace operator (a partial differential equation) governing the flow field. Thus, many global conformal mappings can be viewed as very special methods for designing certain simple shapes in a steady, planar, irrotational, inviscid flow field. Moreover, global conformal mapping is the only example that comes to mind as a method which combines the surface flow design concept and the flow field design concept. It guarantees that the resulting airfoil shapes have the specified surface distribution of the flow parameters while maintaining the irrotationality of the flow field.

In a more general situation, the arbitrary distribution of the surface flow parameters or an arbitrary field distribution of the flow parameters could result in shapes that are not physically meaningful and cannot be manufactured. For example, the lower surface and the upper surface of an airfoil could either cross over ("fish tail shapes") or never meet (open trailing edge shapes) although these solutions are mathematically acceptable (Fig. 1). Obviously, the problem rests in choosing an appropriate surface distribution of the flow parameters that satisfies certain global flow field constraints.[7] Yet, if the flow field design minimizes the entropy generation at every point in the field, the resulting shape would most likely have zero thickness and no stagnation points, that is, the optimal shape would most likely be a flat plate. Certain constraints on the geometry are needed since the final aerodynamic design is often incompatible with heat transfer, structural dynamics, acoustics, or manufacturing requirements.[5-6]

The main objection raised by designers when discussing the inverse (design) methodologies is that these methods create strictly point-designs rather than range-designs. In other words, an aerodynamic shape designed by using a surface flow design method will have the desired characteristics only at the design conditions.[8] If the operating conditions (angle of attack, free stream Mach number, etc.) vary from the design conditions, then the configuration will have to be changed (Fig. 2) in order to maintain the desired surface flow parameters. Moreover, when designing transonic shock-free shapes with any of the surface flow design methods, the resulting configuration could have a mildly concave surface (Fig. 3) partially covered by the supersonic flow. As a result, a "hanging shock" or a "loose-foot" shock will form even at the design conditions.[7] The aerodynamic efficiency of such a configuration will not be satisfactory even at the design operating point. At off-design values for the Mach number or the angle of attack, the hanging shock violently re-attaches itself to the airfoil surface causing a rapid increase in drag due to the boundary layer separation. Consequently, it is more appropriate to design an almost shock-free shape even at the design conditions. Such shapes could have a weak family of shocks[9] that do not increase appreciably in strength at off-design conditions.

SURFACE DATA SPECIFICATION

We must now face the question: what is the appropriate surface pressure distribution? Prevention of uncontrolled flow separation over a wider range of angles of attack, Mach numbers, and Reynolds numbers is the most important goal of an aerodynamic design. The answer to our question is not known. It might not be an appropriate question in light of the fact that the surface pressure distribution alone is not indicative of potentially hazardous flow field features such as an unexpected "hanging shock." Nevertheless, a number of researchers[10-14] have entertained this problem using a classical approach based on boundary layer information. A somewhat speculative approach using a concept of minimal kinetic energy rate has recently been reported.[15] A fast method capable of detecting laminar and turbulent flow separation from the prescribed surface pressure distribution would certainly be very useful. These relatively simple methods can help eliminate those candidate surface pressure distributions that would separate the flow. In addition, these methods leave the designer with the psychologically important feeling that he is still in command, although he realizes that all of his experience is inadequate when compared to a true mathematically constrained optimization.

Among the large number of publications using various surface flow designs, applications to single airfoils[16-24], multi-component airfoils[25], cascades of airfoils[26-32], ducts[34], rotors[35-45], isolated wings[46-47], wing-body combinations[48-50]), nozzles[51-52] and inlets[53], and axisymmetric bodies can be found.[54] Some of these methods have received wider acceptance than others. The general conclusion is that methods which are more versatile, and easier to comprehend and implement are more widely used. There are instances in which three-dimensional methods have been implemented even on personal computers.[50]

MODIFIED GARABEDIAN-McFADDEN METHOD

Methods like Garabedian-McFadden[19] and its modified[20] version are becoming quite popular since they require an extremely simple master code which can call any available flow field analysis code as a subroutine. Thus, as more sophisticated analysis codes become available, they can be directly substituted in the master code that computes corrections to the input geometry. The main drawback of the method is that it converges relatively slowly (Fig. 4). The iterative motion of the surface which is undergoing design can become irregular very quickly if some sort of control over the motion of surface points is not enforced. The surface motion model which treats the surface as an elastic membrane that moves according to a simple linear time dependent damped model[19-20] is quite effective in enforcing a relatively smooth convergence of the surface geometry. A more thorough study on the stability of the surface motion model is necessary, since the choice of coefficients in the model[19-20] can seriously affect the convergence rate and the stability of the entire iterative process.

STREAM FUNCTION BASED METHODS

A very interesting concept, termed Stream-Function-as-a-Coordinate (SFC), is based on a transformed stream function formulation where the vertical coordinate of each stream line is treated as an unknown. Thus, the SFC formulation[32-33] solves directly for the unknown geometric coordinates which are the coordinates of the stream lines (Fig. 5). A three dimensional version of the SFC formulation remains to be developed. A similar concept derived from the boundary element integral method[18] gives a fully converged solution for an airfoil design on a personal computer in 10-20 iterations.

Another method that is based on the interplay of the stream function and the potential function in irrotational subsonic inviscid flows is due to Stanitz.[34] He has obtained fascinating configurations of channels and three-dimensional ducts subject to a specified surface pressure along the duct walls (Fig. 6).

TAYLOR SERIES EXPANSION METHOD

An extremely efficient and simple, although approximate, method has been developed in China[37-38] and reportedly can be used on a pocket programmable calculator. The method is based on prescribing, say, a Mach number distribution along the mid-passage streamline and then deducing values of the Mach number on the top and the bottom of the passage by expanding the prescribed data in the vertical direction using Taylor series. With more terms in the Taylor series, a larger gap-to-chord cascade can be designed. Since the analyticity is carried to an extreme, little work needs to be performed iteratively. As a consequence, the method converges quickly. Errors in this method rapidly increase towards the stagnation points particularly if they are blunt. The method is applicable to radial turbomachinery[39] as well (Fig. 7).

NEW THREE-DIMENSIONAL FORMULATIONS

Highly sophisticated and computationally complex computer codes have been developed and successfully applied in the design of three-dimensional coaxial nozzles[51] and turbomachinery blading[40]. The model includes a complete set of the three-dimensional Euler equations of gas dynamics. Although complex, the method converges quickly since the geometry corrections are calculated using information that propagates along the characteristics.

Several new formulations[41-45] for inviscid quasi three-dimensional and fully three-dimensional turbomachinery using the flow field design approach are analytically novel and interesting. The main drawback of these approaches is that the basic model does not take into account either viscosity or turbulence.

Takanashi[46] successfully applied the general concept of using a small master code to call any available analysis code as a subroutine in the process of surface flow design. The method converges extremely quickly because a small perturbation integral formulation is used to evaluate local geometry corrections of the wing surface.

Recently, surface flow inverse designs of wings[47] and a wing-body combination[48-50] (Fig. 8) were successfully performed using full potential transonic equation solvers[47-48] or higher order surface panel methods[49-50] together with the surface transpiration concept.

Inverse designs of supersonic nozzles[51-52], supercritical jet engine inlets[53], and axisymmetric bodies[54] in incompressible potential flow have also been performed. The approach of Ives[53] is especially innovative and unique.

TRANSONIC SHOCK-FREE DESIGN

Probably the best known method for the flow field design is a hodograph based method[55-58] for designing transonic shock-free shapes. Actually, the method is a combination of both surface flow design (the surface Mach number can be specified on a point-by-point basis) and the flow field design formulations (it can be guaranteed that no shocks occur in the flow field). Consequently, the method suffers from the problems previously mentioned (open trailing edges and fish-tail shapes) that are associated with both general approaches to design. The method was well publicized in the seventies and the resulting software[57] found its way into industry. Nevertheless, methods based on the hodograph transformation are not applicable to three dimensions. Since the hodograph method is based on an elliptic continuation approach[57] it requires two real and two imaginary characteristics. Needless to say, it is a highly complicated method and the resulting software is not easy to modify. The entire method is well described in a textbook by Schrier.[58]

An alternative method is known in the West as Sobieczky's fictitious gas[59-60] or in Japan as Nakamura's gas[61], since both researchers have developed and published the method independently. The concept is based on the fact that shocks can form only in supersonic flow, that is, if the governing partial differential equation is locally hyperbolic. Consequently, if the conditions for possible shock formations are to be eliminated, the governing partial differential equations should never be allowed to become hyperbolic. Sobieczky and Nakamura ensured this by switching from an isentropic expression for density and local speed of sound to an appropriate analytical fictitious density relation at every point in the field and on the boundary where the flow is likely to become supersonic. The resulting computations are acceptable in the subsonic regions (where the isentropic relations are used), but are not acceptable in the supersonic regions (where the fictitious gas relations are used). Nevertheless, the resulting sonic line which separates the two regions, is compatible with both the isentropic and the fictitious gas relations (Fig. 9). We can now use the isentropic relations in the fictitious gas domain, where the governing equations will be locally strictly hyperbolic. Hence, the sonic line values of the stream function can be used as initial data for integration of the locally hyperbolic system. Moreover, the system becomes linear if transformed to a rheograph plane[59-60] characterized by the Prandtl-Meyer function and the local velocity vector angle. The new shape coordinates will be determined from the condition that the stream function should maintain a constant value at every point of the airfoil surface. This method is fairly simple to comprehend and implement in the existing full potential codes. Nevertheless, the fictitious gas method does not allow us the freedom to specify surface values of flow parameters. It only guarantees that if our choice for the fictitious gas density - Mach number relation is not too restrictive, the supersonic bubble will become shallow and stretched along the surface (Fig. 9), which results in an entirely shock-free flow field. The method is suitable for redesinning existing airfoils[59-63], cascades of airfoils[64-66] (Fig. 10), quasi three-dimensional rotors[67], wings[68-70] without having to worry about surface cross-over, fish-tail shapes and hanging shocks.

OPTIMIZATION

Due to the fact that aerodynamic shape design represents only a part of the overall design of a flying vehicle, the need for interdisciplinary[71] optimization arises. Simultaneously, optimization algorithms are finding applicability in pure aerodynamic design[72-88]. The optimization algorithms are presently often used to minimize the difference between the specified and the computed surface flow data. This is obviously not a very imaginative use of computational resources, since optimization codes are known to require a large number of flow field analysis solutions. Such use of an optimizer has nothing to do with optimizing the aerodynamic shape. Noteworthy exceptions involve maximizing lift-to-drag ratio for an isolated airfoil[74] and a multicomponent airfoil[77], minimizing the total pressure loss across the shock waves in a supersonic inlet[80], minimizing the total pressure loss in a viscous flow inside an S-shaped duct[81], and optimizing over a range of operating conditions.[82] Recent publications[83-84] expose interesting and potentially promising sensitivity analysis formulations for the fast evaluation and optimization of off-design conditions. The approach of Rizk[85] is especially welcome since it allows for a stable iterative algorithm where an optimizer is used on each updated configuration even before the flow field has converged to the new geometry. As a consequence, a typical airfoil design involves an equivalent of 5-10 fully converged flow field solutions. A very readable and thorough comparative analysis of optimization-based approaches was performed by Frank and Shubin.[86] They also offer an alternative more economical approach. Besides a wide variety of gradient-based[72] optimization algorithms, it should be pointed out that truly remarkable results were obtained using an evolution type algorithm[77] which seems to be less sensitive to local minimums.

CONTROL THEORY

Control theory applied to systems of partial differential equations was first discussed in the West by Lions[87]. Recently, several researchers[88-94] have looked into using control theory concepts in aerodynamic shape design. In this context, the control theory can be thought of as a minimization process performed in a continuous function space, which is certainly the case when optimizing the large number of variables. The approach is novel and mathematically challenging since most of the fluid flow theory is based on partial differential equations, while the control theory is usually formulated via ordinary differential equations. The treatise by Abergel and Temam[93] appears to be the most complete, while the new publication by Cabuk and Modi[94] offers the most readable text on this subject and provides convincing results (Fig. 11) for design of nozzles with minimum sheer stress at the walls. Their preliminary results dispel earlier reservations that these formulations might not be computationally efficient since they involve solution of an additional set of adjoint equations and several more interface equations.

CONCLUSIONS

We have surveyed a vast number of different aerodynamic shape design concepts and attempted to classify them. Characteristics, both positive and negative, of the more prominent methods were outlined. Future research is expected to concentrate on the use of Navier-Stokes equations and applications to three-dimensional configurations. Interdisciplinary constrained optimization will need to play a more prominent role. Control theory and its variations are the most promising concepts for interdisciplinary aerodynamic shape design which involves a large number of variables.

ACKNOWLEDGEMENTS

I would like to express my deep appreciation to Professor Helmut Sobieczky whose pioneering research on transonic shock-free flow field design sparked my interest in the general field of aerodynamic shape design.

REFERENCES

1. Proceedings of the International Conference on Inverse Design Concepts in Engineering Sciences (ICIDES-I), ed. G.S. Dulikravich, Aero. Eng. and Eng. Mech. Dept., Univ. of Texas, Austin, TX, Oct. 17-18, 1984.

2. Proceedings of the International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-II), ed. G. S. Dulikravich, Aero. Eng. Dept., Penn State Univ., University Park, PA, Oct. 26-28, 1987.

3. Proceedings of the AGARD Specialist's Meeting on Computational Methods for Aerodynamic Design (Inverse) and Optimization, AGARD CP-463, ed. J. W. Slooff, Loen, Norway, May 24-25, 1989.

4. Proceedings of a Special Course on Inverse Methods for Airfoil Design for Aeronautical and Turbomachinery Applications, ed. R. V. Braembusche, AGARD Report No. 780, Rhode-St.-Genese, Belgium, May 14-18, 1990.

5. Slooff, J. W., "Computational Methods for Subsonic and Transonic Aerodynamic Design," Proceedings of ICIDES-I, ed. G. S. Dulikravich, Univ. of Texas, Austin, TX, Oct. 17-18, 1984, pp. 1-68.

6. Sobieczky, H.,"Research on Inverse Design and Optimization in Germany", Appl. Mech. Rev., vol. 41, no. 6, June 1988, pp. 217-238.

7. Volpe, G., "Inverse Design of Airfoil Contours: Constraints, Numerical Methods and Applications," AGARD CP-463, Loen, Norway, May 22-23, 1989, Ch. 4.

8. Chin. W. C.,"Class of Shock-Free Airfoils Producing the Same Surface Pressure", AIAA J.

of Aircraft, Vol. 17, No. 4, April 1980, pp. 286-288.

9. Zhu, Z. and Sobieczky, H.,"An Engineering Approach for Nearly Shock-Free Wing Design", Proceedings of the Internat. Conf. on Fluid Mech., Beijing, China, July 1987.

10. Ives, D. C.," Inverse and Hybrid Compressor Cascade Design Methods", Proceedings of ICIDES-I, ed. G.S. Dulikravich, Univ. of Texas, Austin, TX, Oct. 17-18, 1984, pp. 555-572.

11. Papailiou, K., "An Investigation on Le Foll's Method used for Blade Optimization Based on Boundary Layer Concepts," Proceedings of the XXth Int. Astro. Congress, Aug. 1969.

12. Citavy, J., "Two-Dimensional Compressor Cascades With Optimum Velocity Distribution Over the Blade," J. of Eng. for Power, Jan. 1975, pp. 101-110.

13. Lekoudis, S.G., Sankar, N.L., and Radwan, S.F., "A Method for Designing Three-Dimensional Configurations With Prescribed Skin Friction," J. of Aircraft, Vol.21, No. 11, Nov. 1984, pp. 924-926.

14. Lekoudis, S.G., Sankar, N.L., and Malone, J.B.,"The Application of Inverse Boundary Layer Methods to the Three-Dimensional Viscous Design Problem", Comm. in Appl. Num. Meth., Vol.2, No.1, Jan.-Feb. 1986, pp.57-61.

15. Dulikravich, G. S., "A Criteria for Surface Pressure Specification in Aerodynamic Shape Design," AIAA paper 90-0124, Reno, NV, January 8-11, 1990.

16. Taverna, F., "Advanced Airfoil Design for General Aviation Propellers," AAIA paper 83-1791, Danvers, MA, July 13-15, 1983.

17. Tranen, T., "A Rapid Computer Aided Transonic Airfoil Design Method," McDonnell Douglas Corporation Report No. MDC A2584, Dec. 31, 1973.

18. Soinne, E., and Laine, S., "An Inverse Boundary Element Method for Single Component Airfoil Design," J. of Aircraft, Vol. 22, No. 6, June 1985, pp. 541-543.

19. Garabedian, P., and McFadden, G., "Design of Supercritical Swept Wings," AIAA J., Vol. 20, No. 3, March 1982, pp. 289-291.

20. Malone, J. B., Narramore, J. C., and Sankar, L. N., "An Efficient Airfoil Design Method Using the Navier-Stokes Equations," AGARD-CP-463, Loen, Norway, May 22-23, 1989, Ch. 5.

21. Fray, J. M. J., Slooff, J. W., Boerstoel, J. W., and Kassies, A., "Design of Transonic Airfoils with Given Pressure, Subject to Geometric Constraints," NLR, Amsterdam, The Netherlands, July 13, 1984.

22. Giles, M. B., and Drela, M., "Two-Dimensional Transonic Aerodynamic Design Method," AIAA J., Vol. 25, No. 9, Sept. 1987, pp. 1199-1206.

23. Hassan, A. A., "A Viscous-Inviscid Coupling Method for the Design of Low Reynolds Number Airfoil Sections," Proc. Conf. on Low Reynolds Number Airfoil Aerodyn., June 1985, pp. 53-63.

24. Hirose, N., Takanashi, S., and Kawai, N., "Transonic Airfoil Design Procedure Utilizing a Navier-Stokes Analysis Code," AIAA J., Vol. 25, No. 3, March 1987, pp. 353-359.

25. Kennedy, J. L., and Marsden, D. J., "A Potential Flow Design Method for Multicomponent Airfoil Sections," J. Aircraft, Vol. 15, No. 1, Jan. 1978, pp. 47-52.

26. Jacquotte, O.-P., "A Finite Element Inverse Method for the Design of Turbomachinery Blades," Proc. 7th Int. Conf. Finite Element Meth. in Flow Prob., ed. T. J. Chung, Huntsville, Alabama, April 3-7, 1988, pp. 908-914.

27. Gelder, T. F., Moore, R. D., Sanz, J. M., and McFarland, E. R., "Wind Tunnel Turning Vanes of Modern Design," AIAA paper 86-0044, Reno, NV, Jan. 6-8, 1986.

28. Tong, S. S., and Thompkins, Jr., W. T., "A Design Calculation Procedure for Shock-free or Strong Passage Shock Turbomachinery Cascades," ASME paper 82-GT, London, England, April 18-22, 1982.

29. Miton, H. H., "Properties of a Space-wise Periodic Flow Application to Flow Computation and Design for Blade Cascades," Comm. Appl. Num. Meth., Vol. 2, No. 1, Jan.-Feb. 1986, pp. 101-112.

30. Luu, T. S. and Viney, B., "The Turbomachine Blading Design Achieved by Solving the Inverse Field Problem," ASME paper 87-GT-215, Anaheim, CA, May 31-June 4, 1987.

31. Meauze, G., "On the Use of Inverse Modes of Calculation in Two-dimensional Cascades and Ducts," Comm. in Appl. Num. Meth., Vol.2, no. 1, Jan.-Feb. 1986, pp. 73-81.

32. Chen, N.-X. and Zhang, F.-X.,"A Generalized Numerical Method for Solving Direct, Inverse and Hybrid Problems of Blade Cascade Flow by Using Streamline-Coordinate Equation", ASME paper 87-GT-29, Anaheim, CA, May 31 - June 4, 1987.

33. Huang, C.-Y. and Dulikravich, G.S.,"Stream Function and Stream-Function-Coordinate (SFC) Formulation for Inviscid Flow Field Calculations", Comp. Meth. in Appl. Mech. and Eng., Vol. 59, 1986, pp. 155-177.

34. Stanitz, J.D.,"A Review of Certain Inverse Methods for the Design of Ducts With 2- or 3-Dimensional Potential Flow", Appl. Mech. Rev., Vol. 41, no. 6, June 1988, pp. 217-238.

35. Cedar, R. D., and Stow, P., "The Role of a Mixed Design and Analysis Method in a Turbomachinery Blade Design System," Comm. in Appl. Num. Meth., Vol. 2, No. 1, Jan.-Feb. 1986, pp. 91-99.

36. Ribaut, M., and Martin, D., "A Quasi Three-dimensional Inverse Design Method Using Source and Vortex Integral Equations," Comm. in Appl. Num. Meth., Vol. 2, No. 1, Jan.-Feb. 1986, pp. 63-72.

37. Wang, Z.-M., "A Method of Aerodynamic Design of Blades in Quasi-Three-Dimensional Calculation of Turbomachine," ASME paper 86-GT-192, Dusseldorf, Germany, June 8-12, 1986.

38. Zhao, X.-L., and Qin, L.-S., "An Approximate 3-D Aerodynamic Design Method for Centrifugal Impeller Blades," ASME paper 89-GT-73, Toronto, Canada, June 4-8, 1989.

39. Novak, R. A. and Haymann-Haber, G., "A Mixed-Flow Cascade Passage Design Procedure Based on a Power Series Expansion," ASME paper 82-GT-121, London, England, April 18-22, 1982.

40. Zannetti, L., di Torino, P., and Ayele, T. T., "Time Dependent Computation of the Euler Equations for Designing Fully 3D Turbomachinery Blade Rows, Including the Case of Transonic Shock Free Design," AIAA paper 87-0007, Reno, NV, January 12-15, 1987.

41. Miyazaki, T., and Hirayama, N., "A Theoretical Solution of Three-Dimensional Flows in Subsonic, Transonic and Supersonic Turbomachines: An Exact Solution and Its Numerical Method," ASME paper 86-GT-111, Dusseldorf, Germany, June 8-12, 1986.

42. Borges, J. E., "A Three Dimensional Inverse Method in Turbomachinery; Part 1--Theory," ASME paper 89-GT-136, Toronto, Canada, June 4-8, 1989.

43. Borges, J. E., "A Three Dimensional Inverse Method in Turbomachinery; Part 2--Experimental Verification," ASME paper 89-GT-137, Toronto, Canada, June 4-8, 1989.

44. Hawthorne, W.R. and Tan, C.S.,"Design of Turbomachinery Blading in Three-Dimensional Flow by the Circulation Method; A Progress Report", Proceedings of ICIDES-II, ed. G.S. Dulikravich, Penn State Univ., University Park, PA, Oct. 26-28, 1987, pp. 207-226.

45. Ghaly, W. S., "A Design Method for Turbomachinery Blading in Three-Dimensional Flow," Int. Journal for Num. Methods in Fluids, Vol. 10, 1990, pp. 179-197.

46. Takanashi, S., "Iterative Three-Dimensional Transonic Wing Design Using Integral Equations," J. of Aircraft, Vol. 22, No. 8, August 1985, pp. 655-660.

47. Gally, T. A., and Carlson, L. A., "Transonic Wing Design Using Inverse Methods in Curvilinear Coordinates," J. of Aircraft, Vol. 25, No. 11, November 1988, pp. 1009-1017.

48. Hazarika, N., "An Efficient Inverse Method for the Design of Blended Wing-Body Configurations," Ph.D. thesis, Aero. Eng. Dept., Georgia Institute of Technology, June 1988.

49. Fornasier, L.,"An Iterative Procedure for the Design of Pressure-Specified Three-Dimensional Configurations at Subsonic and Supersonic Speeds by Means of a Higher-Order Panel Method," AGARD-CP-463, Loen, Norway, May 22-23, 1989, Ch. 6.

50. Kubrynski, K., "A Subsonic Panel Method for Design of 3-Dimensional Complex Configurations With Specified Pressure Distribution," GAMM-III Proceedings, Kiel, Germany, 1987, pp. 137-146.

51. Couaillier, V. and Veuillot, J.P.,"Inverse Method for the Euler Equations", Proceedings of ICIDES-II, ed. G.S. Dulikravich, Penn State Univ., University Park, PA, Oct. 26-28, 1987, pp. 131-146.

52. Meauze, G., and Fourmaux, A., "A Coupled Inverse-Inverse Method for Over-Expanded Supersonic Nozzles," paper 109 presented at the 7th ISABE International Symposium on Air Breathing Engines, Beijing, People's Republic of China, Sept. 2-6, 1985.

53. Ives, D. C., "Supercritical Inlet Design," AIAA paper 83-1866, Danvers, MA, July 13-15, 1983.

54. Zedan, M. F., and Dalton, C., "The Inverse Method Applied to a Body of Revolution with an Extended Favourable Pressure Gradient Forebody," Comm. in Appl. Num. Meth., Vol. 2, No. 1, Jan.-Feb. 1986, pp. 113-119.

55. Boerstoel, J.W. and Huizing, G.H.,"Transonic Airfoil Design by an Analytic Hodograph Method", AIAA paper 74-539, Paolo Alto, CA, 1974.

56. Hobson, D.E.,"Shock-Free Transonic Flow in Turbomachinery Cascades", Univ. of Cambridge, UK, CUED/A Turbo/TR 65, 1974.

57. Bauer,F., Garabedian, P., Korn, D. and Jameson, A.,"Supercritical Wing Sections 1,2,3", Lecture Notes in Econ., Math. Syst., No. 66, 108, 150, Springer Verlag, Berlin, 1972, 1975, 1977.

58. Schrier, S.,"Compressible Flow", John Wiley & Sons, New York, 1982.

59. Sobieczky, H., Fung, K.-Y., Seebass, R.A., Yu, N.J.,"New Method for Designing Shock-Free Transonic Configurations", AIAA J., Vol.17, No. 7, 1979.

60. Sobieczky, H.," Related Analytical, Analog and Numerical Methods in Transonic Airfoil Design", AIAA paper 79-1556, Williamsburg, VA, July 23-25, 1979.

61. Nakamura, M., "A Method for Obtaining Shockless Transonic Flows past Two-Dimensional Airfoils whose Profiles are Partially Modified from a Given Arbitrary Profile," Trans. Japan Soc. Aero. Space Sci., Vol. 23, No. 62, Feb. 1981, pp. 195-213.

62. Hassan, A., Sobieczky, H., and Seebass, A. R., "Subsonic Airfoils with a Given Pressure Distribution," AIAA J., Vol. 22, No. 9, Sept. 1984, pp. 1185-1191.

63. Dulikravich, D. S., and Sobieczky, H., "Shockless Design and Analysis of Transonic Cascade Shapes," AIAA J., Vol. 20, No. 11, Nov. 1982, pp. 1572-1578.

64. Dulikravich, G.S. and Sobieczky, H.,"CAS22-FORTRAN Program for Fast Design and Analysis of Shock-Free Airfoil Cascades", NASA CR 165596, 1982.

65. Sobieczky, H. and Dulikravich, G.S.,"A Computational Design Method for Turbomachinery Cascades", ASME paper 82-GT-117, London, UK, April 18-22, 1982.

66. Dulikravich, G.S. and Sobieczky, H.,"Design of Shock-free Compressor Cascades Including Viscous Boundary Layer Effects",ASME paper 83-GT-134, Phoenix, AZ, March 27-31, 1983.

67. Prince, T. C., and Beauchamp, P. P., "Shock-free Transonic Rotor Design," Comm. Appl. Num. Meth., Vol. 2, No. 1, Jan.-Feb. 1986, pp. 21-27.

68. Fung, K-Y., Sobieczky, H., and Seebass, R., "Shock-Free Wing Design," AIAA J., Vol. 18, No. 10, Oct. 1980, pp. 1153-1158.

69. Yu, N. J., "Efficient Transonic Shock-Free Wing Redesign Procedure Using a Fictitious Gas Method," AIAA J., Vol. 18, No. 2, Feb. 1980, pp. 143-148.

70. Raj, P., and Miranda, L. R., "A Cost-Effective Method for Shock-Free Supercritical Wing Design," J. of Aircraft, Vol. 19, No. 4, April 1982, pp. 283-289.

71. Voigt, R. G., "Requirements for Multidisciplinary Design of Aerospace Vehicles on High Performance Computers," ICASE Report No. 89-70, Sept. 1989.

72. Silver, B., and Ashley, H., "Optimization Techniques in Aircraft Configuration Design," Dep. Aero. & Astro., Stanford U., CA, SUDAAR No. 406, June 1970.

73. Lee, K. D. and Eyi, S., "Aerodynamic Design Via Optimization," Proc. of ICAS '90, Stockholm, Sweden, Sept. 9-14, 1990.

74. Reneaux, J., and Thibert, J-J., "The Use of Numerical Optimization for Airfoil Design," AIAA paper 85-5026, Colorado Springs, CO, Oct. 14-16, 1985.

75. Sanger, N. L., "The Use of Optimization Techniques to Design Controlled Diffusion Compressor Blading," NASA TM 82763, April 1982.

76. Ormsbee, A. I., and Chen, A. W., "Multiple Element Airfoils Optimized for Maximum Lift Coefficient," AIAA J., Vol. 10, No. 12, Dec. 1972, pp. 1620-1624.

77. Misegades, K. P., "Optimization of Multi-Element Airfoils," Von Karman Institute for Fluid Dynamics, Belgium, Project Report 1980-5, June 1980.

78. Cosentino, G. B., "Numerical Optimization Design of Advanced Transonic Wing Configurations," AIAA paper 85-0424, Reno, NV, Jan. 14-17, 1985.

79. Lock, R. C., "Comment on 'Numerical Optimization Design of Advanced Transonic Wing Configurations'," J. of Aircraft, Vol. 24, No. 8, Aug. 1987, pp. 575-576.

80. Huddleston, D. H., and Mastin, C. W., "Optimization of Aerodynamic Designs Using Computational Fluid Dynamics," Proceedings of the 7th Internat. Conference on Finite Element Methods in Flow Problems, ed. Chung, T.J. and Karr, G.R., Univ. of Alabama in Huntsville Press, April 3-7, 1989, pp. 899-907.

81. Madabhushi, R.K., Levy, R. and Pinkus, S.M.,"Design of Optimum Ducts Using an Efficient 3-D Viscous Computational Flow Analysis", Proceedings of ICIDES-II, ed. G.S. Dulikravich, Penn State Univ., University Park, PA, Oct. 26-28, 1987, pp.147-166.

82. Ghielmi, L., Marazzi, R. and Baron, A.,"A Tool for Automatic Design of Airfoils in Different Operating Conditions", AGARD-CP-463, Loen, Norway, May 22-23, 1989, Ch. 18.

83. Sharp, H. T., and Sirovich, L., "Constructing a Continuous Parameter Range of Computational Flows," AIAA J., Vol. 27, No. 10, 1989, pp. 1326-1331.

84. Rodman, L., "A Characterization and Search Technique for Unsteady Flow Control Problems," AIAA paper 90-3102-CP, Portland, OR, Aug. 20-22, 1990.

85. Rizk, M. H., "Aerodynamic Optimization by Simultaneously Updating Flow Variables and Design Parameters," AGARD CP-463, Loen, Norway, May 22-23, 1989, Ch. 15.

86. Frank, P. D. and Shubin, G. R., "A Comparison of Optimization-Based Approaches for a Model Computational Aerodynamics Design Problem," Boeing Computer Serv., ECA-TR-136, April 1990.

87. Lions, J. L., "Controle Optimal des Systemes Gouvernes par des Equations aux Derivees Partielles," Dunod, Paris, 1969.

88. Angrand, F., "Optimum Design for Potential Flows," Int. J. Num. Meth. Fluids, Vol. 3, 1983, pp. 265-282.

89. Gu, Chuan-gang, and Miao, Yong-miao, "Blade Design of Axial-Flow Compressors by the Method of Optimal Control Theory--Application of Pontryagin's Maximum Principles, a Sample Calculation and Its Results," ASME paper 86-GT-182, Dusseldorf, Germany, June 8-12, 1986.

90. Gu, Chuan-gang, and Miao, Yong-miao, "Blade Design of Axial-Flow Compressors by the Method of Optimal Control Theory--Physical Model and Mathematical Expression," ASME paper 86-GT-183, Dusseldorf, Germany, June 8-12, 1986.

91. Jameson, A., "Aerodynamic Design Via Control Theory," ICASE Report No. 88-64, Nov. 1988.

92. Cabuk, H. and Modi, V., "Shape Optimization Analysis: First and Second Order Necessary Conditions," Opt. Control Appl. & Meth., Vol. II, 1990, pp. 173-190.

93. Abergel, F. and Temam, R., "On Some Control Problems in Fluid Mechanics," Theor. and Comput. Fluid Dyn., 1, 1990, pp. 303-325.

94. Cabuk, H. and Modi, V., "Optimum Plane Diffusers in Laminar Flow," submitted for publication.

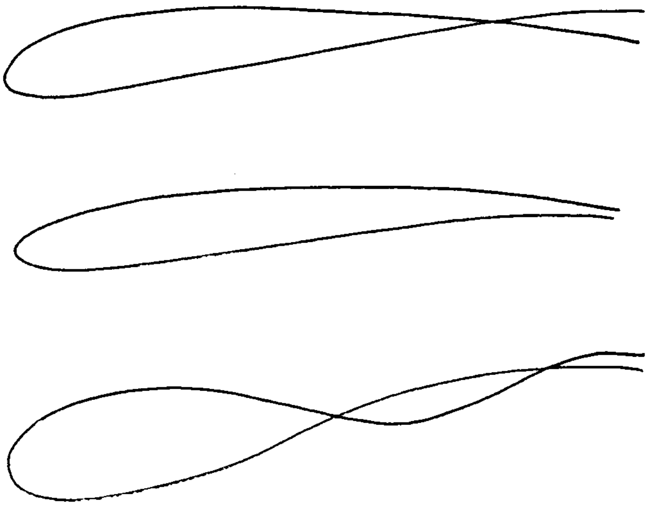
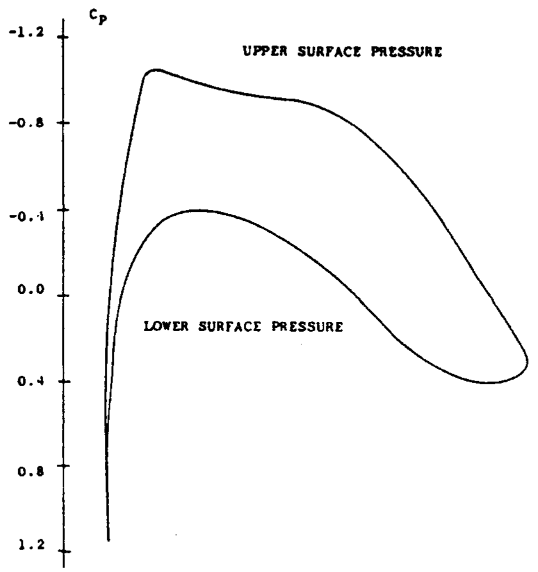


Fig. 1 A sketch of possible airfoil shapes resulting from an inverse design without constraints



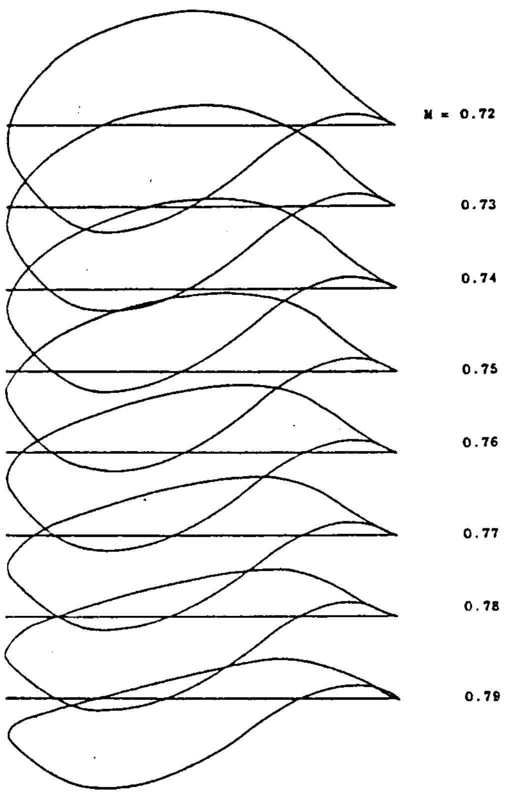


Fig. 2 Airfoil shapes[8] (y-axis magnified five times) having identical surface pressure distributions at different Mach numbers

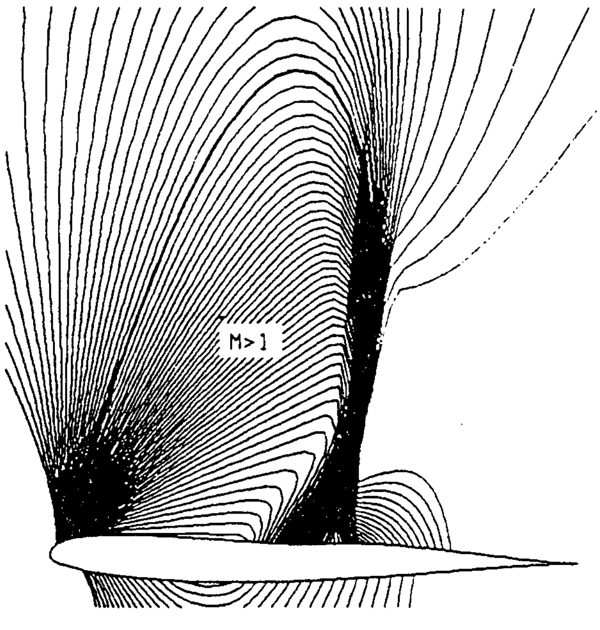


Fig. 3 An example of a "hanging" shock[7] in the flow field when using shock-free surface flow design

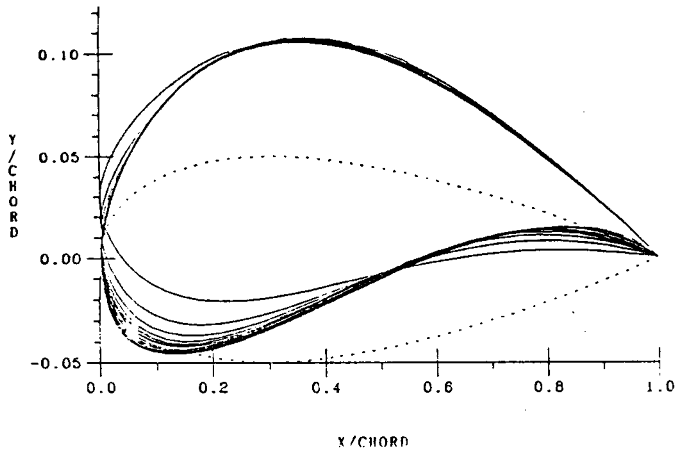


Fig. 4 An example of the convergence history[20] using modified Garabedian-McFadden method and surface panel analysis code. Initial shape was a NACA0010 airfoil. Target pressure distribution corresponds to a 15% thick cambered Zhukovski airfoil

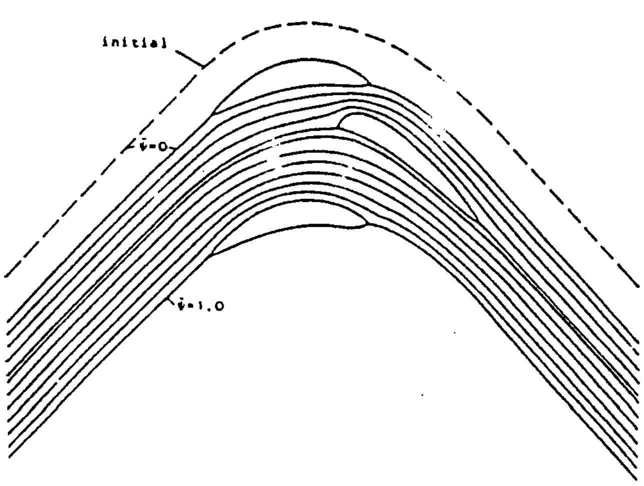


Fig. 5 SFC method[32-33] generates streamline shapes as its solution. Example of a cascade designed simultaneously with a splitter blade

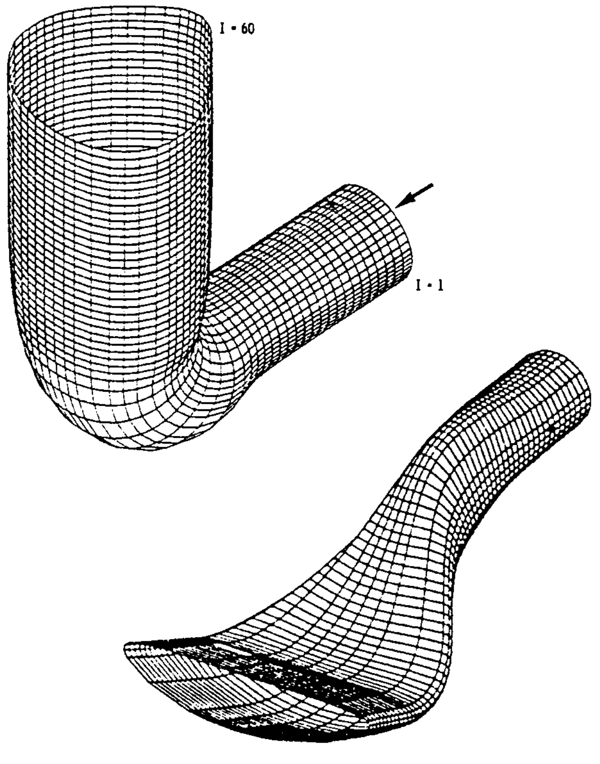


Fig. 6 Stream function - potential function inverse design method (Stanitz[34]) can generate complex realistic three-dimensional configurations

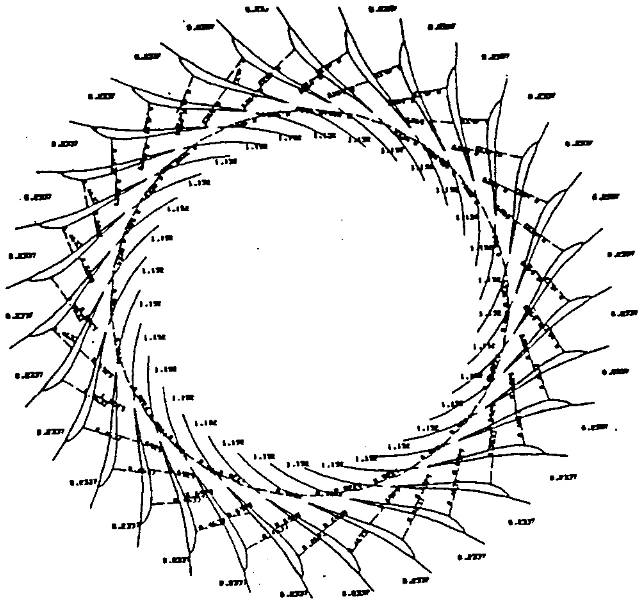


Fig. 7 Taylor series expansion method[37-38] can be used in turbomachinery cascade design[39]

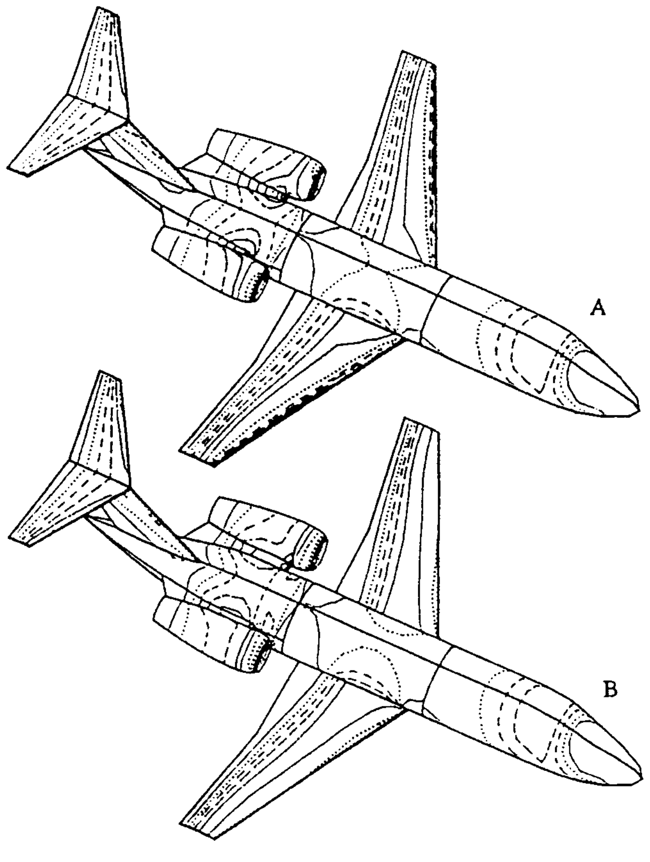


Fig. 8 Entire business jet configuration can be optimized[50] on a personal computer using surface transpiration concept and panel method: a) before, and b) after three optimization cycles

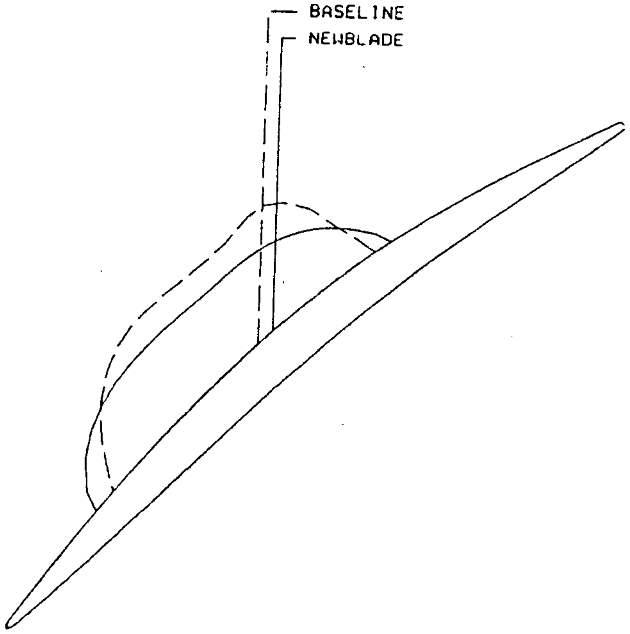


Fig. 9 Sonic line shapes before and after the shock-free[67] design of a compressor cascade using fictitious gas formulation

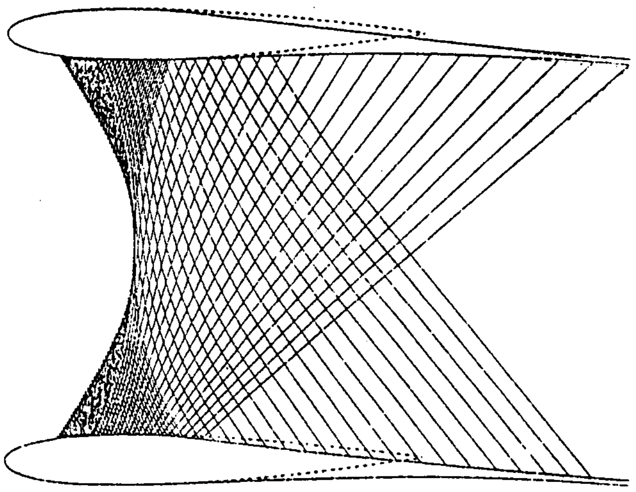


Fig. 10 Shock-free fully choked transonic cascade designed[66] using fictitious gas concept

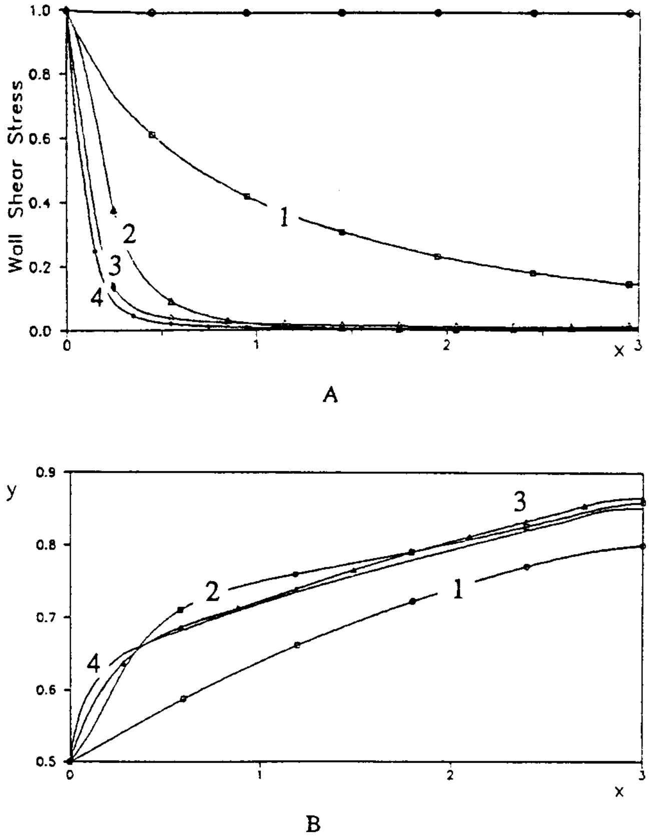


Fig. 11 Two-dimensional nozzle design using Navier-Stokes equations[94] and control theory: a) iteration history for the surface shear stress and, b) iteration history for the nozzle geometry