Proceedings of The First National Congress of Fluid Dynamics
Cincinnati, Ohio, July 1988. AIAA Paper 88-3654

PHYSICALLY CONSISTENT MODELS FOR ARTIFICIAL DISSIPATION IN TRANSONIC POTENTIAL FLOW COMPUTATIONS

George S. Dulikravich[1] Karl W. Mortara[2] Lionel Marraffa[3]
Department of Aerospace Engineering
The Pennsylvania State University
University Park, PA 16802, U.S.A.

ABSTRACT

The effects that artificial dissipation has on numerical solutions of the transonic Full Potential Equation (FPE) are investigated by comparing the artificially dissipative FPE to a Physically Dissipative Potential (PDP) equation. Analytic expressions were derived for the variables C and M_c that are used in the artificial density formulation. It was shown that these new values generate artificial dissipation which is equivalent to the physical dissipation which exists in the PDP equation. The new expressions for the variables C and M_c can easily be incorporated into the existing full potential codes which are based either on the artificial density or on the artificial viscosity formulation. A comparison of Physically Dissipative Potential (PDP), Artificial Density or Viscosity (ADV), Artificial Mass Flux (AMF), and ADV with variable C and M_c formulation (MCC) is also presented.

INTRODUCTION

A mathematical model for nondissipative, irrotational, compressible, inviscid flows is known as the Full Potential Equation (FPE). Numerical techniques used for integrating the FPE in transonic shocked regions required addition of artificial dissipation in an attempt to stabilize these schemes. The numerical dissipation must be added in a fully conservative form if transonic flows with shocks are to be computed accurately. The artificial dissipation usually has the form of artificial viscosity [1], artificial density [2,3] or artificial mass flux [4,5]. Although these schemes have been fairly successful, the amount and the form of the artificial dissipation which is required in specific cases is usually determined in an ad hoc manner [5]. In this work, the artificially dissipative FPE, that is, FPE with an Artificial Density or Viscosity formulation (ADV) and the FPE with an Artificial Mass Flux (AMF) formulation were compared to a recently derived Physically Dissipative Potential (PDP) equation [6]. From these comparisons, a new form of numerical dissipation has been derived which has physical origins and an analytic formulation for the constants presently used in the ADV. This new formulation is termed variable M_c and C or (MCC) formulation.

[1]Associate Professor. Senior Member AIAA.
[2]AFIT Graduate Student. Wright-Patterson AFB, Ohio.
[3]Visiting Research Scientist. ONERA-Chatillon, France

PHYSICALLY DISSIPATIVE POTENTIAL (PDP) EQUATION

Dulikravich and Kennon [6] have derived a new mathematical model which governs irrotational, non-isentropic, viscous flows of calorically perfect gases without body forces, surface tension, radiation heat transfer, internal heat generation and mass sources. This model includes the physical dissipation due to certain effects of shear viscosity, secondary viscosity and heat conductivity. The full three-dimensional version of their Physically Dissipative Potential (PDP) equation can be expressed [6] in a canonical nondimensional form as

$$
\begin{aligned}
\rho\, \{[(1-M^2)\,\phi_{ss} + \phi_{mm} + \phi_{nn}] - \frac{1}{a^2}\,(\phi_{tt} + 2\phi_s\phi_{st})\} \\
= \frac{\mu''}{R_e}\, \{-\frac{\phi_s}{a^2}\,(1 + \frac{\gamma-1}{P_r''})\,(\phi_{sss} + \phi_{smm} + \phi_{snn}) \\
+ \frac{\gamma-1}{a^2}\,(1 - \frac{1}{P_r''})\,(\phi_{ss}^2 + \phi_{mm}^2 + \phi_{nn}^2) \\
+ 2\,\frac{\gamma-1}{a^2}\,(1 - \frac{2}{\mu''})(\phi_{ss}\phi_{mm} + \phi_{ss}\phi_{nn} + \phi_{mm}\phi_{nn}) \\
- 2\,\frac{\gamma-1}{a^2}\,(\frac{1}{P_r''} - \frac{2}{\mu''})(\phi_{sm}^2 + \phi_{sn}^2 + \phi_{mn}^2) \\
- \frac{1}{a^2}\,(\phi_{sst} + \phi_{mmt} + \phi_{nnt})\} \qquad (1)
\end{aligned}
$$

Here, s is the locally streamline aligned coordinate direction and m and n are the mutually orthogonal remaining coordinates of the locally streamline aligned Cartesian coordinate system (s,m,n). The left-hand side of this equation represents the nondissipative FPE and the right-hand side represents physical dissipation due to viscous effects and heat conduction.

Here, ρ is the local fluid density, ϕ is the local velocity potential function, a is the local isentropic speed of sound, t is the time, μ is the coefficient of shear viscosity, λ is the coefficient of secondary viscosity, μ'' is the coefficient of longitudinal viscosity $\mu'' = 2\mu+\lambda$, γ is the ratio of specific heats, M is the local Mach number, R_e is the Reynolds number [7], P_r'' is defined [8] as the longitudinal Prandtl number

$P_r'' = P_r \frac{\mu''}{\mu}$ where $P_r = \frac{\bar{C}_p \bar{\mu}_\infty}{\bar{k}_\infty}$ is the Prandtl number and $\bar{k}$ is the coefficient of heat conductivity. All quantities have been nondimensionalized, that is,

$$\rho = \frac{\bar{\rho}}{\bar{\rho}_*} \; ; \; T = \frac{\bar{T}}{\bar{T}_*} = a^2 = \frac{\bar{a}^2}{\bar{a}_*^2} \; ; \; \phi_s = M_* \; ; \; \lambda = \frac{\bar{\lambda}}{\bar{\mu}_\infty}$$

$$\mu'' = \frac{\bar{\mu}''}{\bar{\mu}_\infty} \; ; \; k = \frac{\bar{k}}{\bar{k}_\infty} \; ; \; R_e^{-1} = \frac{\bar{\mu}_\infty}{\bar{\rho}_* \bar{a}_* \bar{L}_*} \; ; \; x = \frac{\bar{x}}{\bar{L}_*} \qquad (2)$$

where the critical quantities are indicated with the subscript * and the dimensional quantities have an overbar. Coefficients λ, μ and k are treated as constants. If we restrict ourselves to the study of normal shock structure, then the non-dimensional version of (1) for steady flows is

$$\rho(1-M^2)\phi_{xx} + \frac{\mu''}{R_e}\left(1 + \frac{\gamma-1}{P_r''}\right)\frac{\phi_x}{a^2}\phi_{xxx}$$

$$- \frac{\gamma-1}{a^2}\frac{\mu''}{R_e}\left(1 - \frac{1}{P_r''}\right)(\phi_{xx})^2 = 0 \qquad (3)$$

The one-dimensional version (3) of the PDP was numerically integrated using a fourth-order Runge-Kutta integration scheme with $\Delta x = 10^{-7}$ and several values for λ/μ. The results indicate (Figure 1 and Figure 2) that the PDP can produce shocks of various strengths depending on the specified value of the ratio of viscosities λ/μ. Specifically, Stokes hypothesis that $\lambda/\mu = -2/3$ leads to Rankine-Hugoniot shock jumps, and $\lambda/\mu \cong - 2$ leads to isentropic shock jumps [9]. The values used in all test cases were: $P_r = 3/4$; $\gamma = 7/5$; Re = 105.

ENTROPY GENERATION

Dissipation effects in a flowfield can be most rigorously evaluated by computing the entropy generation equation due to viscous effects and heat conduction. The entropy generation equation can be expressed as

$$\bar{\rho}\bar{T}\frac{D\bar{S}}{D\bar{t}} = \bar{\Phi} + \bar{k}\,\bar{\nabla}^2\bar{T} \qquad (4)$$

where $\bar{\Phi}$ is the viscous dissipation function and $\bar{S}$ is the specific entropy. All the variables were consequently nondimensionalized with their critical values. Also, $\bar{\mu}''$ and $\bar{\lambda}$ were nondimensionalized with $\bar{\mu}_\infty$, $\bar{k}$ with $\bar{k}_\infty$ and $\bar{S}$ with $\bar{R}$ where the speed of sound is $\bar{a}^2 = \gamma\bar{R}\bar{T}$. Then

$$\nabla^2 T = \nabla^2\left[\frac{\gamma+1}{2} - \frac{\gamma-1}{2}(\nabla\phi \cdot \nabla\phi)\right] \qquad (5)$$

and the nondimensionalized one-dimensional steady version of (4) becomes

$$R_e\,\rho\,T\,\phi_x\frac{dS}{dx} = \gamma\mu''(\phi_{xx})^2$$

$$- \frac{\bar{k}_\infty(\gamma-1)\gamma}{\gamma\bar{R}\bar{\mu}_\infty}\,k[\phi_x\phi_{xxx} + (\phi_{xx})^2] \qquad (6)$$

Notice that

$$\frac{(\gamma-1)}{\gamma\bar{R}} = \frac{1}{\bar{C}_p} \; ; \qquad P_r'' = \frac{\bar{C}_p\bar{\mu}_\infty''}{\bar{k}_\infty} \qquad (7)$$

Then, the normalized entropy generation equation for one-dimensional steady flows without radiation and internal heat sources is

$$dS = \frac{\gamma\mu''}{\rho T R_e}\left[\left(1 - \frac{1}{P_r''}\right)\frac{(\phi_{xx})^2}{\phi_x} - \frac{\phi_{xxx}}{P_r''}\right]dx \qquad (8)$$

If, for simplicity, we neglect the influence of entropy generation on the variation of density, the nondimensional entropy generation equation becomes

$$\frac{dS}{dx} \cong \frac{\gamma\mu''}{R_e\left[\frac{\gamma+1}{2} - \frac{\gamma-1}{2}(\phi_x)^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\cdot\left[\left(1 - \frac{1}{P_r''}\right)\frac{(\phi_{xx})^2}{\phi_x} - \frac{\phi_{xxx}}{P_r''}\right] \qquad (9)$$

Notice that S in this equation is actually a nondimensional quantity $S = \bar{S}/\bar{R}$. Equation (9) was integrated using fourth order Runge-Kutta scheme and several values of $(\phi_x)_{-\infty}$ and λ/μ, while $R_e = 10^5$. For comparison, the exact values of the total entropy jump across a normal shock satisfying Rankine-Hugoniot conditions can be found from

$$\Delta S = \frac{\gamma}{\gamma-1}\ln\left[\frac{2}{(\gamma+1)M_1^2} + \frac{\gamma-1}{\gamma+1}\right]$$

$$+ \frac{1}{\gamma-1}\ln\left[\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}\right] \qquad (10)$$

where M_1 is the Mach number ahead of the normal shock. Comparison of the numerically computed and the exact values of the total entropy jumps are very good (Figures 3 and 4), despite the fact that we have used an isentropic relation for the density, ρ in equation 9. This result provides a detailed picture of entropy variation through the compression shock showing that it has a strong maximum in the middle of the shock [6].

ARTIFICIAL DENSITY OR VISCOSITY (ADV) FORMULATION

The conventional formulation for artificial density [3] generates (in the one-dimensional steady case) the following terms [5]

$$\rho\{(1-M^2)\,\phi_{xx} + C\,(M^2-M_c^2)\,(M^2)^n\,\phi_{xxx}$$

$$+ C\left[(M^2 - M_c^2)\,(2-(2-\gamma)\,M^2) + \frac{(\gamma+1)}{a^2}\right]$$

$$(M_c^2 + n\,(M^2 - M_c^2))\,(M^2)^n\,(\phi_{xx})^2\}=0 \qquad (11)$$

where the constants C and n, and the constant cut-off [10] Mach number, M_c, are the two user-specified input parameters. This equation is a result of using the following values of artificial density, $\tilde{\rho}$, and artificial viscosity, $\tilde{\mu}$, in the conventional form [3] of the Artificial Density or Viscosity (ADV) formulation

$$\tilde{\rho} = \rho - \tilde{\mu}\, \rho_s \quad \text{where} \quad \tilde{\mu} = C \max\left\{0;\ 1 - \frac{M_c^2}{M^2}\right\}(M^2)^n \tag{12}$$

Both C and M_c are arbitrary constants in the conventional AVD formulation [3]. The coefficient C is usually chosen to be of the order one [5]. The exponent n [5] is usually zero. The cut-off Mach number M_c is usually chosen [5] as having the constant value between 0.8 and 1.0. It should always be less than the post-shock Mach number [5].

Although M_c does not affect the total shock jump, it strongly affects the shock thickness. Since the artificial viscosity formulation is a truncated version of the artificial density formulation and since the directionally biased flux formulation [11] is equivalent [5] to the ADV, only ADV will be discussed. Critical Mach number variations through a normal shock resulting from the numerical integration of the ADV formulation (equation 11) are shown in Figure 5.

EFFECTS OF THE NUMERICAL DISSIPATION BASED ON ADV

Equating the coefficients of like derivatives in (3) and in (11) produces two simultaneous equations, namely

$$-\frac{\gamma-1}{a^2}\frac{\mu''}{R_e}\left(1 - \frac{1}{P''_r}\right) = C\rho\left\{(M^2-M_c^2)(2-(2-\gamma)M^2) + \frac{\gamma+1}{a^2}M_c^2\right\} \tag{13}$$

and

$$\frac{\mu''}{R_e}\left(1 + \frac{\gamma-1}{P_r''}\right)\frac{\phi_x}{a^2} = C\rho\,(M^2 - M_c^2) \tag{14}$$

These two equations can be solved for $(\mu''/R_e)_{eq}$ and for $(1/P_r'')_{eq}$. The result is

$$\left(\frac{\mu''}{R_e}\right)_{eq} = \frac{-C\,\rho\,a}{\gamma}\left\{(M^2-M_c^2)[2-(2-\gamma)M^2) + \frac{\gamma+1}{\frac{2}{a}}M_c^2 - \frac{1}{\phi_x}\right\} \tag{15}$$

and

$$\left(\frac{1}{P_r''}\right)_{eq} = 1 + \frac{C\rho a^2}{(\gamma-1)}\frac{1}{(\mu''/R_e)_{eq}}\left\{(M^2-M_c^2)(2-(2-\gamma)M^2) + \frac{\gamma+1}{a^2}M_c^2\right\} \tag{16}$$

$$\text{Thus}\quad (\mu'')_{eq} = \frac{1/P_r}{(1/P_r'')_{eq}} \tag{17}$$

These expressions provide physically equivalent values for $(\mu''/R_e)_{eq}$ and $(1/P_r'')_{eq}$ generated by the conventional formulations [3] of the artificial density where C and M_c are kept constant. Thus, we could now analyze the physically equivalent dissipative features of the ADV formulation. For this purpose, (15) and (16) were substituted in the entropy generation equation (9) and integrated. The results indicate (Fig. 6) that the ADV formulation generates entropy which is considerably larger than entropy produced by the physical dissipation (Fig. 4). Moreover, when (17) is plotted (Fig. 7) for three different constant values of M_c, it is noticeable that μ''_{eq} generated by the ADV is not constant. Similar results are obtained (Figure 8) when the equivalent Prandtl number, $(P_r'')_{eq}$, is computed from (16).

ARTIFICIAL MASS FLUX (AMF) FORMULATION

In addition to PDP and ADV formulations, it is possible to work with an Artificial Mass Flux (AMF) dissipation. Here, the entire mass flux $(\rho\phi_s)$, instead of just density, ρ, is differentiated in the locally upstream [4,5] direction

$$\overline{\nabla} \cdot (\rho\vec{V}) = \left(\frac{\partial}{\partial s}\hat{e}_s + \frac{\partial}{\partial n}\hat{e}_n\right) \cdot \{[(\rho\phi_s) - (\rho\phi_x)_s]\,\hat{e}_s + (\rho\phi_n)\,\hat{e}_n\} = 0 \tag{18}$$

where $\hat{e}_s$ and $\hat{e}_n$ are the unit vectors in s and n direction, respectively. In the two-dimensional case, the resulting artificially dissipative full potential equation will contain non-physical dissipation [5].

$$\rho\,[(1-M^2)\,\phi_{ss} + \phi_{nn}] + A\left\{\frac{\phi_s}{a^2}\phi_{sss} + \frac{1}{a^2}\phi_{ss}\,\phi_{nn} + [3 + 2(\gamma-1)M^2 - \frac{\gamma+1}{a^2}]\,\frac{(\phi_{ss})^2}{a^2}\right\} = 0 \tag{19}$$

where

$$A = -\frac{\mu''}{R_e}(\gamma-1)2\left(1 - \frac{2}{\mu''}\right) \quad \text{and} \quad \mu'' = \frac{(\gamma-1)(4 - \frac{1}{P_r})}{1 + 2(\gamma-1)} \tag{20}$$

The switching function $\tilde{\mu}$ and the constant C in the AMF formulation were evaluated [5] by equating the coefficients multiplying ϕ_{sss} and $\phi_{ss}\,\phi_{nn}$ terms in the full two-dimensional AMF equation and in the PDP equation. Figure 9 shows that AMF formulation provides only shock jumps corresponding to isentropic shocks and that it produces positive entropy change across a shock (Figure 10). This formulation requires Reynolds number as the only input parameter [5].

VARIABLE C AND M_c (MCC) FORMULATION

If (13) and (14) are solved simultaneously for C and M_c, the result is an analytic expression for variable values of C and M_c that are consistent with the physical dissipation generated by the PDP equation. Hence

$$M_c^{\,2} = \frac{M^2 \left\{ \left(1 + \frac{\gamma-1}{P_r''}\right)\left[2-(2-\gamma)\, M^2\right]\phi_s + (\gamma-1)\left(1 - \frac{1}{P_r''}\right) \right\}}{\left(1 + \frac{\gamma-1}{P_r''}\right)\left[2-(2-\gamma)M^2 - \frac{\gamma+1}{a^2}\right]\phi_s + (\gamma-1)\left(1 - \frac{1}{P_r''}\right)} \tag{21}$$

$$C = \frac{\left(1 + \frac{\gamma-1}{P_r''}\right) \frac{\phi_s}{a^2}\, \mu''}{\rho(M^2 - M_c^{\,2})\, R_e} \tag{22}$$

The new <u>variable</u> values for C and M_c in the ADV formulation are <u>analytically</u> defined so that they duplicate the physical dissipation from the one-dimensional version of the PDP equation. Substituting (21) and (22) in (11) and using the fourth order Runge-Kutta integration scheme produced the results shown in Figures 11 and 12. Since the variable M_c and C formulation (MCC) duplicates exactly the terms from one-dimensional PDP equation, it is capable of producing Rankine-Hugoniot shock jumps if $\lambda/\mu = -2/3$ and isentropic shock jumps if $\lambda/\mu \cong -2$.

SUMMARY

The Artificial Density or Viscosity (ADV) formulation as used in the existing computer codes for the solution of the transonic full potential equation utilizes constant, ad hoc values for the cut-off Mach number, M_c, and a constant, C, in the switching function μ. Numerical results show that ADV formulation generates entropy which is almost an order of magnitude larger than the entropy generated by the Physically Dissipative Potential (PDP) equation. Analytic expressions for both M_c and C were then derived that introduce the effects of physical dissipation. The existing full potential codes that use artificial density formulation can easily accommodate this physically consistent numerical dissipation by evaluating C and M_c analytically at every point in the flow field. Moreover, full potential codes can now be used for computing flows with both isentropic shocks and with Rankine-Hugoniot shocks, since values of R_e and λ/μ are the input parameters. Consequently, the strengths and thicknesses of the resulting shocks and the amount of entropy generated can be controlled with the physically known coefficients: P_r'', μ'' and R_e.

Finally, Table 1 summarizes the analytic forms of all three artificial dissipation formulations (ADV, MCC and AMF) and compares them with the Physically Dissipative Potential (PDP) flow formulation.

ACKNOWLEDGEMENTS

The authors would like to thank Ms. Amy Myers for her careful typing and to Apple Computer, Inc. and Sun Microsystems, Inc. for the computing equipment. This work was performed in part while the lead author worked as an invited Senior Research Scientist visiting the Institute for Computational Mechanics in Propulsion (ICOMP) which is administered by the NASA Lewis Research Center and the Case Western Reserve University.

REFERENCES

1. Jameson, A., "Transonic Flow Calculations," Computational Fluid Dynamics, Vol. 1, Von Karman Institute for Fluid Dynamics, VKI-LS-87-VOL-1, 1976, pp. 1.1-5.84.

2. Eberle, A. "Eine Method Finiter Elements Berrechnungder Transsoniken Potential-Stromung un Profile," MBB Berich Nr, UFE 1352 (0), 1977.

3. Hafez, M., J. South, and E. Murman, "Artificial Compressibility Methods for Numerical Solution of Transonic Full Potential Equation," AIAA Journal, Vol. 17, No. 8, August 1979, pp. 838-844.

4. Dulikravich, G. S. and P. Niederdrenk, "Artificial Mass Concept and Transonic Viscous Flow Equation," ARO Report 84-1, Jan. 1984, (presented at the First Army Conference on Applied Mathematics and Computing, Washington D.C., May 9-11, 1983).

5. Dulikravich, G. S., "Analysis of Numerical Dissipation Models for Transonic Full Potential Equation," AIAA paper no. 88-0711, presented at the Aerospace Sciences Meeting, Reno, NV, Jan. 1988; also AIAA Journal, August 1988.

6. Dulikravich, G. S. and Kennon, S. R., "Theory of Irrotational Unsteady Compressible Flows Including Heat Conductivity and Longitudinal Viscosity," AIAA paper no. 88-0709 presented at the Aerospace Sciences Meeting, Reno, NV, 1988; to appear in Internat. J. of Mathematical Modelling, 1988.

7. Ryzhov, O. S., "Viscous Transonic Flows," Annual Review of Fluid Mechanics, Vol. 10, 1978, pp. 65-92.

8. Sichel, M., "Structure of Weak Non-Hugoniot Shocks," Physics of Fluids, 6, No. 5, May 1963, pp. 653-662.

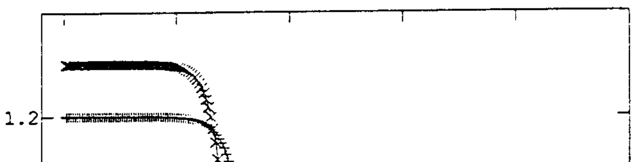
9. Dulikravich, G. S. and Sobieczky, H., "Shockless Design and Analysis of Transonic Cascade Shapes," AIAA Journal, Vol. 20, No. 11, Nov. 1982, pp. 1572-1578.

10. Jameson, A., "Acceleration of Transonic Potential Flow Calculations on Arbitrary Meshes by the Multiple Grid Method," AIAA Paper 79-1458, 1979.

11. Osher, S., Whitlow, W., and Hafez, M. M, "Improved Finite Difference Schemes for Transonic Potential Calculations," AIAA Paper 84-0092, 1984.

	ϕ_{ss}	ϕ_{nn}	ϕ_{sss}	ϕ_{snn}	ϕ_{ss}^2	ϕ_{nn}^2	$\phi_{ss}\phi_{nn}$	ϕ_{sn}^2
FPE-PDP	$\rho(1-M^2)$	ρ	$\frac{\mu^*}{R_e}\frac{\phi_s}{a^2}(1+\frac{\gamma-1}{P_r^*})$	$\frac{\mu^*}{R_e}\frac{\phi_s}{a^2}(1+\frac{\gamma-1}{P_r^*})$	$\frac{\mu^*}{R_e}\frac{\gamma-1}{a^2}(\frac{1}{P_r^*}-1)$	$\frac{\mu^*}{R_e}\frac{\gamma-1}{a^2}(\frac{1}{P_r^*}-1)$	$\frac{\mu^*}{R_e}\frac{\gamma-1}{a^2}2(\frac{2}{\mu^*}-1)$	$\frac{\mu^*}{R_e}\frac{\gamma-1}{a^2}\cdot 2(\frac{1}{P_r^*}-\frac{2}{\mu^*})$
FPE-ADV	$\rho(1-M^2)$	ρ	$\rho\ C\ (M^2-M_c{}^2)$	0	$\rho C[(M^2-M_c{}^2)\cdot(2-(2-\gamma)M^2)-(\gamma+1)\frac{M_c{}^2}{a^2}]\frac{1}{\phi_s}$	0	$\rho C\ (\frac{M^2-M_c{}^2}{\phi_s})$	0
FPE-AMF	$\rho(1-M^2)$	ρ	$\frac{\mu^*}{R_e}\frac{\phi_s}{a^2}(1+\frac{\gamma-1}{P_r^*})$	0	$\frac{\mu^*}{R_e}\frac{[1+(\gamma-1)/P_r^*]}{a^2}\cdot[3+2(\gamma-1)M^2-\frac{\gamma+1}{a^2}]$	0	$\frac{\mu^*}{R_e}\frac{(1+\frac{1}{P_r^*})}{a^2}$	0
FPE-MCC	$\rho(1-M^2)$	ρ	$\frac{\mu^*}{R_e}\frac{\phi_s}{a^2}(1+\frac{\gamma-1}{P_r^*})$	0	$\frac{\mu^*}{R_e}\frac{\gamma-1}{a^2}(\frac{1}{P_r^*}-1)$	0	$\frac{\mu^*}{R_e}\frac{1}{a^2}(1+\frac{\gamma-1}{P_r^*})$	0

TABLE 1. COMPARISON OF COEFFICIENTS MULTIPLYING CORRESPONDING DERIVATIVES OF ϕ IN FOUR DISSIPATION MODELS FOR THE NUMERICAL INTEGRATION OF THE STEADY TWODIMENSIONAL FULL POTENTIAL EQUATION



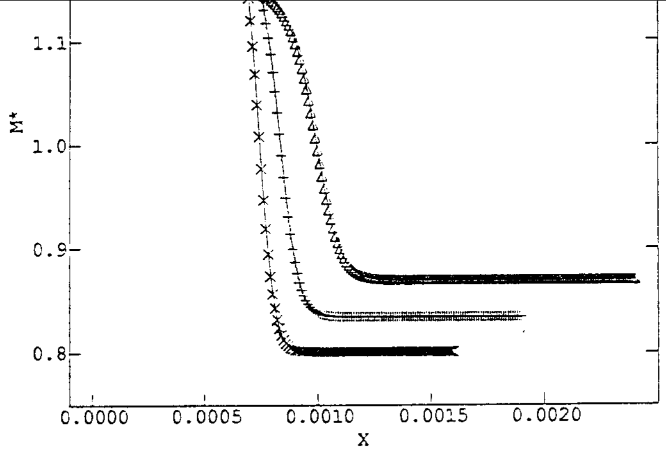


Figure 1 PDP Formulation: Variation of Critical Mach Number Through a Normal Shock Using $\lambda/\mu = -0.666$ Which Produces Rankine-Hugoniot Shock Jumps: $(\phi_x)_{-\infty} = 1.15$, $(\phi_x)_{-\infty} = 1.20$; $(\phi x)_{-\infty} = 1.25$.

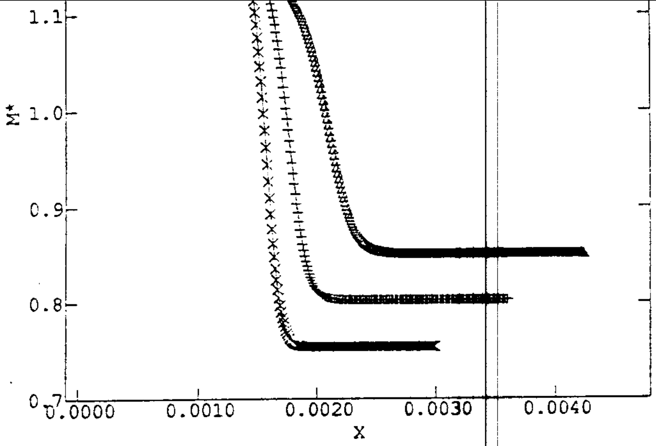


Figure 2 PDP Formulation: Variation of Critical Mach Number Through a Normal Shock Using $\lambda/\mu = -2.118$ Which Produces Isentropic Shock Jumps: $(\phi_x)_{-\infty} = 1.15$, $(\phi x)_{-\infty} = 1.20$; $(\phi_x)_{-\infty} = 1.25$.

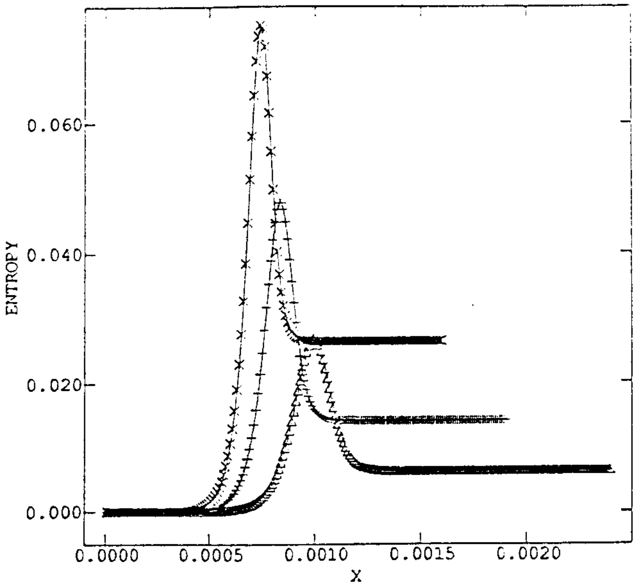


Figure 3 PDP Formulation: Variation of Nondimensional Entropy Through a Normal Shock Using λ/μ = -0.666
—△—△—△— $(\phi x)_{-\infty}$ = 1.15;
—|—|—|— $(\phi x)_{-\infty}$ = 1.20;
—x—x—x— $(\phi x)_{-\infty}$ = 1.25.

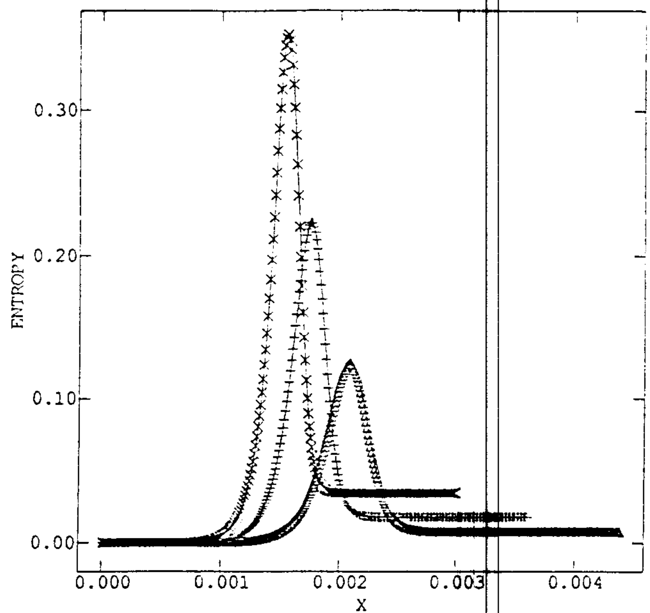


Figure 4 PDP Formulation: Variation of Nondimensional Entropy Through a Normal Shock Using λ/μ = -2.118:
—△—△—△— $(\phi x)_{-\infty}$ = 1.15,
—|—|—|— $(\phi x)_{-\infty}$ = 1.20;
—x—x—x— $(\phi x)_{-\infty}$ = 1.25.

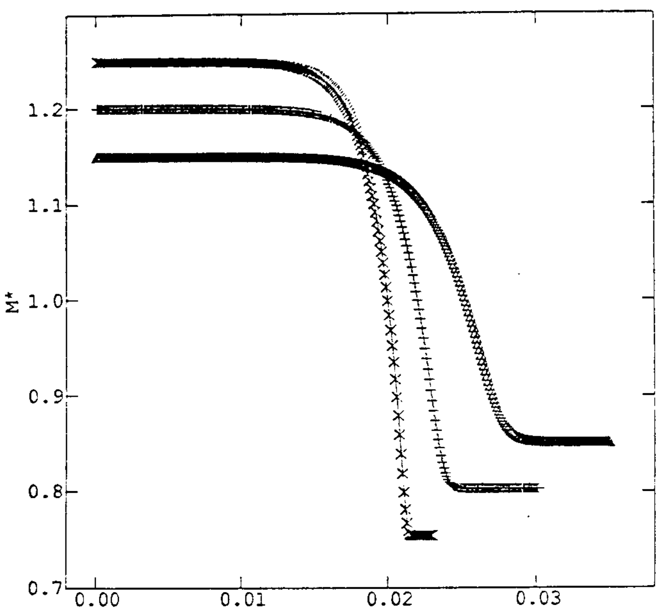


Figure 5 ADV Formulation: Variation of Critical Mach Number Through a Normal Shock Using C = 0.001 and M_c = 0.70: —△—△—△— $(\phi_x)_{-\infty}$ = 1.15;
—|—|—|— $(\phi_x)_{-\infty}$ = 1.20;
—x—x—x— $(\phi_x)_{-\infty}$ = 1.25.

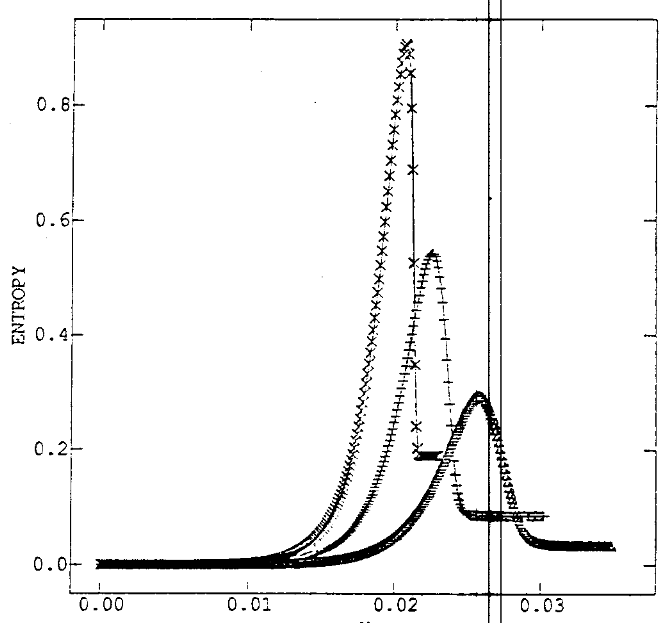


Figure 6 ADV Formulation: Variation of Nondimensional Equivalent Entropy Through a Normal Shock Using C = 0.001 and Mc = 0.70:
—△—△—△— $(\phi x)_{-\infty}$ = 1.15;
—|—|—|— $(\phi x)_{-\infty}$ = 1.20;
—x—x—x— $(\phi x)_{-\infty}$ = 1.25.

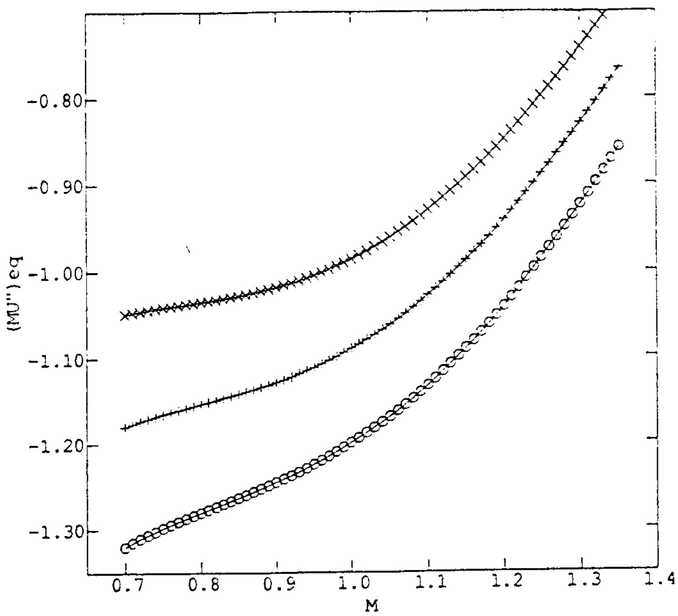


Figure 7 ADV Formulation: Variation of Equivalent Nondimensional Longitudinal Viscosity $(\mu'')_{eq}$ Using C = 1.0: –o–o–o– Mc = 0.80; –+–+–+– Mc = 0.75; –x–x–x– Mc = 0.70.

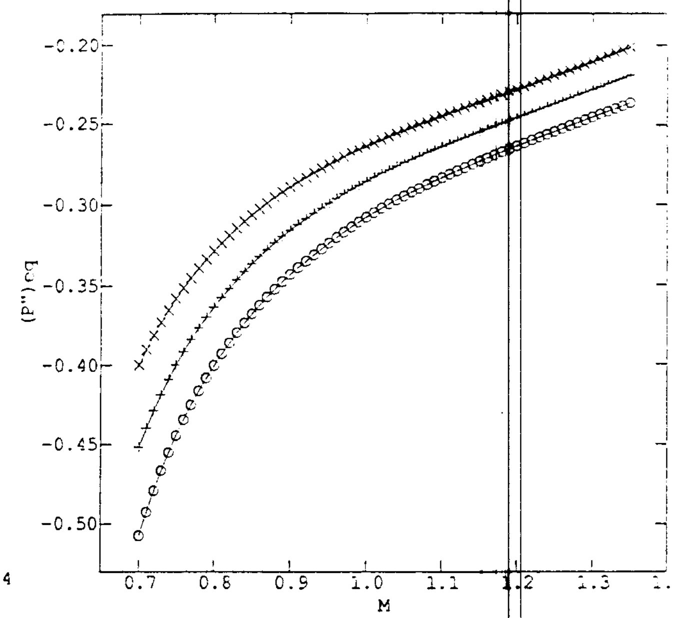


Figure 8 ADV Formulation: Variation of Equivalent Longitudinal Prandtl Number $(P_r'')_{eq}$ Using C = 1.0; –o–o–o– M_c = 0.80; –+–+–+– Mc = 0.75; –x–x–x– Mc = 0.70.

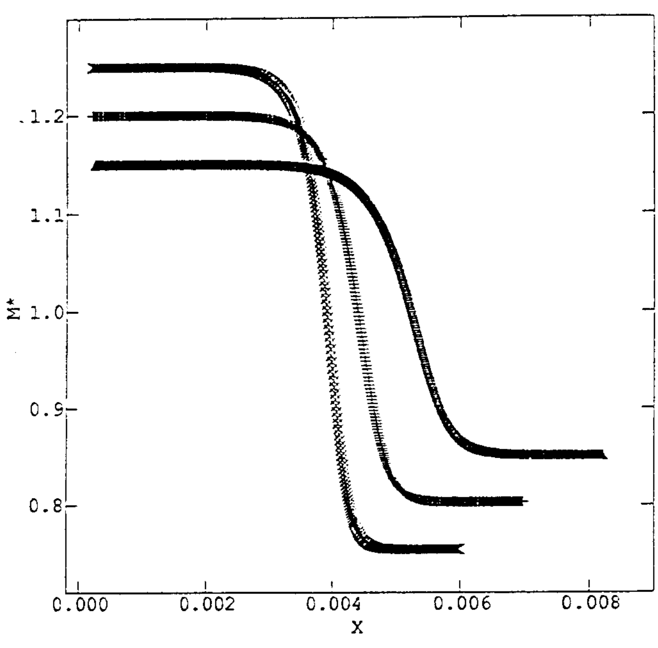


Figure 9 AMF Formulation: Variation of Critical Mach Number Through A Normal Shock: –Δ–Δ–Δ– $(\phi_x)_{-\infty}$ = 1.15; –+–+–+– $(\phi x)_{-\infty}$ = 1.20; –x–x–x– $(\phi x)_{-\infty}$= 1.25.

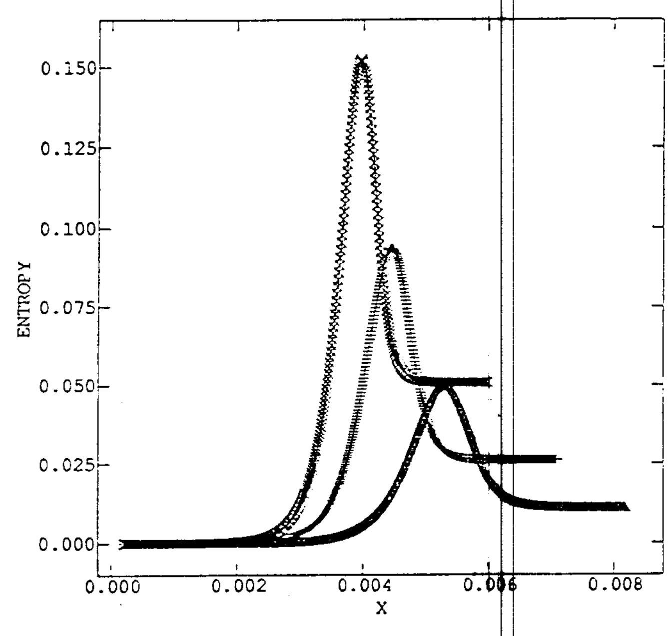


Figure 10 AMF Formulation: Variation of Nondimensional Equivalent Entropy Through a Normal Shock : –Δ–Δ–Δ–Δ– $(\phi x)_{-\infty}$ = 1.15; –+–+–+– $(\phi_x)_{-\infty}$ = 1.20; –x–x–x– $(\phi x)_{-\infty}$ = 1.25.

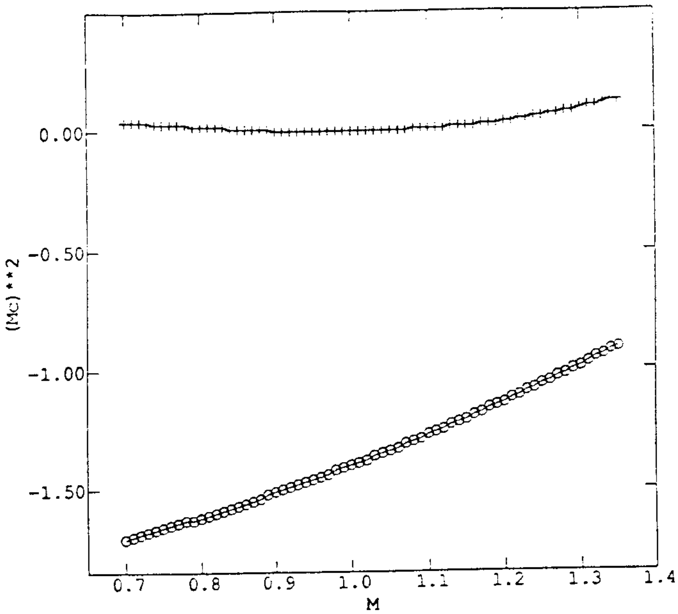


Figure 11 MCC Formulation: Variation of M_c^2 With M: —o—o—o— $\lambda/\mu = -0.666$ (Producing Rankine-Hugoniot Shock Jumps); —+—+—+—+— $\lambda/\mu \cong -2$ (Producing Isentropic Shock Jumps).

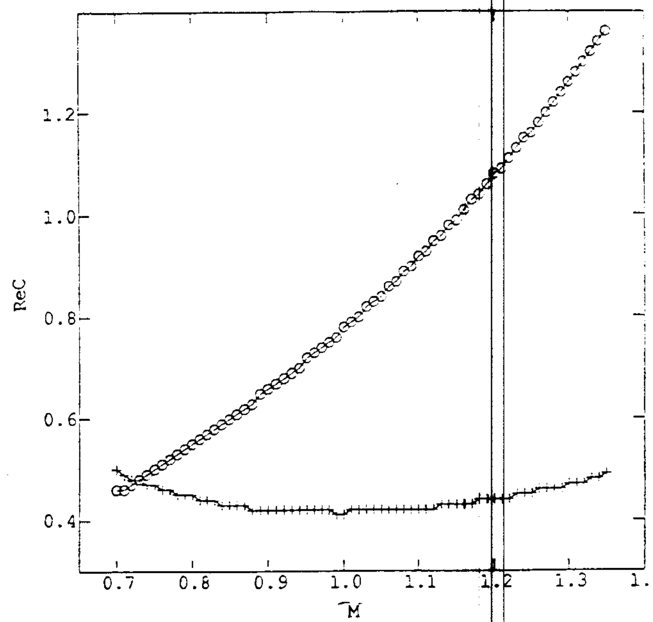


Figure 12 MCC Formulation: Variation of C Multiplied With Re: —o—o—o— $\lambda/\mu = -0.666$; Producing Rankine-Hugoniot Shock Jumps; —+—+—+— $\lambda/\mu \cong -2$ Producing isentropic Shock Jumps.