ABSTRACT

The effects that artificial dissipation has on numerical solutions of the transonic Full Potential Equation (FPE) are investigated by comparing the artificially dissipative FPE to a Physically Dissipative Potential (PDP) equation. Analytic expressions were derived for the variables C and M_e that are used in the artificial density formulation. It was shown that these new values generate artificial dissipation which is equivalent to the physical dissipation which exists in the PDP equation. The new expressions for the variables C and M_e can easily be incorporated into the existing full potential codes which are based either on the artificial density or on the artificial viscosity formulation. A comparison of Physically Dissipative Potential (PDP), Artificial Density or Viscosity (ADV), Artificial Mass Flux (AMF), and ADV with variable C and M_e formulation (MCC) is also presented.

INTRODUCTION

A mathematical model for nondissipative, irrotational, compressible, inviscid flows is known as the Full Potential Equation (FPE). Numerical techniques used for integrating the FPE in transonic shocked regions required addition of artificial dissipation in an attempt to stabilize these schemes. The numerical dissipation must be added in a fully conservative form if transonic flows with shocks are to be computed accurately. The artificial dissipation usually has the form of artificial density [1], artificial density [2,3] or artificial mass flux [4,5]. Although these schemes have been fairly successful, the amount and the form of the artificial dissipation which is required in specific cases is usually determined in an ad hoc manner [5]. In this work, the artificially dissipative FPE, that is, FPE with an Artificial Density or Viscosity formulation (ADV) and the FPE with an Artificial Mass Flux (AMF) formulation were compared to a recently derived Physically Dissipative Potential (PDP) equation [6]. From these comparisons, a new form of numerical dissipation has been derived which has physical origins and an analytic formulation for the constants presently used in the ADV. This new formulation is termed variable M_e and C or (MCC) formulation.

PHYSICALLY DISSIPATIVE POTENTIAL (PDP) EQUATION

Dulikravich and Kennon [6] have derived a new mathematical model which governs irrotational, non-isentropic, viscous flows of calorically perfect gases without body forces, surface tension, radiation heat transfer, internal heat generation and mass sources. This model includes the physical dissipation due to certain effects of shear viscosity, secondary viscosity and heat conductivity. The full three-dimensional version of their Physically Dissipative Potential (PDP) equation can be expressed [6] in a canonical nondimensional form as

\[ \rho \left[ \begin{array}{c}
\frac{1}{2} \left( \frac{1}{M_e^2} \dot{e}_{ss} + \dot{e}_{mn} + \dot{e}_{nn} \right) + \frac{1}{a} \left( \dot{e}_{tt} + 2 \dot{e}_{es} \right)
\end{array} \right] = 
\]

\[ \frac{\gamma - 1}{\gamma} \left( \frac{1}{F} \right) \left( \dot{e}_{ss} + \dot{e}_{mn} + \dot{e}_{nn} \right)
\]

\[ + \frac{\gamma - 1}{\gamma} \left( \frac{1}{F} \right) \left( \dot{e}_{ss} + \dot{e}_{mn} + \dot{e}_{nn} \right)
\]

\[ + \frac{2 \gamma - 1}{\gamma} \left( \frac{1}{F} \right) \left( \dot{e}_{ss} + \dot{e}_{mn} + \dot{e}_{nn} \right)
\]

\[ - \frac{2 \gamma - 1}{\gamma} \left( \frac{1}{F} \right) \left( \dot{e}_{ss} + \dot{e}_{mn} + \dot{e}_{nn} \right)
\]

\[ + \frac{1}{2} \left( \dot{e}_{ss} + \dot{e}_{mn} + \dot{e}_{nn} \right)
\]

Here, s is the locally streamline aligned coordinate direction and m and n are the mutually orthogonal remaining coordinates of the locally streamline aligned Cartesian coordinate system (s,m,n). The left-hand side of this equation represents the nondissipative FPE and the right-hand side represents physical dissipation due to viscous effects and heat conduction.

Here, \( p \) is the local fluid density, \( \dot{e} \) is the local velocity potential function, \( s \) is the local isentropic speed of sound, \( t \) is the time, \( \rho \) is the coefficient of shear viscosity, \( \lambda \) is the coefficient of secondary viscosity, \( \mu^* \) is the coefficient of longitudinal viscosity \( \mu^* = 2 \mu + \lambda \), \( \gamma \) is the ratio of specific heats, \( M \) is the local Mach number, \( Re \) is the Reynolds number [7], \( Pr^* \) is defined [8] as the longitudinal Prandtl number

\[ \frac{Pr^*}{Pr} = \frac{\frac{\tau}{\rho}}{\mu} \text{ where } Pr = \frac{\tau}{\mu} \text{ is the Prandtl number} \]
and $\bar{k}$ is the coefficient of heat conductivity.

All quantities have been nondimensionalized, that is,

$$
\bar{p} = \frac{p}{p_0}; \quad \bar{T} = \frac{T}{T_0}; \quad \bar{a}^2 = \frac{a^2}{a_0^2}; \quad \bar{\phi} = \frac{\phi}{M_c}; \quad \bar{\lambda} = \frac{\lambda}{\bar{\lambda}_0};
$$

$$
\bar{\mu} = \frac{\mu}{\mu_0}; \quad k = \frac{k}{k_0}; \quad \bar{R} = \frac{R}{R_0}; \quad \bar{x} = \frac{x}{x_0}; \quad \bar{e} = \frac{e}{e_0}
$$

(2)

where the critical quantities are indicated with the subscript * and the dimensional quantities have an overbar. Coefficients $\lambda$, $\mu$ and $k$ are treated as constants. If we restrict ourselves to the study of normal shock structure, then the nondimensional version of (1) for steady flows is

$$
(1 - \bar{M}^2)\bar{\phi}_{xx} + \bar{\mu} \left(1 + \frac{Y-1}{P_r} \right) \bar{\phi}_x + \bar{\phi}_{xxx} = 0
$$

(3)

The one-dimensional version (3) of the PDP was numerically integrated using a fourth-order Runge-Kutta integration scheme with $\Delta x = 10^{-7}$ and several values for $\nu/\mu$. The results indicate (Figure 1 and Figure 2) that the PDP can produce shocks of various strengths depending on the specified value of the ratio of viscosities $\lambda/\mu$. Specifically, Stokes hypothesis that $\lambda/\mu = 2/3$ leads to Rankine-Hugoniot shock jumps, and $\lambda/\mu = 2$ leads to isentropic shock jumps [9]. The values used in all test cases were: $P_r = 3/4$; $Y = 7/5$; $Re = 105$.

ENTROPY GENERATION

Dissipation effects in a flowfield can be most rigorously evaluated by computing the entropy generation due to viscous effects and heat conduction. The entropy generation equation can be expressed as

$$
\bar{S} = \frac{\partial \bar{S}}{\partial \bar{T}} = \bar{\Phi} + \bar{\kappa} \bar{V} \bar{T}
$$

(4)

where $\bar{\Phi}$ is the viscous dissipation function and $\bar{S}$ is the specific entropy. All the variables were consequently nondimensionalized with their critical values. Also, $\bar{\Phi}$ and $\bar{S}$ were nondimensionalized with $\bar{\Phi}_0$ and $\bar{S}_0$ with $R$ where the speed of sound is $\bar{a}^2 = \sqrt{R\bar{T}}$. Then

$$
\bar{a}^2 \bar{T} = \bar{a}^2 \left[ \frac{Y + 1}{2} - \frac{Y - 1}{2} (\bar{\phi}_x + \bar{\phi}_x) \right]
$$

(5)

and the nondimensionalized one-dimensional steady version of (4) becomes

$$
\frac{\partial \bar{S}}{\partial \bar{T}} = \frac{\partial \bar{S}}{\partial \bar{T}} = -\frac{\bar{\mu}}{\bar{\mu}_0} \bar{\mu} \bar{\phi}_x^2 + \bar{\phi}_{xxx}^2 + \bar{\phi}_x \bar{\phi}_{xxx}
$$

(6)

Notice that

$$
\bar{S} = \frac{Y \bar{\mu}^2}{\bar{R} \bar{e}} \left[ 1 + \frac{Y-1}{P_r} \left( \frac{\phi_{xx}}{\phi_x} \right)^2 - \frac{\phi_{xxx}}{\phi_x} \right]
$$

(7)

Then, the normalized entropy generation equation for one-dimensional steady flows without radiation and internal heat sources is

$$
\frac{d\bar{S}}{d\bar{x}} = \frac{Y \bar{\mu}^2}{\bar{R} \bar{e}} \left[ (1 - \frac{1}{P_r}) \left( \frac{\phi_{xx}}{\phi_x} \right)^2 - \frac{\phi_{xxx}}{\phi_x} \right]
$$

(8)

If, for simplicity, we neglect the effect of entropy generation on the variation of density, the nondimensional entropy generation equation becomes

$$
\frac{d\bar{S}}{d\bar{x}} = \frac{Y \bar{\mu}^2}{\bar{R} \bar{e}} \left[ (1 - \frac{1}{P_r}) \left( \frac{\phi_{xx}}{\phi_x} \right)^2 - \frac{\phi_{xxx}}{\phi_x} \right]
$$

(9)

Notice that $S$ in this equation is actually a nondimensional quantity $S = S/R$. Equation (9) was integrated using fourth order Runge-Kutta scheme and several values of $(\phi_{xx})$, and $\lambda/\mu$ while $Re = 10^5$. For comparison, the exact values of the total entropy jump across a normal shock satisfying Rankine-Hugoniot conditions can be found from

$$
\frac{d\bar{S}}{d\bar{x}} = \frac{\partial \bar{S}}{\partial \bar{x}} = \frac{2}{\bar{Y}+1} \left[ \frac{\partial \bar{S}}{\partial \bar{T}} \right]_0
$$

(10)

where $\bar{M}$ is the Mach number ahead of the normal shock. Comparison of the numerically computed and exact values of the total entropy jumps are very good (Figures 3 and 4), despite the fact that we have used an isentropic relaxation for the density, $\phi$ in equation 9. This result provides a detailed picture of entropy variation through the compression shock showing that it has a strong maximum in the middle of the shock [6].

ARTIFICIAL DENSITY OR VISCOSITY (ADV) FORMULATION

The conventional formulation for artificial density [3] generates (in the one-dimensional steady case) the following terms [5]

$$
\bar{p} \left[ (1 - \bar{M}^2) \bar{\phi}_{xx} + C (\bar{M}_c^2 - \bar{M}^2) \bar{\phi}_{xxx}^2 \right] + \left[ (\bar{M}_c^2 - \bar{M}^2) (2 - (2 - \bar{Y}) \bar{M}_c^2)^N + \frac{\phi_{xx}}{\phi_x} \right]
$$

(11)
where the constants $C$ and $n$, and the constant
cut-off [10] Mach number, $M_c$, are the two
user-specified input parameters. This equation is
a result of using the following values of
artificial density, $\tilde{\rho}$, and artificial viscosity,
$\tilde{\nu}$, in the conventional form [3] of the Artificial
Density or Viscosity (ADV) formulation

$$\tilde{\rho} = \rho - \tilde{\nu} \rho_e \quad \text{where} \quad \tilde{\nu} = C \max \left( 0; 1 - \frac{M^2}{M_c^2} \right)^n$$

(12)

Both $C$ and $M_c$ are arbitrary constants in the
conventional ADV formulation [3]. The
coefficient $C$ is usually chosen to be of the
order one [5]. The exponent $n$ [5] is usually
zero. The cut-off Mach number $M_c$ is usually
chosen [5] as having the constant value between
0.8 and 1.0. It should always be less than the
post-shock Mach number [5].

Although $M_c$ does not affect the total shock
jump, it strongly affects the shock thickness.
Since the artificial viscosity formulation is a
truncated version of the artificial density
formulation and since the directionally biased
ADV, only ADV will be discussed. Critical Mach
number variations through a normal shock
resulting from the numerical integration of the
ADV formulation (equation 11) are shown in
Figure 5.

EFFECTS OF THE NUMERICAL DISSIPATION BASED ON ADV

Equating the coefficients of like
derivatives in (3) and in (11) produces two
simultaneous equations, namely

$$\frac{Y-1}{a^2} \frac{\rho''}{\rho_e} \left( 1 - \frac{1}{\gamma} \right) = C \rho \left( M^2 - M_c^2 \right)^2 \left( 2 - (2 - \gamma) M^2 \right)$$

$$+ \frac{Y+1}{a^2} M_c^2$$

(13)

and

$$\frac{\rho''}{a^2} \left( 1 - \frac{Y-1}{2} \frac{\rho''}{\rho_e} \right) = C \rho \left( M^2 - M_c^2 \right)^2 \left( 2 - (2 - \gamma) M^2 \right)$$

(14)

These two equations can be solved for $(\rho''/\rho_e)_{eq}$
and for $(1/\rho_e')_{eq}$. The result is

$$(\rho''/\rho_e)_{eq} = \frac{C \rho \rho}{Y} \left( M^2 - M_c^2 \right)^2 \left( 2 - (2 - \gamma) M^2 \right)$$

$$+ \frac{Y+1}{a^2} M_c^2$$

(15)

and

$$\frac{1}{\rho_e'} = 1 + C \rho \rho \left( Y-1 \right) \frac{1}{(\rho''/\rho_e)_{eq}}$$

$$+ \frac{Y+1}{a^2} M_c^2$$

(16)

Thus

$$(\rho'')_{eq} = \frac{1}{\rho_e'} \frac{1}{(\rho''/\rho_e)_{eq}}$$

(17)

These expressions provide physically equivalent
values for $(\rho''/\rho_e)_{eq}$ and $(1/\rho_e')_{eq}$ generated by
the conventional formulations [3] of the
artificial density where $C$ and $M_c$ are kept
constant. Thus, we could now analyze the
physically equivalent dissipative features of the
ADV formulation. For this purpose, (15) and (16)
were substituted in the entropy generation
equation (9) and integrated. The results
indicate (Fig. 6) that the ADV formulation
generates entropy which is considerably larger
than entropy produced by the physical dissipation
(Fig. 4). Moreover, when (17) is plotted (Fig.
7) for three different constant values of $M_c$, it
is noticeable that $(\rho'')_{eq}$ generated by the ADV
is not constant. Similar results are obtained
(Figure 8) when the equivalent Prandtl number,
$(\rho''/\rho_e')_{eq}$, is computed from (16).

ARTIFICIAL MASS FLUX (AMF) FORMULATION

In addition to PDP and ADV formulations, it
is possible to work with an Artificial Mass Flux
(AMF) dissipation. Here, the entire mass flux
$(\rho \dot{\phi}_g)$, instead of just density, $\rho$, is
differentiated in the locally upstream [4, 5]
direction

$$\nabla \cdot (\rho \cdot V) = \frac{\partial}{\partial s} \dot{s}_s + \frac{\partial}{\partial n} \dot{n}_n = 0$$

(18)

where $\dot{s}_s$ and $\dot{n}_n$ are the unit vectors in $s$ and $n$
direction, respectively. In the two-dimensional
case, the resulting artificially dissipative full
potential equation will contain non-
physical dissipation [5].

$$\rho \left( 1 - M^2 \right) \dot{\phi}_{ss} + \dot{\phi}_{nn} = A \left( \frac{\dot{s}_s}{ \dot{s}_s} \right) \dot{\phi}_{ss} + \frac{\dot{\phi}_{nn}}{\dot{n}_n}$$

(19)

where

$$A = \frac{\rho''}{\rho_e'} \left( Y-1 \right) \frac{1}{(\rho''/\rho_e)_{eq}}$$

(20)

The switching function $\tilde{\nu}$ and the constant $C$ in
the AMF formulation were evaluated [5] by
equating the coefficients multiplying $\dot{\phi}_{ss}$ and
$\dot{\phi}_{nn}$ terms in the full two-dimensional AMF
equation and in the PDP equation. Figure 9 shows
that AMF formulation provides only shock jumps
corresponding to isentropic shocks and that it
produces positive entropy change across a shock
(Figure 10). This formulation requires Reynolds
number as the only input parameter [5].

VARIABLE C AND M C (MCC) FORMULATION

If (13) and (14) are solved simultaneously
for $C$ and $M_c$, the result is an analytic
expression for variable values of $C$ and $M_c$ that
are consistent with the physical dissipation
generated by the PDP equation. Hence
\[
\begin{align*}
M_c^2 &= \frac{(1 + \frac{1}{\rho \frac{\partial}{\partial x}}) 2(1 - (2 - \gamma) \frac{1}{\gamma - 1} \frac{1}{\gamma - 1})}{(1 + \frac{1}{\rho \frac{\partial}{\partial x}}) 2(1 - (2 - \gamma) \frac{1}{\gamma - 1} \frac{1}{\gamma - 1})}\frac{\partial}{\partial x} \phi + \gamma - 1 - \frac{1}{\gamma - 1}) \\
C &= \frac{(1 + \frac{1}{\rho \frac{\partial}{\partial x}}) \frac{\partial}{\partial x} \phi}{\rho (\gamma - 1) \frac{\partial}{\partial x} \phi + \gamma - 1 - \frac{1}{\gamma - 1}) \frac{\partial}{\partial x} \phi}
\end{align*}
\]

The new variable values for \(C\) and \(M_c\) in the ADV formulation are analytically defined so that they duplicate the physical dissipation from the one-dimensional version of the PFP equation. Substituting (21) and (22) in (11) and using the fourth order Runge-Kutta integration scheme produced the results shown in Figures 11 and 12. Since the variable \(M_c\) and \(C\) formulation (MCC) duplicates exactly the terms from one-dimensional PFP equation, it is capable of producing Rankine-Hugoniot shock jumps if \(1/\rho = -2/3\) and inviscid shock jumps if \(1/\rho = -2\).

**SUMMARY**

The Artificial Density or Viscosity (ADV) formulation as used in the existing computer codes for the solution of the transonic full potential equation utilizes constant, ad hoc values for the cut-off Mach number, \(M_c\), and a constant, \(C\), in the switching function \(p^*\). Numerical results show that ADV formulation generates entropy which is almost an order of magnitude larger than the entropy generated by the Physically Dissipative Potential (PDP) equation. Analytic expressions for both \(M_c\) and \(C\) were then derived that introduce the effects of physical dissipation. The existing full potential codes that use artificial density formulation can easily accommodate this physically consistent numerical dissipation by evaluating \(C\) and \(M_c\) analytically at every point in the flow field. Moreover, full potential codes can now be used for computing flows with both isentropic shocks and with Rankine-Hugoniot shocks, since values of \(C\) and \(M_c\) are the input parameters. Consequently, the strengths and thicknesses of the resulting shocks and the amount of entropy generated can be controlled with the physically known coefficients: \(p^*\), \(\mu^*\) and \(R_e^*\).

Finally, Table 1 summarizes the analytic forms of all three artificial dissipation formulations (ADV, MCC and AMP) and compares them with the Physically Dissipative Potential (PDP) flow formulation.

**ACKNOWLEDGEMENTS**

The authors would like to thank Ms. Amy Myers for her careful typing and to Apple Computer, Inc. and Sun Microsystems, Inc. for the computing equipment. This work was performed in part while the lead author worked as an invited Senior Research Scientist visiting the Institute for Computational Mechanics in Propulsion (ICOMP) which is administered by the NASA Lewis Research Center and the Case Western Reserve University.

**REFERENCES**


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**Table 1. Comparison of Coefficients Multiplying Corresponding Derivatives of $\phi$ in Four Dissipation Models for the Numerical Integration of the Steady Two-dimensional Full Potential Equation**
Figure 1 PDP Formulation: Variation of Critical Mach Number Through a Normal Shock Using $\lambda/\mu = -0.666$ Which Produces Rankine-Hugoniot Shock Jumps: $\Delta - \Delta - (\phi_x)_- = 1.15, \mid \mid \mid (\phi_x)_- = 1.20; \mid \mid \mid (\phi_x)_- = 1.25$.

Figure 2 PDP Formulation: Variation of Critical Mach Number Through a Normal Shock Using $\lambda/\mu = -2.118$ Which Produces Isentropic Shock Jumps: $\Delta - \Delta - (\phi_x)_- = 1.15, \mid \mid \mid (\phi_x)_- = 1.20; \mid \mid \mid (\phi_x)_- = 1.25$. 
Figure 3  PDP Formulation: Variation of Nondimensional Entropy Through a Normal Shock Using $\lambda/\mu = -0.666$

- ······ $(\phi x)_{+\infty} = 1.15$
- ······ $(\phi x)_{-\infty} = 1.20$
- ······ $(\phi x)_{+\infty} = 1.25$

Figure 4  PDP Formulation: Variation of Nondimensional Entropy Through a Normal Shock Using $\lambda/\mu = -2.118$

- ······ $(\phi x)_{+\infty} = 1.15$
- ······ $(\phi x)_{-\infty} = 1.20$
- ······ $(\phi x)_{+\infty} = 1.25$

Figure 5  ADV Formulation: Variation of Critical Mach Number Through a Normal Shock Using $C = 0.001$ and $M_C = 0.70$

- ······ $(\phi x)_{+\infty} = 1.15$
- ······ $(\phi x)_{-\infty} = 1.20$
- ······ $(\phi x)_{+\infty} = 1.25$

Figure 6  ADV Formulation: Variation of Nondimensional Equivalent Entropy Through a Normal Shock Using $C = 0.001$ and $M_C = 0.70$

- ······ $(\phi x)_{+\infty} = 1.15$
- ······ $(\phi x)_{-\infty} = 1.20$
- ······ $(\phi x)_{+\infty} = 1.25$
Figure 7  ADV Formulation: Variation of Equivalent Nondimensional Longitudinal Viscosity ($\nu'$)_{eq} Using $C = 1.0$; —— $Mc = 0.80$; —— $Mc = 0.75$; —— $Mc = 0.70$.

Figure 8  ADV Formulation: Variation of Equivalent Longitudinal Prandtl Number ($P_{\alpha}$)_{eq} Using $C = 1.0$; —— $Mc = 0.80$; —— $Mc = 0.75$; —— $Mc = 0.70$.

Figure 9  AMF Formulation: Variation of Critical Mach Number Through A Normal Shock: $\phi_x = 1.15$; (\phi_x)_{\infty} = 1.20; (\phi_x)_{\infty} = 1.25$.

Figure 10  AMF Formulation: Variation of Nondimensional Equivalent Entropy Through a Normal Shock: (\phi_x)_{\infty} = 1.15; (\phi_x)_{\infty} = 1.20; (\phi_x)_{\infty} = 1.25$. 
Figure 11 MCC Formulation: Variation of $M_c^2$
With M: $\lambda/\mu = -0.666$
(Producing Rankine-Hugoniot Shock Jumps); $\lambda/\mu = -2$
(Producing Isentropic Shock Jumps).

Figure 12 MCC Formulation: Variation of C
Multiplied With Re:
$\lambda/\mu = -0.666$; Producing
Rankine-Hugoniot Shock Jumps;
$\lambda/\mu = -2$ Producing
Isentropic Shock Jumps.