HTD-Vol. 62

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Numerical Methods in Heat Transfer

presented at

THE WINTER ANNUAL MEETING OF
THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
ANAHEIM, CALIFORNIA
DECEMBER 7–12, 1986

sponsored by

THE HEAT TRANSFER DIVISION, ASME

edited by

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THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
United Engineering Center 345 East 47th Street New York, N.Y. 10017

Paper presented at the ASME 1986 Winter Annual Meeting, Anaheim, CA., December 7-12, 1986. (in NUMERICAL METHODS IN HEAT TRANSFER, edited by Chen and K. Vafai, ASME HTD-Vol.62)

INVERSE DESIGN OF COOLANT FLOW PASSAGES IN CERAMICALLY COATED SCRAM JET COMBUSTOR STRUTS

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ABSTRACT

An inverse design method is presented which determines the proper size, shape, and location of coolant flow passages in composite ceramic/metal scram jet combustor struts. During normal operation, strong thermal stresses frequently cause cracking and/or melting of the struts. Coolant flow passages in the struts are currently used in an attempt to alleviate these problems and to allow higher gas temperatures; however, the proper locations, shapes and sizes of these passages are often determined in an ad hoc manner.

In this work, an optimization technique is coupled with the boundary element method to produce an inverse design procedure for multi-holed, internally cooled scram jet combustor struts. Both single material homogeneous struts and non-homogeneous struts with a ceramic coating can be considered. Constraints can be enforced to assure minimum prescribed strut wall thickness and minimum spacing between the walls of neighboring holes.

The initial sizes, shapes and locations of the holes are adjusted iteratively in order to match temperature distributions which can be specified a priori on the combustor strut outer surface and on the surface of the inner coolant passages. In addition, heat flux distribution on the combustor strut outer surface can be prescribed and iteratively enforced during the optimization procedures. During each optimization iteration, a two-dimensional, steady state heat conduction equation is solved using a direct boundary element method with a linear temperature singularity distribution on each flat surface element.

This inverse design technique is demonstrated for several examples of internally cooled scram jet combustor struts in order to prove the feasibility and accuracy of the method. This numerical technique readily applies to other inverse design problems and configurations involving heat transfer.

NOMENCLATURE

- C_i scaled internal angle of the boundary at node i
- d_0 prescribed minimum strut wall thickness (m)
- d_i prescribed minimum spacing between walls of neighboring holes
- E_0 error function without penalty
- E error function with penalty function
- H^{-1} inverse Hessian matrix of the error function E
- k thermal conductivity (W/m°C)
- M number of circular holes
- N_1 number of nodes on the outer surface
- n outward unit normal vector on the boundary Γ
- q_j^R specified heat flux at node j (W/m^2)
- q_j^c calculated heat flux at node j (W/m^2)
- q normal heat flux
- $\bar{q}$ boundary conditions for heat flux
- r distance from i^{-th} point to the point under consideration
- r_i radius of i^{-th} circular hole (m)
- T temperature (°C)
- $\bar{T}$ boundary conditions for temperature
- T^* fundamental solution of Laplace's equation
- x_i the x coordinate of the center of i^{-th} circular hole (m)

y_i the y coordinate of the center of the i^{-th} circular hole (m)

Γ boundary of the domain of interest

δ_i Dirac delta function at i^{-th} point

θ interior angle at a boundary node

ρ independent variable of the error function

Ω domain of interest

Subscripts

E essential

N Neumann

INTRODUCTION

In order to avoid the melting and burn-through of scram jet combustor struts and to allow for higher combustion temperatures, internal coolant flow passages can be placed in the strut and/or a ceramic coating can be applied to the outer surface of the strut. The temperature (or heat flux) distribution on the outer surface of the strut can be determiend from the analysis of the gas flow field in the combustor. The temperature distribution on the unknown coolant flow passage walls can be prescribed in advance. Therefore, the design problem is to determine the proper size, shape and location of the internal coolant flow passages so as to satisfy the design requirements of prescribed temperature and heat flux distribution on the fixed outer surface of the strut and prescribed temperatures on the unknown coolant flow passage walls. The idea of coupling an optimiation technique with the boundary element method to develop an inverse design procedure for multi-holed, internally cooled turbine blades was developed by Kennon and Dulikravich [1,2,3,4] and further refined by Chiang and Dulikravich [5,6]. This previous work used an indirect boundary element method and incorporated a single material. The present work uses a direct boundary element technique in which the temperature appears as a primary variable. It has been extended to include multi-materials, thus allowing for ceramically coated blades, and it allows for manufacturing constraints such as minimum wall thickness and minimum spacing between coolant flow passages.

In this work, the desired temperature distribution on both the outer surface of the strut and inner surface of the coolant flow passages are specified a priori. In addition, a desired flux distribution is also prescribed in the outer wall. Normally, this would be an over-specified boundary value problem. However, the calculated heat flux distribution on the outer surface is then adjusted iteratively by changing the sizes, shapes, and locations of the coolant flow passages until the resulting heat flux distribution agrees with the prescribed heat flux distribution. This optimization problem is solved using the Davidon-Fletcher-Powell (DFP) technique and, during each optimization iteration, a two-dimensional steady heat conduction equation is solved using the direct boundary element method with a linear temperature singularity distribution on each flat surface element. As in all general optimization problems, several local minimums may exist and the DFP method attempts to detect these and to move towards a global minimum. Final configurations may depend on the initial guess for the hole sizes and locations; however, this study shows that this technique behaves very well for a variety of initial guesses. In practice, one would expect the designer to be able to choose a reasonable initial strut configuration.

Two design constraints which are applicable to a practical strut manufacturing process are included. A minimum distance which will be allowed between any hole and the ceramic coated surface can be specified, and a minimum distance which will be allowed between the walls of any two adjacent holes can also be specified. In the present study, only circular shaped coolant flow passages are considered since circular shaped holes are very acceptable from the manufacturing point of view. The numerical procedure presented here can easily accommodate arbitrary shaped passages [1,2,4].

This study is designed to show the potential benefit which can be derived using inverse design techniques. For simplicity, only the heat conduction phase of the problem is considered. In an actual application, the total simulation would included heat conduction, heat convection, radiative transfer, gas dynamic flow analysis, coolant flow analysis [7], and a structural analysis.

MATHEMATICAL MODEL

The differential equation governing two-dimensional steady state heat conduction is

$$\nabla\cdot k\nabla T = 0 \qquad \text{in } \Omega \tag{1}$$

with temperatures specified on the surface

$$T(\underset{\sim}{X}) = T_o(\underset{\sim}{X}) \qquad \text{on } \Gamma_E$$

or heat flux specified on the surface

$$k\nabla T \cdot n = \bar{q} \qquad \text{on } \Gamma_N \tag{2}$$

Since in reality the domain is multiply-connected and highly irregular, the simplest approach is to use the boundary element (surface panel) formulation.

The direct formulation of the boundary element method can be derived from the weighted residual method [8]. The weighted residual statement of the Laplace equation can be written as

$$\int_\Omega (\nabla^2 T)T^* d\,\Omega = \int_{\Gamma_N} (q - \bar{q})T^* d\,\Gamma - \int_{\Gamma_E} (T - \bar{T})q^* d\,\Gamma \tag{3}$$

where $\bar{q}$ and $\bar{T}$ are the specified boundary conditions and T^* is the fundamental solution of the Laplace equation on a domain Ω, that is,

$$\nabla^2 T^* + \delta_i = 0 \tag{4}$$

For an isotropic two-dimensional medium,

$$T^* = \frac{1}{2\pi} \ln\frac{1}{r} \tag{5}$$

where r is the distance from point i to the point under consideration. Then

$$q^* = \frac{\partial T^*}{\partial n}, \qquad q = \frac{\partial T}{\partial n} \tag{6}$$

Integrating by parts and substituting Eq. (4) into the left-hand side of Eq. (3), the final form of the boundary integral equation is

$$C_i T_i + \int_{\Gamma_N} T q^* d\Gamma = \int_{\Gamma_E} q T^* d\Gamma \qquad (7)$$

This equation provides a functional relationship between T and q over Γ which ensures their compatibility as boundary data. Here, C_i is the value of the scaled internal angle of the boundary Γ at the point i, that is,

$$C_i = \theta/2\pi \qquad (8)$$

Equation (7) can be discretized using a series of straight elements on the surface Γ with the variation of T and q assumed to be linear along each element. Equation (7) can be written for the n elements as:

$$C_i T_i + \sum_{j=1}^{n} \hat{H}_{ij} T_j = \sum_{j=1}^{n} G_{ij} q_j \qquad (9)$$

or, more simply,

$$\sum_{j=1}^{n} H_{ij} T_j = \sum_{j=1}^{n} G_{ij} q_j \qquad (10)$$

where

$$H_{ij} = \hat{H}_{ij} + C_i \qquad \text{for } i = j$$

$$H_{ij} = \hat{H}_{ij} \qquad \text{for } i \neq j \qquad (11)$$

Combustor struts with ceramic coating have two regions of considerably different thermal conductivities. Therefore, two coupled Laplace equations for the temperature fields need to be solved subject to the continuity of temperatures and heat fluxes at the intermaterial surface between the coating and the main material [5,6].

THE OPTIMIZATION PROCEDURES AND CONCEPT

In this optimization procedure, the designer guesses the initial size and location for the coolant flow passages. The size and location of the internal coolant flow passages are then iteratively adjusted in an attempt to match the desired heat flux distribution on the outer surface of the strut. The geometry of each circular hole can be described by three independent variables: the two coordinates of the center and the radius of the hole. Therefore, if there are M coolant flow passages, there will be $3M$ independent variables in the simulation. A global error function $E(\underset{\sim}{\rho})$ is defined which is a function of the $3M$ independent variables. It measures the error in heat flux distribution on the surface and enforces the design constraints using a penalty approach [10].

$$E_0(\underset{\sim}{\rho}) = E_0(X_i, y_i, r_i) = \frac{\left[\sum_{j=1}^{N_1} (Q_j^c - Q_j^R)^2\right]^{1/2}}{\left[\sum_{j=1}^{N_1} (Q_j^R)^2\right]^{1/2}} \quad \text{for } i=1,\dots,M \qquad (12)$$

where Q_j^R is the required heat flux and Q_j is the calculated heat flux on the surface Γ_1 and N_1 is the number of nodes on the discretized surface. A penalty function must be added to E_0 to impose the two manufacturing constraints [10,11] and the final error function is

$$E(\underset{\sim}{\rho}) = E_0(\underset{\sim}{\rho}) + \text{penalty function} \qquad (13)$$

The penalty function used here is of the interior method type with the inverse barrier function [6]. In the Davidon-Fletcher-Powell (DFP) optimization technique, a search direction S at a point $\underset{\sim}{\rho}$ is defined as

$$S = -H^{-1} \nabla E(\underset{\sim}{\rho}) \qquad (14)$$

where H is the Hessian matrix associated with the error function. The value of $E(\underset{\sim}{\rho})$ can always be reduced in this search direction. H^{-1} is difficult to evaluate directly, so it is approximated iteratively in the DFP method [9]. A new value of $\underset{\sim}{\rho}$ is chosen which minimizes $E(\underset{\sim}{\rho})$ in the direction S.

The iterative constrained optimization procedure used to modify the sizes and locations of the guessed coolant flow passages can be summarized using the following steps:

(1) Specify the temperature distributions on the outer and inner surfaces Γ_1 and Γ_3 (Fig. 1):

(2) Specify the required heat flux distribution q_j^R on surface Γ_1, and the thermal conductivities k_1 and k_2 in domains Ω_A and Ω_B, respectively;

(3) Specify the manufacturing constraints (i) the minimum distance d_0 allowed between the holes and the surface Γ_2, and (ii) the minimum distance d_1 allowed between any two neighboring hole surfaces Γ_3;

(4) Specify the number of holes M required, and the initial guess of the radii and location of centers of the holes. Specify the number of boundary elements to be used on surfaces Γ_1, Γ_2, and on each of the holes Γ_3;

(5) With the boundary element method [8], solve Laplace's equation for a steady temperature field inside Ω_A and Ω_B subject to the specified surface temperatures. Then calculate the heat flux Q_j^c $(j=1,\dots N_1)$ on the surface Γ_1;

(6) Use the calculated heat flux Q_j^c to evaluate the discrete flux error function, E, of the optimization problem; and

(7) Use the Davidson-Fletcher-Powell [9] optimization technique to find the new values of the independent variables which minimize $E(\underset{\sim}{\rho})$ in the direction S. Test to see if $E(\underset{\sim}{\rho})$ is below a prescribed value; otherwise, return to step (5).

NUMERICAL RESULTS

Three test cases are presented to illustrate the feasibility of the inverse design concept. A model problem with the geometry shown in Figure 1 was solved.

The thermal conductivity in the coating area was set to 1 W/(m°C) and in the main area was set to 20 W/(m°C). First, an 'exact' solution was created by specifying the temperature distribution (solid line) on the inner and outer surfaces and then calculating the resulting 'exact' heat flux distribution on the outer surface (dashed line in Fig. 1). Then, an initial guess for the size and location of the three coolant flow passages was created by moving and scaling the holes. The inverse design technique was used iteratively to adjust their size and location in order to match the prescribed, 'exact' heat flux on the outer surface. The initial positions of 3 holes are given in Table 1 for the first two cases. The final configuration should converge to the configuration which was originally used to generate the 'exact' solution. The coordinates of the center and the radius of the holes in the 'exact' solution are given in Table 1. Two test cases with different initial configurations as stated in Table 1 were considered. The initial holes were placed asymmetrically in the strut in order to illustrate the robustness of this technique. Figures 2 and 3 show the sizes and locations of the three holes after each iteration. In each case, the final configuration reduced the error, $E(\rho)$, to approximately 0.5%, and the size and location of the final configuration agrees well with the exact solution. The error convergence histories of the two cases can be seen in Figure 4.

In the third test case, prescribed surface temperatures and desired heat flux distribution on the outer surface were chosen corresponding to a single, large, irregularly shaped coolant flow passage as shown in Figure 5. The thermal conductivities were the same as in the previous case. Four circular holes were used in the inverse design procedure in order to investigate the ability of this technique to position the four holes so as to match the flux distribution generated by a single large hole. The initial configuration of the four holes is given in Table 2. Again, the holes were initially placed asymmetrically in the strut in order to illustrate the ability of this technique to correctly position and size the holes. The final configuration shown in Figure 6 reduced the size of the error, $E(\rho)$, to 1.861% in 15 iterations. The specified and calculated heat fluxes as a function of position on the strut outer surface are given in Figure 7, and they agree very well.

CONCLUSION

A reliable inverse procedure for designing multiple coolant flow passages in composite, ceramically coated scram jet combustor struts has been developed. This is accomplished by coupling the direct boundary element method and a nonlinear constrained minimization technique. A specified heat flux distribution on the outer surface of the strut is iteratively approached while satisfying the prescribed temperature distributions on the outer surface of the strut and on the surfaces of the coolant flow passages by successively adjusting the sizes and locations of the coolant flow passages. Manufacturing constraints regarding the minimal allowable strut wall thickness and hole spacing are also incorporated into the model. This method can easily be modified in order to allow for noncircular coolant flow passages, multiple struts and for unsteady heating or cooling regimes.

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TABLE 1. Results for the First Two Cases: Using Linear BEM

CASE		Initial Guess x	y	r	Converged Solution x	y	r	Norm Error of $E(\underset{\sim}{\rho})$	No. of Iterations
1	Hole 1	-9.	0.1	0.1	-8.623	-0.006	0.229	0.554%	9
	Hole 2	-7.	0.2	0.1	-6.127	0.000	0.661		
	Hole 3	-3.	-1.	0.3	-1.013	-0.002	0.992		
2	Hole 1	-8.	0.2	0.1	-8.250	0.002	0.490	0.506%	9
	Hole 2	-5.	0.5	0.1	-4.976	-0.000	0.403		
	Hole 3	-2.	-1.	0.3	-0.930	-0.003	0.965		
					Exact Solution				
	Hole 1				-8.5	0.	0.3		
	Hole 2				-6.0	0.	0.6		
	Hole 3				-1.0	0.	1.0		

TABLE 2. Initial and Converged Circular Holes, Case 3

Hole	Initial Guess: Center x	Center y	Radius r	Converged Solution: Center x	Center y	Radius r	No. of Iterations
1	-8.5	0.1	0.1	-8.289	0.0003	0.2878	15
2	-7.0	0.2	0.1	-6.956	-0.0008	0.7816	
3	-4.5	-0.1	0.1	-4.517	0.0030	1.1802	
4	-1.5	-0.2	0.1	-1.605	0.0047	1.5893	

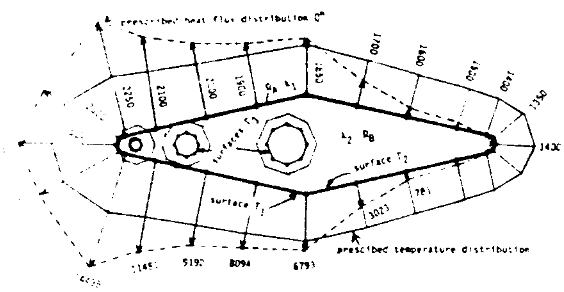

Figure 1 Discretized ceramically coated scram jet combustor strut with prescribed temperatures and outer surface heat flux chord length of the strut: 19. maximum thickness of the strut: 5.

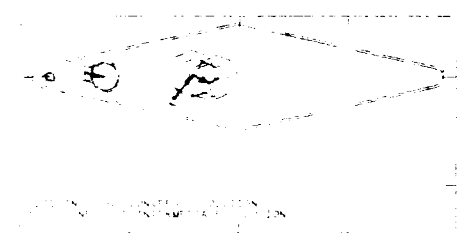

Figure 2 Iteration sequence of case 1 (norm error = 0.554%)

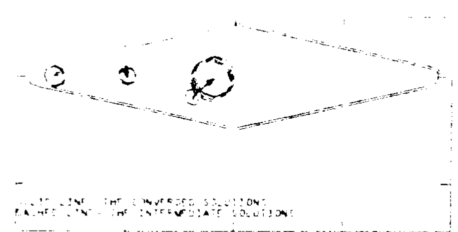

Figure 3 Iteration sequence of case 2 (norm error = 0.506%)

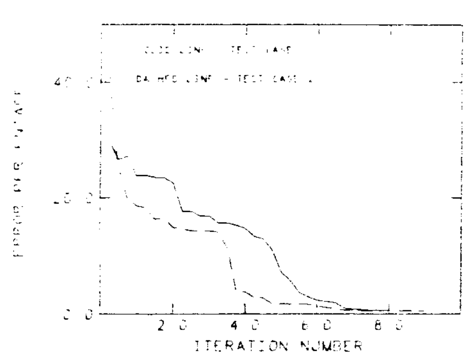

Figure 4 Error convergence histories

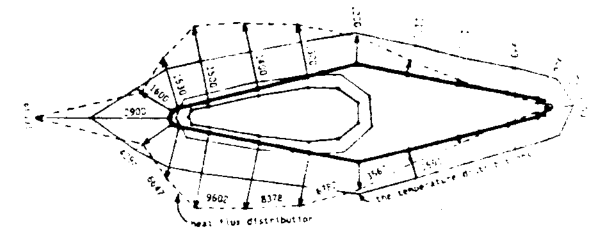

Figure 5 The prescribed temperatures and outer surface heat flux, case 3 chord length of the strut: 19. Maximum thickness of the strut: 5.

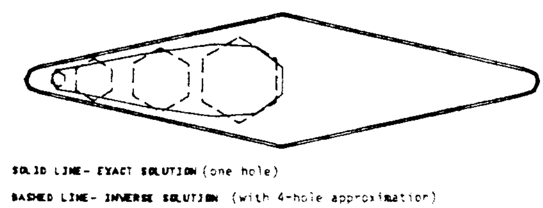

Figure 6 Case 3 (norm error = 1.861%)

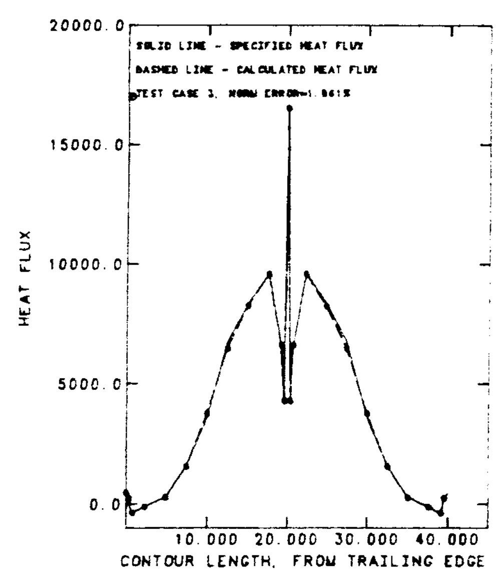

Figure 7 The prescribed and the calculated outer surface heat flux