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GSD28-Fortran Program for Analysis and Design of Shock-Free Transonic Airfoils and Turbomachinery Cascades Including Viscous/Inviscid Interaction

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ABSTRACT

The user-oriented computer program, GSD28, is applicable to aerodynamic analysis and shock-free redesign of existing two-dimensional transonic choked and unchoked cascades of airfoils. GSD28 can be used in two separate modes of operation: (1) as an analysis code for steady, transonic, shocked or shock-free choked or unchoked flows, (2) as a redesign code for shock-free unchoked cascades that uses fictitious-gas concept. GSD28 generates its own four-level, boundary-conforming computational grid of C-type. Viscous effects are included through an iterative viscous/inviscid interaction with a complete boundary layer/wake computer code.

The mathematical model used for the inviscid flow is a full-potential equation. Its artificially time-dependent version is solved in a fully conservative form by using a finite area technique; rotated, type-dependent differencing; successive line over relaxation; and consecutive grid refinement. Isentropic shocks are captured by using artificial viscosity or artificial density in a fully conservative form. Rankine-Hugoniot shocks can be captured by enforcing correct shock entropy jump conditions.

The shock-free design is performed by implementing fictitious-gas concept of elliptic continuation from subsonic into supersonic flow domains. Recomputation inside each supersonic zone is performed using method of characteristics in the rheograph plane involving isentropic gas relations. The new shock-free airfoil contour is determined from the condition that stream function must equal zero on the airfoil surface. Besides being capable of converting existing cascade shapes with multiple shocked supersonic regions into shock-free cascades, GSD28 can also unchoke previously choked cascades and make them shock free at the same time.

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INTRODUCTION

Transonic flow generally involves shocks, which are the main generators of vorticity and the aerodynamic drag force (wave drag) and the high aerodynamic noise level. When the shocks are strong enough, the shock - boundary layer interaction leads to boundary layer separation and potentially dangerous unsteady flow and mechanical vibrations.

Besides these general features, transonic flow, especially through the turbine cascades, is often characterized by the flow-choking phenomenon. Choking occurs when two neighboring airfoils in a cascade are connected by a single sonic line. Choking represents an upper limit on the mass flow through a given cascade. It inevitably occurs in accelerated turbine cascades with supersonic exit conditions.

Shock-free or shockless flow decelerates from supersonic speed to subsonic speed not discontinuously (through a shock) but smoothly over a finite distance (isentropic recompression). There are only a limited number of design techniques [1] for obtaining entirely shock-free cascade flow fields. The technique for designing shock-free cascade flows presented in this report is based on the fictitious-gas concept of Sobieczky [2] and the full-potential, steady, transonic cascade analysis codes of Dulikravich [3,4].

The present work has two main objectives. One is to eliminate all shocks from the flow field. The other objective is to unchoke already choked cascades. Both tasks are achieved by slightly reshaping ("shaving off") existing cascade and airfoil shapes without changing their stagger angle or gap-to-chord ratio.

INVISCID ANALYSIS

For the purpose of designing shock-free transonic flow fields Sobieczky developed the idea [2] of a fictitious-gas elliptic continuation. The method consists of three basic steps. The first step is an iterative determination of the shape of a sonic line bounding a supersonic shock-free flow domain. This is attained by utilizing non-physical (fictitious) analytic expressions for the fluid density and the speed of sound whenever the flow is locally supersonic. The second step of the design procedure recomputes the flow field inside each supersonic domain using now physical (isentropic) relations for the fluid density and the speed of sound. The third step then searches through each supersonic shock-free domain for the points where stream function has a zero value and thus determines a shock-free airfoil shape that differs from the input airfoil contour only along the sections wetted by the supersonic shock-free flow. Computer program GSD28 unifies all three steps of the design procedure.

The continuity equation for a steady, compressible flow normalized by the critical values of density and the speed of sound can be expressed in its canonical full-potential form as

$$(\rho u)_x + (\rho v)_y = \rho\,[(1-M^2)\phi_{ss} + \phi_{nn}] = 0 \qquad (1)$$

Here ρ is the normalized density, u and v are the normalized local velocity vector components along the x and y coordinate axes correspondingly, M is the local Mach number, s is the local streamline aligned coordinate and n is the coordinate, locally orthogonal on the streamline. When the flow is locally subsonic $(M < 1)$, equation (1) is nonlinear but elliptic. This means that the normalized mass flow (ρM_*) will continue to increase with the increasing of M until the critical flow conditions are met.

Once the flow field becomes locally supersonic ($M > 1$), equation (1) hanges its type and becomes nonlinear and hyperbolic. With the further increase of M, mass flow starts to decrease. Supersonic flows generally revert to subsonic conditions discontinuously through a shock wave (or an isentropic discontinuity in the case of a full-potential flow model). To prevent this from ever happening, each time the flow is locally supersonic ($M > 1$) a fictitious-gas relation should be introduced so that the governing equation (1) continues to retain its elliptic character.

Isentropic and fictitious-gas relations coincide only along the sonic line where $M = 1$. The limitations of the form of the otherwise arbitrary analytic expressions for the fictitious density and the fictitious speed of sound are summarized in ref. 4. The simplest form of the fictitious gas density relation involves general polytropic [5] low

$$\frac{\rho_f}{\rho_*} = \left[\frac{GAS+1}{2} - \frac{GAS-1}{2} M_*^2\right]^{\frac{1}{GAS-1}} \tag{2}$$

where $GAS = \gamma = \frac{Cp}{Cv}$ is the ratio of specific heats when the flow is locally subsonic, and $GAS < -1$ when the flow is locally supersonic. The corresponding fictitious speed of sound is then

$$\left(\frac{a_f}{a_*}\right)^2 = \frac{GAS+1}{2} - \frac{GAS-1}{2} M_*^2 \tag{3}$$

The parameter, GAS, has the purpose of making the fictitious gas more or less incompressible. This parameter directly determines how much longer and flatter the resulting supersonic shock-free flow region will be as compared to the original shocked supersonic zone.

Therefore, airfoil surface shock-free pressure distribution cannot be influenced in a detailed fashion, but only in a global way by different choices of parameter GAS.

After the iterative process for numerically solving the full-potential equation using fictitious gas relation in the supersonic flow regions has converged, the entire flow field is searched for the location of the shock-free sonic lines. GSD28 is capable of determining the shapes of two supersonic shock-free regions. Once the x^*,y^* coordinates of a sonic line enveloping the particular shock-free supersonic region are found by interpolation, values of the sonic velocity vector components are determined together with the corresponding values of the velocity potential function and then integrated along the sonic line. These data serve as initial data for the two-dimensional method of characteristics marching integration (recomputation) of the full-potential equation inside the supersonic zones, where now only isentropic relations are used.

With a very poor choice of fictitious gas parameter GAS it could happen that the mathematical domain of dependence is smaller than the physical supersonic zone. Such a case is signaled automatically as an error, and the further execution of the GSD28 code is terminated. If the limit lines occur inside the supersonic flow domain, the entire calculation should be repeated with a different (usually smaller negative) value of the input parameter GAS. Coordinates of the partially new shape of the shock-free airfoil are determined from the condition that the stream function must have constant value everywhere on the surface of the airfoil.

VISCOUS ANALYSIS

The boundary layer is assumed to be divided into laminar, transitional and turbulent regions. The wake is assumed to be turbulent. Integral or finite-difference methods can be used in either the direct mode with the inviscid velocity on the airfoil or wake centerline specified or in the inverse mode with the mass flux defect Q specified. Here $Q = \rho_{inv} u_{inv} \delta^*$, where ρ_{inv}, u_{inv} are the inviscid density and speed on the airfoil or wake centerline, respectively, and δ^* is the displacement thickness.

First the integral method will be described. The boundary layer near the leading edge is assumed to be laminar. It is computed in the direct mode with a modified version of a compressible Thwaites method [6]. A specified transition point (no transition region) can be enforced or the transition region can be computed by the empirical method of Abu-Ghannam and Shaw [7] modified for compressibility. If laminar separation or a shock occurs inside the transition region, transition to turbulent flow is enforced at that point. The turbulent boundary layer and wake is computed in either the direct or inverse mode by the method of Green et al. [8] modified by East et al. [9]. Surface transpiration effects were included. Turbulent separated flow can be calculated. The boundary layer and wake on the upper and lower sides of the airfoil and wake centerline are computed separately.

The laminar and transitional regions can also be comnputed in the direct or inverse mode with a modified version of the finite-difference code of Drela [10]. The scaling for the normal coordinate was changed to that of Drela and Thompkins [11]. Surface transpiration effects were incorporated. The intermittency factor of Abu-Ghannam and Shaw [7] is used in the transitional region. The inner eddy viscosity was modified [12] so that both laminar and transitional thin separated flow regions can be calculated. When the flow becomes fully turbulent the turbulent integral method is initiated.

The transpiration coupling concept [13] boundary layer and wake into the inviscid code. This involves computing an equivalent transpiration velocity. The advantage of this approach is that the inviscid grid only has to be generated once, if the wake centerline and airfoil coordinates do not change. Wake curvature effects due to the jump in the tangential velocity component along the wake centerline are also included [13].

The semi-inverse method is applied to successively iterate between the viscous and inviscid calculations. In this approach part or all of the boundary layer code can be operated in the inverse mode. Either Carter's [14] or Wigton's [15] formulas to update the mass flux defect can be used. An initial guess for the mass flux defect must be supplied.

The passive effects of a perforated airfoil surface with a cavity or plenum chamber located underneath can also be simulated [16]. The total mass flux through the porous region is equal to zero. Darcy's law is used to determine the physical transpiration velocities

$$v_w = \sigma\,(p_p - p_w)$$

$$\sigma = \bar{\sigma}/(\rho_\infty\, u_\infty)$$

where v_w is the physical transpiration velocity, p_w is the airfoil surface pressure, p_p is the plenum pressure, σ is the permeability factor, $\bar{\sigma}$ is the nondimensional permeability factor and ρ_∞, u_∞ are the upstream density and speed, respectively. A value of $\bar{\sigma} = 0.6$ corresponds to a geometrical porosity of about 10 percent [17]. It is assumed that the density of the transpired fluid is the adiabatic wall density. This report represents a complimentary publication resulting from the work of Olling and Dulikravich [18] and the final successful work of Olling [19].

COMPUTER PROGRAM

General Description

Computer program GSD28 was developed with the intention to create a single computer program that can operate either as an analysis code or as a shock-free redesign code for transonic isolated airfoil and cascade flows.

Computer program GSD28 consists of the MAIN routine and subroutines. Subroutine MAIN reads input data and generates computational C-type grid by calling subroutines GRIDC, SPLIF, INTPL, XYINF. The grid is generated very efficiently by utilizing conformal mapping and nonorthogonal coordinate stretching and shearing transformations. GSD28 can generate automatically a maximum of four consecutively refined grids. Subroutine XYINF modifies the positions of the grid points at upstream and downstream infinity and generates x,y coordinates of imaginary grid points and periodic grid points. If the user already has the computational grid, this separate grid can be read in and the entire grid generation procedure in GSD28 easily bypassed.

Subroutine ANALYS performs an iterative line overrelaxation of the continuity equation in its conservative form. ANALYS generates coefficients of the three-diagonal correction-to-the-potential matrix obtained from the artificially time-dependent form of the full-potential equation. The vector of residuals is evaluated by applying a finite area technique to equation (1). When executing GSD28 in analysis mode, isentropic relations are used everywhere. ANALYS then captures all possible existing shocks (isentropic discontinuities) by performing type-dependent, rotated finite differencing.

Boundary and periodicity conditions are applied explicitly in subroutine BOUND. When executing GSD28 in shock-free design mode, isentropic-gas relations are used at all the grid points where the flow is subsonic. At the points where $\dot{M} > 1$, subroutine ANALYS uses the fictitious-gas relations for fluid density and the speed of sound. In such a way the full-potential equation is prevented from ever becoming locally hyperbolic, and only the shock-free supersonic zones are created. Subroutine SHAPE searches for the existence of shock free supersonic zones and determines the values, of x^*, y^*, θ^*, ϕ^*, and ψ^* along the sonic lines. SONIC can perform this task on two separate supersonic zones on a one-by-one basis starting at the trailing edge and moving clockwise around the airfoil. Subroutine SONIC searches for the values of x_s and y_s where $\psi = 0$. These are the new coordinates of the shock-free airfoil in the intervals covered by the supersonic flow. SHAPE also searches for the existence of possible limit lines inside the supersonic zones.

After the fictitious-gas, shock-free redesign procedure is finished, the GSD28 program proceeds with the closing subroutine RESULT. RESULT calculates and prints values of surface Mach number, critical local Mach number, surface fluid density, surface fluid temperature, and the coefficient of surface pressure. RESULT also plots (with the line printer) a rough plot of the Mach number distributions on the surface of the airfoil and the trailing gridline. If the iterative calculation process is to be repeated on one or more additionally refined grids, subroutine RESULT is circumvented until the iterative process is completed on the finest specified grid. During each grid refinement, GSD28 automatically doubles the number of grid cells in each computational direction and interpolates values of the potential function obtained on the previous coarser grid onto the new grid. Thus, an improved initial guess is automatically created for each potential field calculation on the refined grid.

BOUNDARY LAYER/WAKE CODE

The boundary-layer code consists of the following subroutines.

Subroutine VISCO is the first subroutine and is called by subroutine MAIN of the inviscid code. It controls the branching to subroutines BL and BC and prints out the boundary-layer parameters computed in subroutine BL.

Subroutine BL controls the calculation of the boundary layer and wake by either integral or finite-difference methods. If the integral method is to be used the following three subroutines are called. Subroutine ROTT computes the laminar boundary layer by a modified compressible Thwaites method [6], subroutine STRAN computes the transitional boundary layer by the method of Abu-Ghannam and Shaw [7] and subroutine RUNGE4 computes the turbulent boundary and wake by the method of Green et al. [8] modified by East et al. [9] with a 4th order Runge-Kutta integrator. Alternatively, subroutine SHBOX computes the laminar and transitional regions by a modified version of the finite-difference code of Drela [10] utilizing a shifted box method.

Subroutine BC calculates and prints out the updated values of the mass flux defect to be used in the next viscous-inviscid interaction cycle by either the method of Carter [14] or Wigton [15]. It also computes and prints out the boundary conditions to be imposed on the inviscid code. These are the equivalent transpiration velocity on the airfoil and the jumps in the normal and tangential velocity components along the wake centerline. Subroutine WAKE, which is called by subroutine MAIN, converts these jump conditions along the wake centerline into the boundary conditions in the computational space of the inviscid code.

INPUT

The entire input is written and read in an unformatted form. The first card of the input data set (see examples on figs. 1 and 2) contains an arbitrary text with up to 80 arbitrary characters describing the airfoil or the test case. The following input lines contain:

NX	number of grid cells to be used on the surface of airfoil when using first (coarse) grid. The number of grid cells must be an even integer. The suggested minimum value is NX = 20.
NW	number of grid cells on the wake, that is, number of the grid cells to be used on the first (coarse) grid between trailing edge and the downstream cut-off boundary. The suggested value is NW = 5 + XEX/2.

NY — number of C- layers of grid cells enveloping airfoil, that is, half the number of grid cells between two neighboring airfoils to be used on the first (coarse) grid. The suggested value is NY = 4. * ln (GTC + 4.).

NP — number of input points on surface of the airfoil. Points must be numbered in a clockwise direction starting at the lower trailing edge and ending at the upper trailing edge.

I1,I2,I3,I4 — maximum number of iterations (ITRMAX) on each of the four grids, respectively. These parameters are important because they serve as a convergence criterion in the case of nonlifting flows. The suggested values that will provide results with engineering accuracy in the case of shocked flows are

I1 = 150

I2 = 100

I3 = 150

I4 = 50

If only one grid is to be used, then I2 = I3 = I4 = 0. If only two grids are to be used, then I3 = I4 = 0. If only three grids are to be used, then I4 = 0.

NS — number of computational cells (length intervals) that every existing sonic line will automatically be divided into in the case that GSD28 executes as a shock-free design code. Suggested value is NS = 160, although the maximum value with the present array sizes is NS = 242.

MP — boundary identification parameter. MP = 0 means that the surface of the airfoil and the wake is treated as a solid boundary. Also, when MP = 0, upper and lower boundaries are treated as solid walls. MP = 1 means that only airfoil and wake surfaces are treated as having been transpired. MP = 2 means that only tunnel walls are treated as having been transpired. MP = 3 means that all boundary surfaces (airfoil, wake, and tunnel walls) are treated as having been transpired.

XS1,YS1 — (x',y') coordinates of leading edge reference point, length. The location of this reference point is arbitrary but it must lie on a straight line from which the stagger (or twist) angle of the cascade is measured (see fig. 4).

XS2,YS2 — (x',y') coordinates of trailing edge reference point, length. Note that (x',y') input coordinate system does not have to coincide (fig. 4) with the physical (cascade) coordinate system (x,y).

EM1 — free stream Mach number at upstream infinity.

EM2 — free stream Mach number at exit boundary. EM2 = 0 means that the actual exit Mach number is unknown and will be automatically iteratively determined by the computer program GSD28. EM2 > 0.0001 means that the imputed value of EM2 will be actually enforced by GSD28.

GTC — gap-chord ratio, h/c (fig. 5). The iterative determination of certain parameters in the grid-generating routine might fail for small values of GTC because of the computer-dependent accuracy. Therefore, it is suggested that GSD28 be used for cascades with GTC > 0.55. On the other hand, GTC can have values as high as GTC = 100 thus simulating free-air conditions.

BET — cascade stagger angle, (in degrees) between x-axis and airfoil chord line (fig. 5). Chord line is defined as a line passing through the points originally defined by (XS1,YS1) and (XS2,YS2).

AL1,AL2 — angles, α_1 and α_2 (in degrees) between x-axis and free stream at up- and-downstream infinity, respectively (fig. 5).

MBC — index indicating type of the boundary conditions. MBC = 0 means that cascade periodic boundary conditions and the imputted value of the exit flow angle AL2 will be enforced. MBC = 1 means that cascade periodic boundary conditions will be enforced, but the inputted value of AL2 will not be used. Instead, trailing edge Kutta-Zhukovski condition will be enforced and the corresponding value of AL2 will be iteratively determined. MBC = 2 means that isolated airfoil far field boundary conditions will be enforced including Prandtl-Glauert compressibility correction. MBC = 3 means that the upper and the lower boundary will be treated not as periodic boundaries, but as tunnel walls and that the inputted value of AL2 will be used. MBC = 4 means that the upper and lower walls will be treated as tunnel walls and that inputted value of AL2 will not be used. Instead, Kutta-Zhukovski trailing edge condition will be used and the corresponding value of AL2 will be iteratively found.

MXY grid generation index. MXY = 0 means that computer program GSD28 should automatically generate and use its own computational grids. MXY = 1 means that a separately generated and stored grid should be read in by GSD28 in formatted form. MXY = 2 means that the reading of the separate grid should be in unformatted form. The separate grid can be generated using any technique available. MXY = 3 means that the code GSD27 was used to generate the separate grid. Nevertheless, the grid must be of C- type. Grid points must be numbered in a clockwise direction, starting with I = 2 at the exit boundary and J = 2 at the airfoil/wake surface.

MPF index for the initial guess used for the reduced potential function value at each grid point. MPF = 0 means that GSD28 will automatically use zero value of the reduced potential function as an initial guess at each grid point. MPF = 1 means that GSD28 will read (in unformatted form) the entire reduced potential field that was automatically generated and stored during some previous run. Considerable savings could be achieved in the cases where a particular airfoil or a cascade is analyzed (or redesigned) for a range of, say, inlet Mach numbers, angles of attack, gap-to-chord ratios etc. In such a situation, only the first run should be done by using MPF = 0 and a sequence of automatically refined computational grids. Each next increment in, say, free stream Mach number can be then tested by using MPF = 1 and only one (the last previous) grid.

GAS — fictitious gas constant, that is, the value of parameter GAS in eq. 2 and eq.3. If GAS = GAM, the program GSD28 executes as an analysis code. If, GAS < -1 , the same program will automatically execute as a shock-free design code. Suggested value for GAS in the case of a shock-free mode of GSD28 is GSD = -10.

GAM — ratio of specific heats of the working fluid. For diatomic gases GAM = 1.4. For monoatomic gases GAM = 1.666 .

RLX — overrelaxation factor on first grid for the case of locally subsonic flow. The suggested value is RLX = 1.70. The maximum value must be less than 2. The value of RLX will be automatically increased by 4 percent on each consecutively finer grid. In the locally supersonic flow, the relaxation factor will automatically be set to RLX = 2.

XEX — length of the wake expressed as the number of chord lengths between the trailing edge and the exit boundary. Suggested value is XEX = 2 + 0.3 * GTC * COS(AL1).

XX — stretching (clustering) parameter for grid points to be generated on the airfoil surface. IF XX = 0.0 all surface grid points will be equidistantly distributed with respect to the surface coordinate (arc length). If XX > 0.0 , the surface grid points will be symmetrically clustered towards the leading edge and the trailing edge. Suggested value for cascades is XX < 0.08 and the maximum value is XX = 0.14 .

IAVIS — index for artificial viscosity. IAVIS = 0 means that artificial density will be used. IAVIS = 1 means that first-order artificial viscosity will be used. IAVIS = 2 means that second-order artificial viscosity will be used. The suggested value is IAVIS = 1.

IADEN — index for artificial density. IADEN = 0 means that artificial viscosity will be used. IADEN = 1 means that first-order artificial density will be used. IADEN = 2 means that second-order artificial density will be used. The suggested value is IADEN = 0.

IENTRO — index for entropy correction at the shock. IENTRO = 0 means that no entropy correction at the shock will be made. IENTRO = 1 means that an entropy correction at the shock will be made. The suggested value is IENTRO = 0.

ISENTRO — the number of inviscid iterations after with which the entropy correction at the shock will be applied when computing on the first grid, when IENTRO = 1. The suggested value is ISENTRO = 10.

SIGMAXU — maximum value of the nondimensional permeability factor $\bar{\sigma}$ on the upper side of the airfoil. The suggested range is 0 to 0.6.

XSU — the chordwise coordinate x/c of location of the maximum nondimensional permeability factor on the upper side of the airfoil.

X1U — the chordwise coordinate x/c of the beginning of the porous region on the upper side of the airfoil.

X2U — the chordwise coordinate x/c of the end of the porous region on the upper side of the airfoil.

SIGMAXL — maximum value of the nondimensional permeability factor $\bar{\sigma}$ on the lower side of the airfoil. The suggested range is 0 to 0.6.

XSL — the chordwise coordinate x/c of the location of the maximum nondimensional permeability factor on the lower side of the airfoil.

X1L — the chordwise coordinate x/c of the beginning of the porous region on the lower side of the airfoil.

X2L — the chordwise coordinate x/c of the end of the porous region on the lower side of the airfoil.

IPF — index for the permeability factor. IPF = 0 means that the airfoil is solid. IPF = 1 means that the nondimensional permeability factor $\bar{\sigma}$ is uniform. IPF = 2 means that the permeability factor is zero at the ends of the porous region and has a maximum inside the porous region.

INTRMAX — the maximum number of viscous-inviscid interaction cycles to be performed. If INTRMAX = 1 only a pure inviscid calculation is made.

ITRMAX1 — the number of inviscid sweeps to be performed for each viscous sweep. The suggested range is 1 to 5. For cases with well-defined shock-induced separation the suggested value is ITRMAX1=1.

ISTART1 — index for restarting the viscous-inviscid interaction calculations. Suppose a certain number of viscous-inviscid interaction cycles have been completed and after examining the output it is determined that more interaction cycles would be desirable. ISTART1=1 means that the calculation is to resume where it was previously terminated. ISTART1=0 means that the calculation is not restarting from previous calculations. If ISTART1=1 then also set MXY=2 and MPF=1.

MIM1 — number of iterations of the inviscid flow solver during the viscous-inviscid interaction during which the inviscid relaxation factor is equal to unity. MIM1=0 means that the relaxation factor will be larger than unity. The suggested value is MIM1=0.

INW — index for wake displacement and wake curvature effects. INW=0 means that there are no wake displacement and wake curvature effects. INW=1 means that there is only wake displacement. INW=2 means that there are both wake displacement and wake curvature effects. The suggested value is INW=2.

NPR — the number of viscous-inviscid interaction cycles after which the output will be saved. The suggested value is NPR=10.

VCON — the viscous-inviscid velocity convergence criterion. The velocity error measure is $u_{vis}/u_{inv}-1$, where u_{vis} is the viscous velocity at the edge of the boundary layer and u_{inv} is the inviscid velocity on the airfoil or wake centerline. When the error measure is less than VCON for all points the calculation terminates. The suggested value is VCON=0.01.

INV — index for starting the inverse mode for the boundary layer code on the initial sweep. INV=0 means that the boundary layer code will compute in the direct mode until separation is encountered, at which point the inverse mode is initiated. INV=1 means that on the upper surface the inverse mode will be switched on when x/c=XU and on the lower surface the direct mode will be used until separation is encountered and then the inverse mode will be used. INV=2 means that on the upper surface the inverse mode will be switched on when x/c=XU and on the lower surface the inverse mode will be switched on when x/c=XL.

ISP — index for the initial value of the mass flux defect. ISP=1 means that the mass flux defect will be computed using specified initial values of the displacement thickness δ^*/c and the inviscid velocity and density.

ISP=2 means that the initial values of the mass flux defect are specified in the input.

XU	chordwise coordinate x/c on the upper side of the airfoil where the boundary layer code will switch to the inverse mode.
XL	chordwise coordinate x/c on the lower side of the airfoil where the boundary layer code will switch to the inverse mode.
T1	static temperature (K) at upstream infinity.
CH	chord length (m)
ORLX	relaxation factor for Wigton's update method. The suggested value is ORLX=1.0.
QRLX	relaxation factor for Carter's update method. The suggested value is QRLX=0.5.
ISI	index for the method to update the mass flux defect calculated by the turbulent integral method. ISI=1 means that Carter's method is used. ISI=2 means that Wigton's method is used. Carter's method is used during calculations by the finite-difference boundary-layer method.
RAD	nose radius normalized by the chord length.
TAU	turbulence level (%) at upstream infinity.
ITRAN	index for the transition calculation. ITRAN=1 means that a natural transition calculation will be made. ITRAN=2 means that transition will be enforced at a specified point (no transition region). ITRAN=3 means that transition will be enforced at a specified point

on the upper surface but natural transition will occur on the lower surface.

REI — Reynolds number based on the chord length at upstream infinity.

ILAM — index for laminar boundary layer calculation. ILAM=1 means that a compressible Thwaites method will be used. ILAM=2 means that the finite difference method will be used.

XTRANI — chordwise location x/c for enforced transition on the upper side of the airfoil.

XTRAN2 — chordwise location x/c for enforced transition on the lower side of the airfoil.

EPS — convergence criteria for the finite-difference boundary-layer code. The suggested value is EPS=10^{-3}.

ITMAX — maximum number of iterations allowed at each station for the finite-difference boundary-layer code. The suggested value is ITMAX=30.

NSTR — index for streamwise output from the finite-difference boundary-layer code in file STR.DAT. NSTR=0 means that no streamwise data will be saved. NSTR=1 means that streamwise data will be saved at every NPR-th viscous-inviscid interaction cycle.

NPFL — index for boundary layer velocity profile output from the finite-difference boundary-layer code in file PFL.DAT. NPFL=0 means that no profile data will be saved. NPFL=1 means that profile data will be saved at every NPR-th viscous-inviscid interaction cycle.

PR	Prandtl number for the finite-difference boundary-layer code. The suggested value is PR=0.72.
PRT	turbulent Prandtl number for the finite-difference boundary- layer code. The suggested calue is PRT=1.0
PRAP	pressure gradient parameter $(x/u_e)\ du_e/dx$ at the leading edge for the finite-difference boundary-layer code. The suggested value is PPAR=1.0.
UGUESS	initial edge velocity guess at the leading edge for the finite-difference boundary-layer code. The suggested value is UGUESS = 0.0001.
JJ	number of grid cells normal to the airfoil for the finite-difference boundary-layer code. The suggested value is JJ=30.
GESO	geometric grid stretching constant in the finite-difference boundary-layer code. The suggested value is GEO=1.2
ETAE	the edge value of the nondimensional normal coordinate in the finite-difference boundary-layer code. The suggested value is ETAE=14.

The x' and y' coordinates of input points on the surface of an airfoil or the cascade are given on the following input cards. These coordinates will be automatically scaled with respect to the airfoil chord length. The input coordinate system (x',y') could be arbitrarily positioned with respect to the airfoil (fig. 4). Translation and rotation into (x,y) coordinate system will be performed automatically.

Note that the number of the input points on the airfoil lower surface does not have to be the same as the number of the input points on its upper surface. The input points are numbered in the clockwise direction starting from the trailing-edge point. The numbering must end with the same trailing-edge point. The airfoil (x',y') coordinates should be printed on input cards in such a way that x' coordinates of all the input points are given in the first column, followed by the corresponding y' coordinates in the second column.

The initial guess for the displacement thickness normalized by the chord length or mass flux defect (with dimensions of kg/ms)) is now ready to be inputted. First a line of four indices and two scaling parameters is given. The first number is the index of the trailing edge on the upper side of the airfoil (the leading edge has the index unity). The next number is the index of the end of the wake on the upper side of the wake centerline. The next two numbers are the corresponding indices on the lower side of the airfoil and wake centerline. The next two numbers are the scaling factors for the displacement thickness or mass flux defect on the upper and lower sides of the airfoil and wake centerline, respectively. Then in two columns are inputted first the chordwise coordinate normalized by the chord length and then the corresponding displacement thickness normalized by the chord or the mass flux defect, starting with the leading edge on the upper side of the airfoil and continuing to the end of the wake. After that the corresponding columns for the lower side of the airfoil and wake centerline are inputted.

OUTPUT

It starts with an echo printout of the input data followed by the values of the flow parameters at the downstream infinity. They are obtained from an iterative procedure involving the input flow parameters and the global mass conservation principle.

The iterative process of GSD28 can be monitored by analyzing the following parameters, which are printed after each complete iterative sweep through the flow field. These parameters are:

ITER	number of iteration sweeps just completed
IR,JR	(X,Y) index coordinates of a grid point where residual had largest absolute value during the last iterative sweep.
MAX RESIDUE	maximum residue (with artificial dissipation included) in the flow field during the last iterative sweep. Its location is at the point (IR,JR).
IC,JC	(X,Y) index coordinates of a grid point where correction to the reduced potential had maximum absolute value during the last iterative sweep.
MAX. CORRECT.	maximum value of calculated correction to the reduced potential during the last sweep. This correction was introduced at the point (IC,JC).
RELAX	absolute value of relaxation factor RLX used in the inviscid solver during the last iteration sweep.
CIRCULATION	value of circulation after the last sweep completed.
NSUP	total number of supersonic points in the flow field after the last sweep completed.
CHOKE	choked flow warning parameter showing if the flow was choked during the last iterative sweep.
ISTG	index number (X coordinate) of the grid point closest

to the leading-edge stagnation point. From that point the next iteration sweep will start proceeding along the airfoil suction surface to the exit boundary and then again from ISTG along the pressure surface to the exit boundary. In such a way the problems of marching upstream in the locally supersonic flow and the consequent introduction of negative artificial viscosity are avoided.

When the absolute value of the normalized convergence rate becomes smaller than 1×10^{-7} the iterative process on that particular grid will terminate. When the flow is nonlifting the iterative process on each grid will terminate after ITER = ITRMAX on that particular grid.

If I2 is given a different value from I2 = 0 in the input data, GSD28 will automatically refine the first grid so that the new (second) grid will have roughly twice as many grid cells in the X and Y directions as the previous grid. The printout will continue with a listing of ITER, IR, JR, etc., on that new grid and so on (if I3 is non-zero and/or I4 is non-zero).

Finally, when the last iteration sweep is completed on the last specified grid, the printout will conclude with the printer plot of the Mach number distribution (x10) in the entire flow field. The Mach number chart is followed by the listing of airfoil/wake surface flow parameter:

X/C	x coordinate of airfoil/wake surface point normalized with chord
X,Y	(x,y) coordinates of points on airfoil surface.
U/A*,V/A*	local values of x-component and of

y-component of the critical Mach number.

CP — coefficient of surface pressure

$$CP = \frac{p - p_1}{\frac{1}{2}\rho_1 q_1^2}$$

T/T* — local fluid absolute temperature divided with the critical temperature assuming isentropic process.

D/D* — local fluid density divided with critical density assuming isentropic process.

MACH — local fluid speed divided with the local speed of sound (Mach number).

MACH* — local fluid speed divided with the critical speed of sound (critical Mach number).

Output from GSD28 continues with a printer-plot of the surface Mach number distribution. GSD28 does not incorporate any actual computer graphics. GSD28 automatically continues the output by writing X/C, X, Y, CP, T/T*, D/D*, MACH, MACH* on a tape. These values will be written for all the wake points and airfoil surface points (I=2 until I=MAXX). The WRITE statement can be found in the subroutine RESULT.

When INTRMAX > 1 viscous-inviscid interaction will occur. First the solution of the boundary layer parameters for the upper side of the airfoil and wake centerline and then those for the lower side will be printed out. These parameters include

X/C — chordwise coordinate normalized by the chord length.

UVIS — viscous velocity at the edge of the boundary layer/wake normalized by the critical speed of sound.

UINV — inviscid velocity on the airfoil and wake centerline normalized by the critical speed of sound.

ERROR — velocity error measure $u_{vis}/u_{inv}-1$.

THETA/C — momentum thickness θ normalized by the chord length.

DEL*/C — displacement thickness δ^* normalized by the chord length.

H — shape factor δ^*/θ.

CF — skin friction coefficient

Q — mass flux defect $Q=\rho_{inv}u_{inv}\delta^*$ with dimensional units kg/(m s)

After the boundary layer parameter are printed out the mass flux defect is updated for the next viscous-inviscid interaction cycle. These updated Q values are next printed out along with the associated displacement thickness DEL*/C and the following parameters.

VBL — equivalent transpiration velocity normalized by the critical speed of sound

CK — correction to the pressure coefficient C_p due to wake curvature effects

SLOPE — slope of the displacement thickness surface plus airfoil/wake centerline surface

DY/DX — slope of the airfoil/wake centerline surface

After the boundary layer and wake on both the upper and lower sides of the airfoil and wake centerline have been calculated, the following jumps in the velocity components along the wake centerline to be imposed on the inviscid code are printed out starting with the trailing edge

DELV	jump in the normal velocity component normalized by the critical speed of sound.
DELU	jump in the tangential velocity component normalized by the critical speed of sound

ASSUMPTIONS, LIMITATIONS, AND APPLICABILITY OF COMPUTER PROGRAM GSD28

Computer program GSD28 is directly applicable for the analysis of transonic, shocked, steady, inviscid, and irrotational flow through two-dimensional, static cascades of given airfoils. GSD28 is also directly applicable for the redesign (determination of coordinates of the points defining an airfoil shape) of shock-free airfoils for transonic, steady, inviscid and irrotational flow. It should be remembered that in both of these applications the identical program GSD28 is used. Only the inputs to GSD28 in these two operation modes vary. The difference in the inputs is confined to a value of a single input parameter, GAS.

In its analysis mode GSD28 can predict flow fields ranging in local speeds from incompressible through transonic, including the possible isentropic discontinuities in the solution of the full-potential equation. These isentropic discontinuities do not represent physical shock waves. This suggests that all possible aerodynamic shocks must be weak (Mach number just ahead of the discontinuity should be $M < 1.3$) so that the entropy (actually vorticity) generated by them is negligible. In order to get better agreement with the solution of Euler equations of gasdynamics, entropy correction can be automatically added at the shock. GSD28 can also calculate the boundary layer/wake in an automatic viscous/inviscid iterative coupling loop if specified so in the input. Cascades of closely spaced, highly cambered, thick blades cannot be always handled by the present GSD28 grid-generation technique.

On the other hand, GSD28 can be successfully used for the analysis and shockless design of isolated airfoils by giving the gap-chord input parameter a value of up to GTC = 100.

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