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USER'S MANUAL

WBCTG31 - FORTRAN PROGRAM FOR EFFICIENT THREE-DIMENSIONAL COMPUTATIONAL GRID GENERATION FOR WING-BODY-CANARD-TAIL REALISTIC AIRCRAFT CONFIGURATIONS

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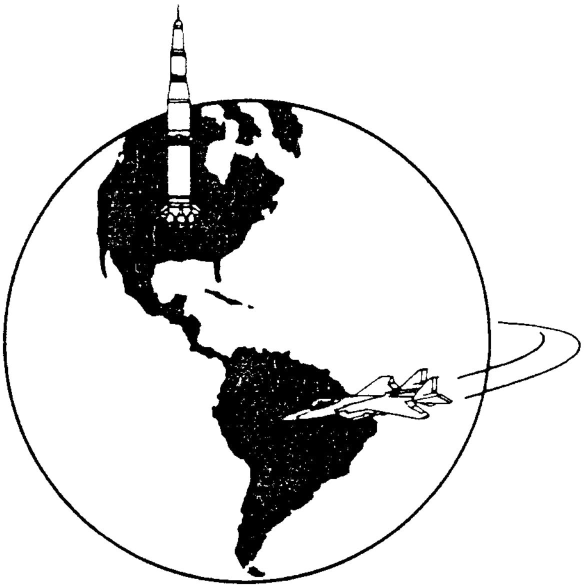

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ABSTRACT

A fast computer program has been developed for accurately generating boundary conforming, three-dimensional computational grids for realistic aircraft configuarations. The code can successfully treat configurations with an arbitrarily shaped and positioned wing, horizontal tail and vertical tail attached to an arbitrarily shaped fuselage. The grid generation technique is based on a conformal transformation and nonorthogonal coordinate stretching and shearing. The physical space is first stretched radially so that the fuselage becomes a circular cylinder. After finding the intersection contours of the wing and horizontal and vertical tails with the co-axial cylindrical cutting surfaces, these surfaces are unwrapped onto two-dimensional planes. A two-dimensional computational grid is generated on each of these planes by way of conformal mapping. Each two-dimensional grid is then transformed back to physical space to form a separate three-dimensional computational surface. A series of these congruent surfaces results in a three-dimensional boundary conforming computational grid which is C-type about the leading wing and of an H-type about the trailing lifting surfaces. Clustering of the surfaces is possible in the spanwise direction so that better resolution near the surface of the fuselage, tips of the lifting surfaces, or outer boundary can be achieved. Grid clustering is possible near all leading and trailing edges and at selected points along the fuselage. The shape

of the outer boundary can be prescribed making the grids suitable for aircraft-in-wind-tunnel applications.

APPLICABILITY OF THE COMPUTER PROGRAM

The program is capable of generating a boundary-conforming grid for wing-body-tail configurations with the following characteristics:

(1) Arbitrarily shaped lifting surfaces arbitrarily positioned on the fuselage. The lifting surfaces can have arbitrary sweep and dihedral angles with any type of airfoil sections.

(2) Arbirarily shaped fuselages.

(3) Vertical tail need not be on the fuselage centerline.

(4) Up to three lifting surfaces may be treated.

(5) The outer boundary can be arbitrary in shape, but must be constant along the length of the aircraft.

(6) The program can also produce grids for isolated wings.

GRID GENERATION CONCEPT

The coordinate system used is as follows. The x-coordinate direction is the centerline of the fuselage with the positive direction running from the nose to the tail (fig. 1). The y-coordinate direction is in the spanwise direction, while the z-coordinate direction points downward from the fuselage centerline.

The first step in the generation of the three-demensional computational grid is to radially shear the physical geometry so that the fuselage becomes a circular cylinder. This means that at each x=constant station along the aircraft axis, the lifting surfaces are radially sheared and stretched. It should be noted that in the program, the input points that define the fuselage are not actually changed, only those points that define the lifting surfaces are changed. This radial coordinate normalization depends on the local radius of the fuselage and the local radius of the outer boundary corresponding to the angular position of the point in question. The resulting geometry consists of a distorted wing and horizontal and vertical tail mounted on a circular cylinder shaped fuselage with the outer boundary also becoming a circular cylinder.

The next step is to find the intersection between the cutting (or computational) surfaces and the distorted lifting surfaces. The cutting surfaces are coaxial circular cylinders varying in radius from the surface of the fuselage to the outer boundary (wind tunnel wall) surface. Therefore, the problem of finding the intersection contours of the cylindrical cutting surfaces with the distorted geometry of the lifting surfaces becomes the problem of finding the intersection of a circle (the cutting surface) and a spanwise cubic spline that represents the shape of the distorted lifting surface. Also, the spanwise cubic spline is a function of three space variables, but it is considered to lie only in one plane since the cutting surfaces do not vary with x. The intersection point of the two curves is determined using Newton's iteration scheme. The initial guess is choosen as the intersection of the straight line passing through the pair of neighboring points in the spanwise direction, and the circle. The Newton iteration requires only a few iterations to converge. After the y and z coordinates of the intersection point have been determined, the corresponding x coordinate is found by cubic spline interpolation in the x-direction.

Since each lifting surface is represented by a number of spanwise splines, this process is repeated for each spanwise spline and for each cylindrical surface. The cutting surfaces containing the intersection contours are then unwrapped into a $(X,R\theta)$ plane to form the input for the two-dimensional grid generation scheme. Once the grid has been generated in each two-dimensional unwrapped plane, the final three-dimensional grid is formed by re-wrapping and re-stretching the cutting surfaces into a series of congruent computational surfaces.

The two-dimensional grid generation is performed as follows. After relocating and clustering the input points near the leading and trailing edges of the airfoil contours, points on the wake are generated. Two horizontal boundaries which correspond to the intersection of each cutting surface and the aircraft plane of symmetry are then generated. Along both upper and lower boundaries the points are clustered at desired locations. The clustering of these points can be performed automatically along the fuselage where the fuselage has the highest curvature. The location of the clustering can also be specified by the user.

Each unwrapped cylindrical cutting surface is then non-orthogonally sheared and stretched in the vertical direction to a width of two pi. The resulting uniform strip containing the airfoil contours and wakes is then conformally mapped to the computational plane with the leading contour and accompanying wake becoming the curved upper boundary (fig. 2). The upper and lower walls of the strip plane become the straight lower boundary of the computational plane, while the trailing airfoil contours are mapped to the left and right-hand sides of the strip plane between the upper and lower boundaries.

The corresponding points along the boundaries of the computational plane are connected by a combination of linear and sine functions. Grid lines that are emanating from points on the lower boundary will have to be connected to points on the trailing contours and wakes. Separate grid lines will have to connect the trailing airfoil contours and wakes and the upper boundary. The resulting grid is mapped back to the uniform strip plane and after restretching becomes the grid in figure 3. This figure can be recognized as a plot of a two-dimensional grid on an unwrapped three-dimensional cylindrical surface intersecting a wing-body-tail configuration.

Each two-dimensional grid is wrapped back into a cylinder and then restretched to obtain a three-dimensional computation surface. A series of such surfaces starting with the fuselage surface and ending with the outer boundary combine to form a fully three-dimensional boundary conforming computational grid. The surfaces can be clustered in the spanwise direction to obtain better resolution of areas of interest: fuselage surface, tips of the lifting surfaces and the outer boundary which could possibly be a wind tunnel wall.

COMPUTER PROGRAM

The computer program consists of a main program and seven subroutines. Input is accomplished by a separate file. The program makes wide use of IF-THEN-ELSE statements and unformatted read and write statements.

The main program performs the following functions. It reads in all input data and performs the radial stretching. It also determines the airfoil contours to be used in the two-dimensional grid generation. Lastly, the main program restretches the cylindrical computational surfaces into their three-dimensional shapes.

The subroutines do the following tasks. Subroutine TSPLIN performs all spline fittings and interpolations. Subroutine SPLIF2 determines the coefficients for the cubics used to define the lifting surfaces. Subroutine NEWT determines the intersection between the cutting surface and the spanwise cubic splines, found in SPLIF2, using a Newton procedure. Subroutine YZGRID determines the clustering of the cutting surfaces in the radial direction. Subroutine WING2D performs all functions associated with the two-dimensional grid generation. Subroutine WALLS generates the upper and lower horizontal boundaries used in the two-dimensional grid generation. Subroutine XSPACE clusters points, such as the spacing of the cutting surfaces or the spacing of grid points along the airfoil contours and wakes, by use of a series of single or double sine waves.

Input

The first part of the input is contained in the namelist IYZGRID. The following input parameters should be included in the namelist.

MAXK

The total number of cutting surfaces, should be less than

26.

AMPKG

Clustering parameter used in determining the number of cutting surfaces on each lifting surface. Increasing AMPKG increases the number of cutting surfaces that will be near the fuselage. The range for AMPKG is greater than or equal to zero and less than 0.31.

MAXY

Number of C-layers for the two-dimensional grid (including the airfoil and outer boundary) minus one. MAXY should be less than 51. MAXY may be increased by the program for configurations having more than one lifting surface.

XMINF

X-coordinate of the first fuselage station.

XPINF

X-coordinate of the last fuselage station.

END

Length of two-dimensional plane (approximately 10.).

AMP1

Clustering parameter for the grid points on the airfoil and wake for all contours in the two-dimensional plane. The range for AMP1 is greater than or equal to zero and less than 0.15. Increasing AMP1 causes more clustering of the grid points near the leading and trailing edges.

AMPY1

Clustering parameter for the C-layers along the right side of the mapped plane. The range for A is greater than or equal to zero and less than 0.31.

AMPY2

Similar to AMPY1, AMPY2 clusters C-layers along the left side of the strip plane. The range for AMPY2 is greater than or equal to zero and less than 0.32.

AMPYC

Clustering parameter for the cutting surfaces. AMPYC clusters the surfaces closer to the fuselage and tips of the lifting surfaces. Increasing AMPYC causes more clustering. The range for AMPYC is greater than or equal to zero and less then 0.15

NOB

Number of input points on the outer boundary, should be less than 51.

NWLEFT

Number of points on the upstream boundary between the upper and lower boundaries in the two dimensional plane. NWLEFT should be less than 22.

NLS

Number of lifting surfaces (present version needs NLS less than 5).

NCELLT

Total number of grid cells about the leading lifting surface and wake in the two-dimensional plane, should be less than 90.

AYY1

Controls C-layer curvature near leading edge of leading lifting surface. AYY1 should be between 0.0 and 0.20.

CFACT

Provides additional C-layer curvature control near the leading edge of the leading lifting surface. CFACT should be between 0 and 1.0.

AMPG

Clustering parameter that is used to determine from NCELLT the number of grid points on the airfoil contours and wakes in the two-dimensional plane. Increasing AMPG causes more grid points to be on the airfoils. The range for AMPG is greater than or equal to zero and less than 0.31.

ITERMAX

The maximum number of iterations (typically less than 25) for subroutine NEWT.

TAILCL

Logical variable that indicates if the vertical tail is in the aircraft plane of symmetry (TAILCL is TRUE) or not (TAILCL is FALSE).

ALPHDWN

Slope of the wake in degrees at the exit boundary in the two-dimensional plane.

NVTMIN

Minimum number of grid cells for the vertical tail when in the aircraft plane of symmetry.

VTAIL

Logical variable that indicates if a vertical tail exists on the configuration. If one is present, VTAIL is TRUE, otherwise VTAIL is FALSE.

ISOWING

Logical variable that indicates if the grid to be generated is for an isolated wing.

YMAX and ZMAX

When ISOWING is TRUE, these parameters should be provided. They set the locations of the square outer boundary for the isolated wing case.

EXPO

Exponential factor that influences the C-layer clustering in the divisions of the mapped plane. As the divisions become smaller, EXPO can be used to make the clustering decrease rapidly by having the value of EXPO greater than 1.

The following input parameters are not in the namelist.

NR

Number of fuselage cross-section to be input, less than 101.

NTH

Number of radial coordinates that define the fuselage cross-sections. NTH is the same for all cross-sections and should be less than 31.

ICON

Determines how clustering points on the fuselage are determined. If ICON equals zero, the determination is automatic. If ICON is greater than zero, the clustering locations need to be read in.

XCONL1

First clustering point on the lower centerline of fuselage.

XCONL2

Second clustering point on the lower centerline.

XCONU1

First clustering point on the upper centerline.

XCONU2

Second clustering point on the upper centerline.

When more than two lifting surfaces are present, the following six parameters are needed.

ZVLE(I)

Z location of the leading edge of the tip of the third (and fourth) lifting surface.

ZVTE(I)

Z location of the trailing edge of the tip of the third (and fourth) lifting surface.

XVLE(I)

X location of the leading edge of the tip of the third (and fourth) lifting surface.

XVTE(I)

X location of the trailing edge of the tip of the third (and fourth) lifting surface.

YVLE(I)

Y location of the leading edge of the tip of the third (and fourth) lifting surface.

YVTE(I)

Y location of the trailing edge of the tip of the third (and fourth) lifting surface.

AMPUW

Clustering parameter for grid points along the upper centerline of the fuselage. Increasing AMPUW causes more clustering of the grid points. The range for AMPUW is greater than or equal to zero and less 0.31.

AMPLW

Clustering parameter for grid points along the lower centerline. The range for AMPLW is greater than or equal to zero and less than 0.31

EXPL

Exponential factor that controls the decrease of clustering along the lower boundary in the strip plane as the cutting surface radii increases. EXPL will make the clustering decrease slower if EXPL is made less than 1, or faster if it is greater than 1.

EXPU

EXPU is the same as EXPL, but for the upper boundary of the strip plane.

DXLE(I),DYLE(I),DZLE(I)

X, Y, and Z coordinates of the break points of the lifting surface planform. The arrays are used several time to define the different lifting surfaces. It is important to note that the leading lifting surface should be read in first and the vertical tail should be read in last.

DXTE(I),DZTE(I)

X and Z coordinates of the trailing edge of the lifting surface.

NDA(I)

The number of input points for the DXLE, DYLE, DZLE, DXTE, and DZTE arrays for each of the lifting surfaces. NDA(I) should be less than 26.

NSTAT(I)

The number of span stations for each lifting surface that are to be generated by the program from the input parameters to describe the shape of the lifting surface.

XFUS(I)

X locations of the fuselage cross-sections.

RFUS(I,J)

The radius of the fuselage at the I,J location. The indice I refers to the x location, while the J indice refers to the angular position. Note that the cross-section is defined by cylindrical and not cartesian coordinates.

THETA(I,J)

Angular position of RFUS(I,J). The value of THETA(I,J) should start at negative one-half pi and vary monotonically to positive one-half pi.

ROB(I)

Radius of the outer boundary at the angular position I.

THEOB(I)

Angular position of ROB(I).

DX(I,J),DZ(I,J)

The coordinate of the input airfoil shapes. DX is the chordwise direction and DZ gives the thickness. The indice I refers to the position on the airfoil. The airfoil points should start at the trailing edge and move counter-clockwise about the airfoil. The program assumes the airfoil to be at zero degrees angle-of-attack. The airfoils need only be of the desired thickness-to-chord ratio, the chord length and twist are determined from the arrays DXLE, DXTE, DZLE, and DZTE. The indice J indicates which airfoil in the wing planform.

NAFXC

The number of different airfoils in the planform (NAFXC=1 o

NDA(I)).

MAXX

Number of points that define the input airfoil. MAXX shoul

be less than 90.

Structure of Input File

The first line contains a header describing the input.

The first section should contain the namelist IYZGRID.

NDA(I), I=1, NLS
NSTAT(I), I=1, NLS
FORMAT(I4)

NR, NTH, ICON
FORMAT(3X, I3, 4X, I2, 5X, I3)

ZVLEI(I), ZVTEI(I)
XVLE(I), XVTE(I)
YVLE(I), YVTE(I)
I=1, NLS-2
FORMAT(2F15.0)

AMPUW, EXPU
AMPLW, EXPL
FORMAT(6X, F10.0, 5X, F10.0))

XCONL1

XCONL2

XCONU1

XCONU2

FORMAT(7X,F10.0)

(XFUS(I),(RFUS(I,J),THETA(I,J),J=1,NTH),I=1,NR)

UNFORMATTED

(ROB(I),THEOB(I),I=1,NOB)

UNFORMATTED

The following are read for each lifting surface.

(DXLE(I),DYLE(I),DZLE(I),I=1,ND)

(DXTE(I),XXS,DZTE(I),I=1,ND)

NAFXC,MAXX

((DX(I,J),DZ(I,J),I=2,MAXX),J=1,NAFXC)

ALL UNFORMATTED

XXS is a dummy variable

Output

Numerous values besides the actual grid points are printed out in the program. A sample output is included to show what a correct output should be. A brief explanation of the output is as follows.

The main program first prints out an echo check of th header. The wing input planform break points for the firs two lifting surfaces are then printed, followed by th computed planform definition. The sting tips of the firs two lifting surfaces are then printed. Output fro subroutine YZGRID contains the y and z coordinates of th stretched lifting surface tips and the radii of th stretched cutting surfaces. When more than two liftin surfaces are present, the break points, planform definitio and sting tip locations of the other lifting surfaces ar then printed. Subroutine WING2D prints coordinates of eac airfoil contour and the singular point for the conforma mapping

The coordinates of the three-dimensional grid ar printed out on TAPE13. Shown in figure 4 is the surfac grid for an F-15 type configuration that was produced by th program.

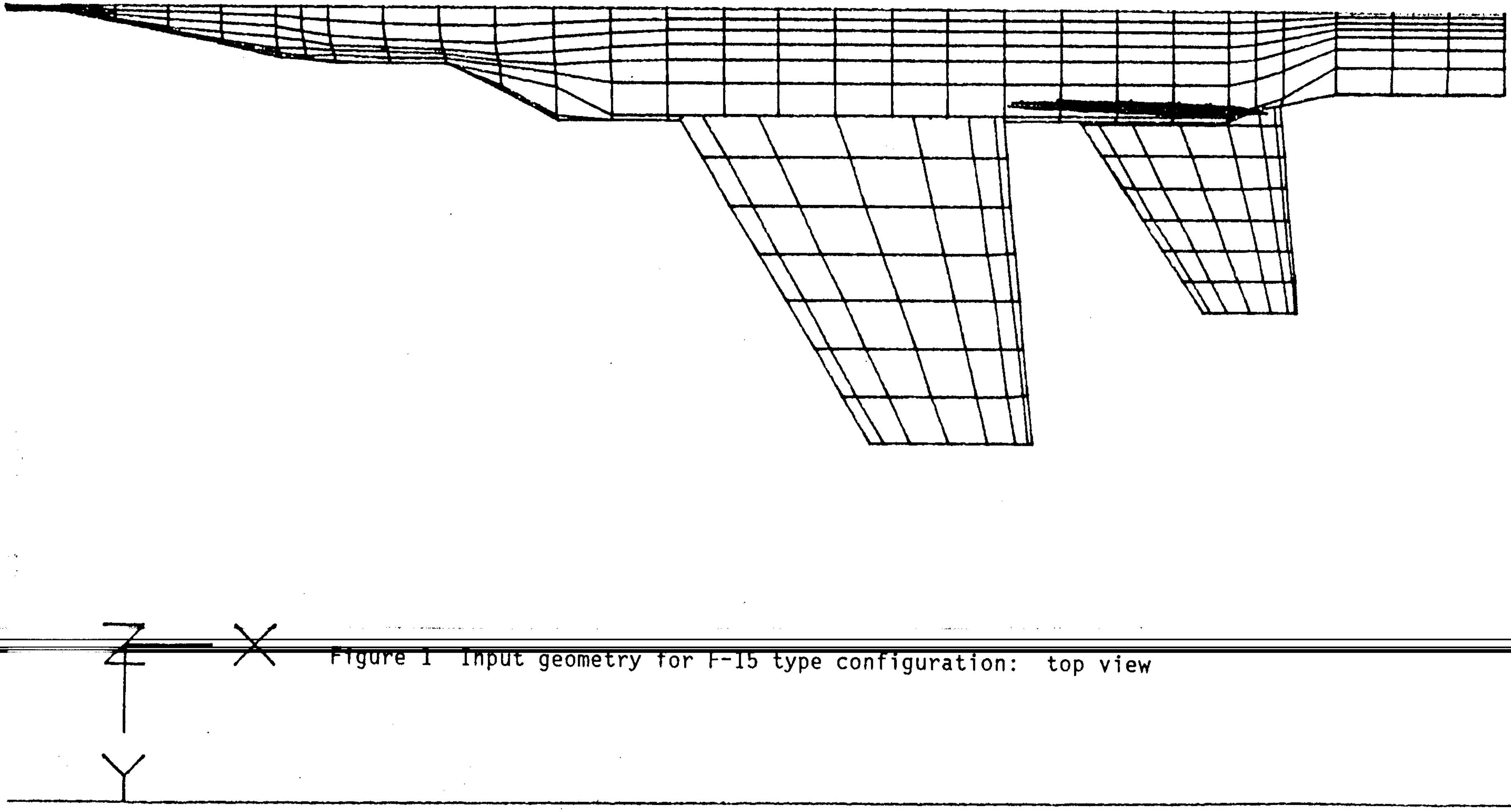


Figure 1 Input geometry for F-15 type configuration: top view

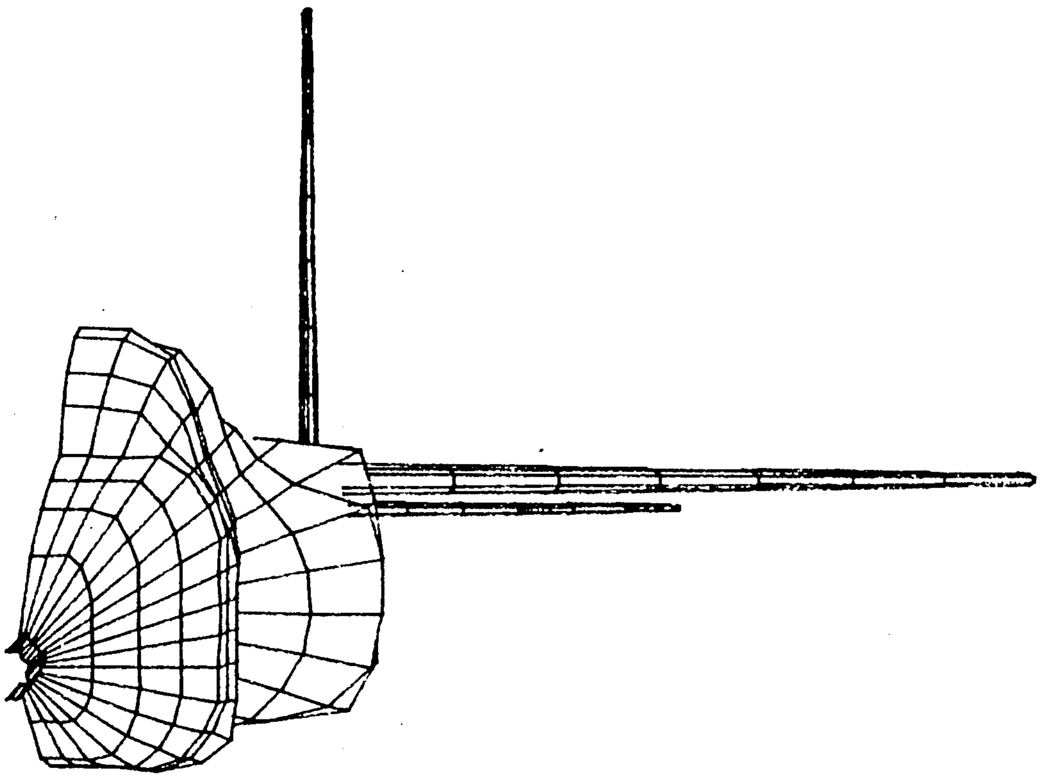


Figure 1 Input geometry for F-15 type configuration: front view

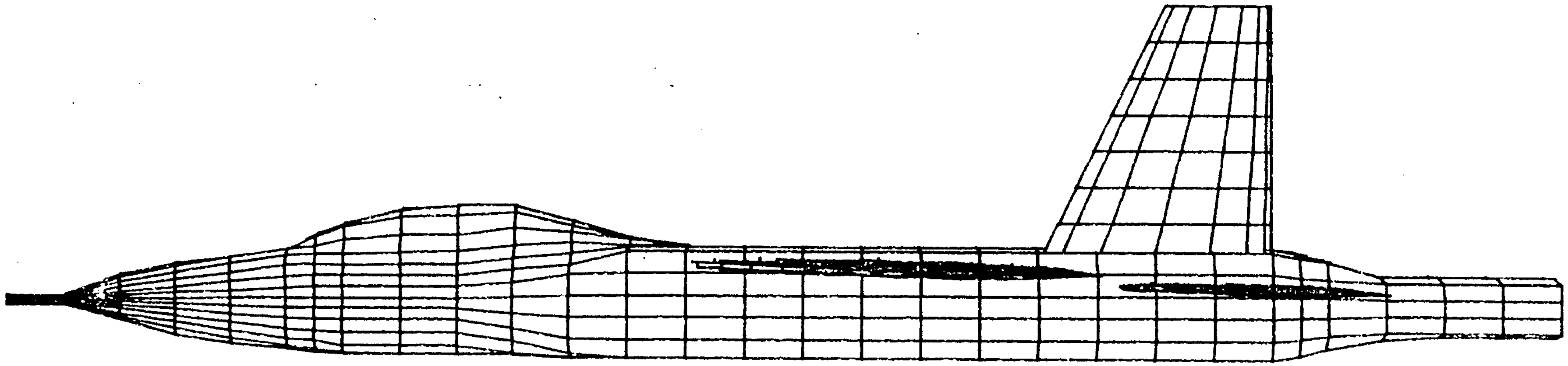


Figure 1 Input geometry for F-15 type configuration: side view

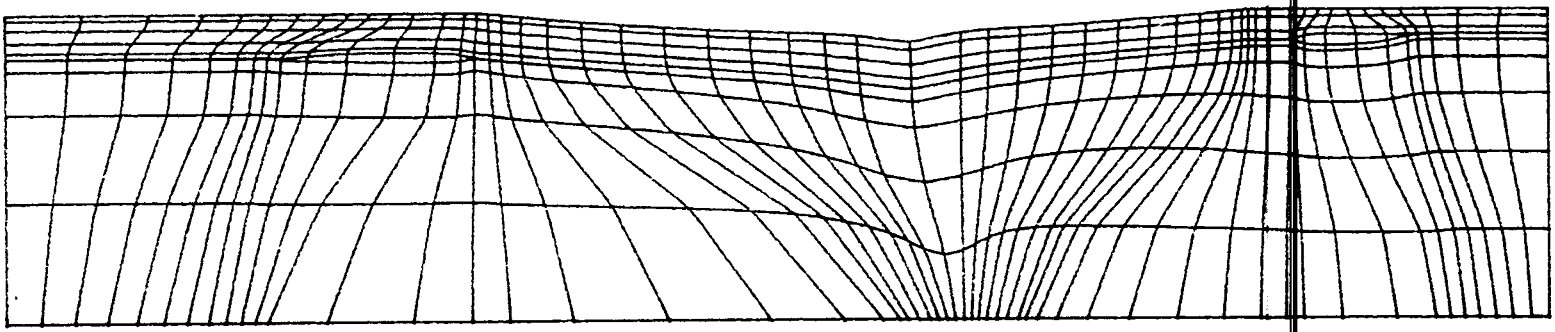


Figure 2 Computational plane with internal grid points

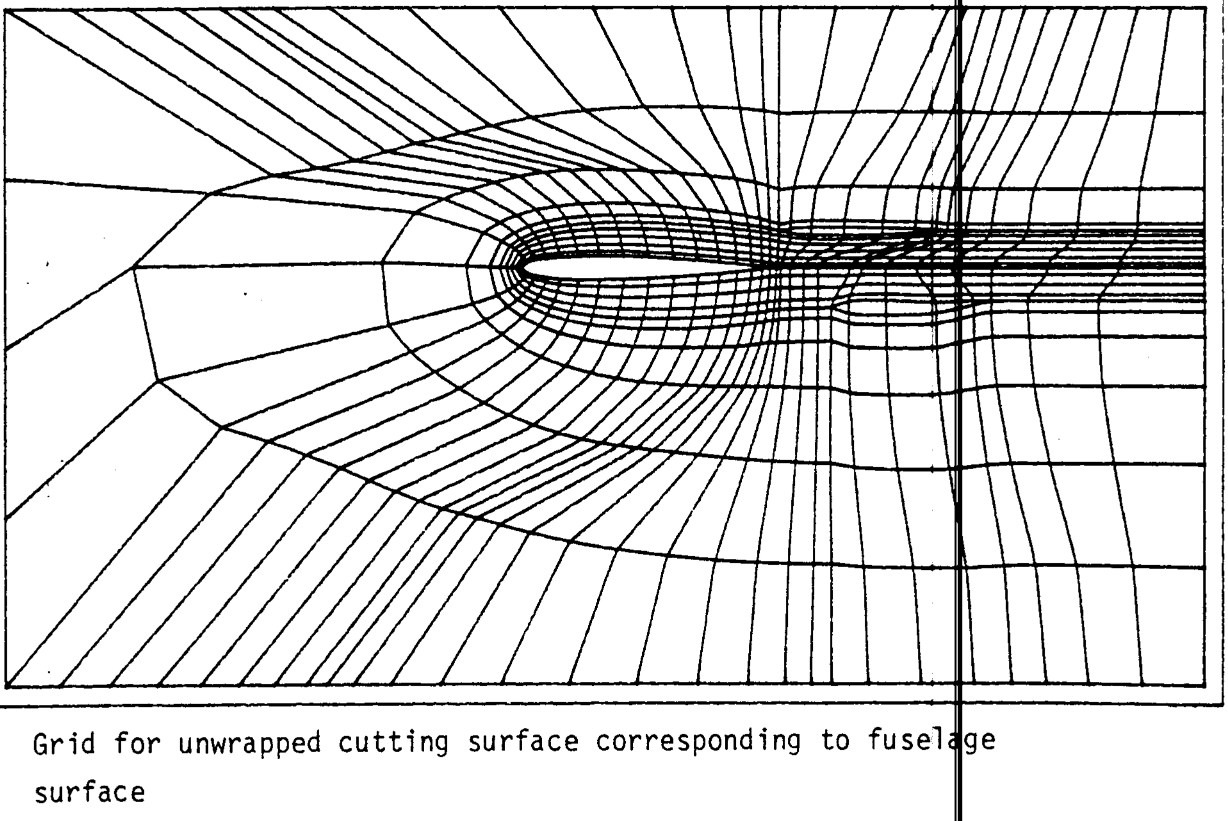


Figure 3 Grid for unwrapped cutting surface corresponding to fuselage surface

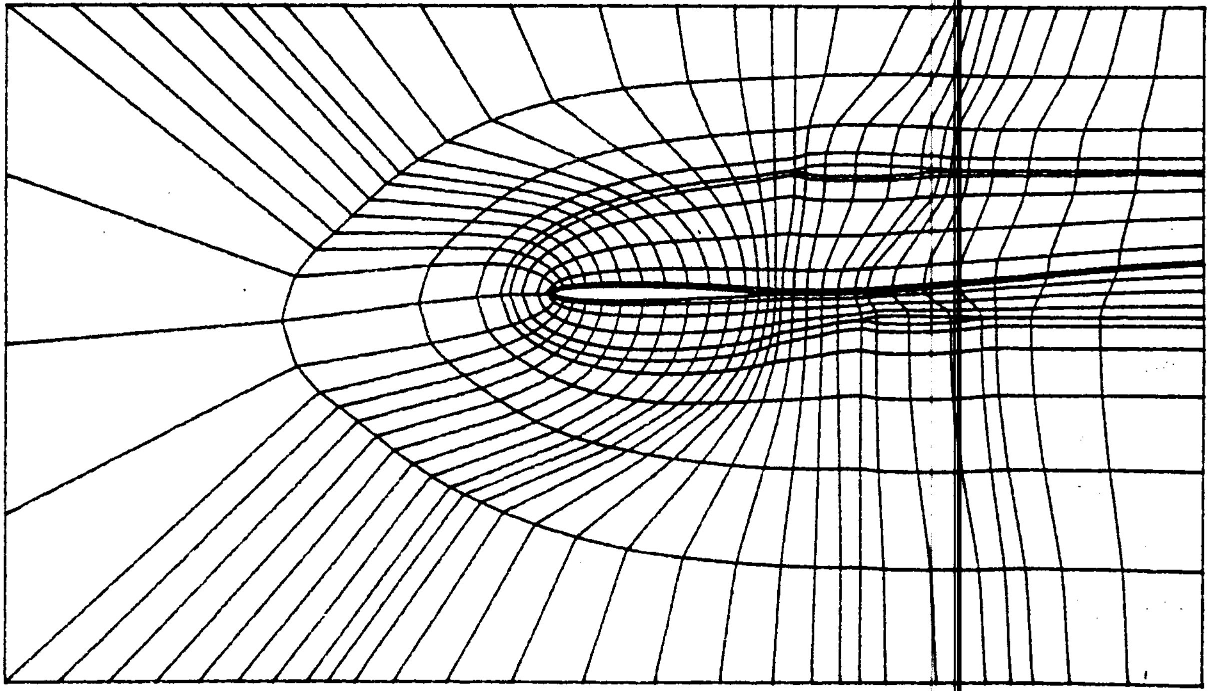


Figure 3 Grid for unwrapped cutting surface corresponding to surface beyond the fuselage

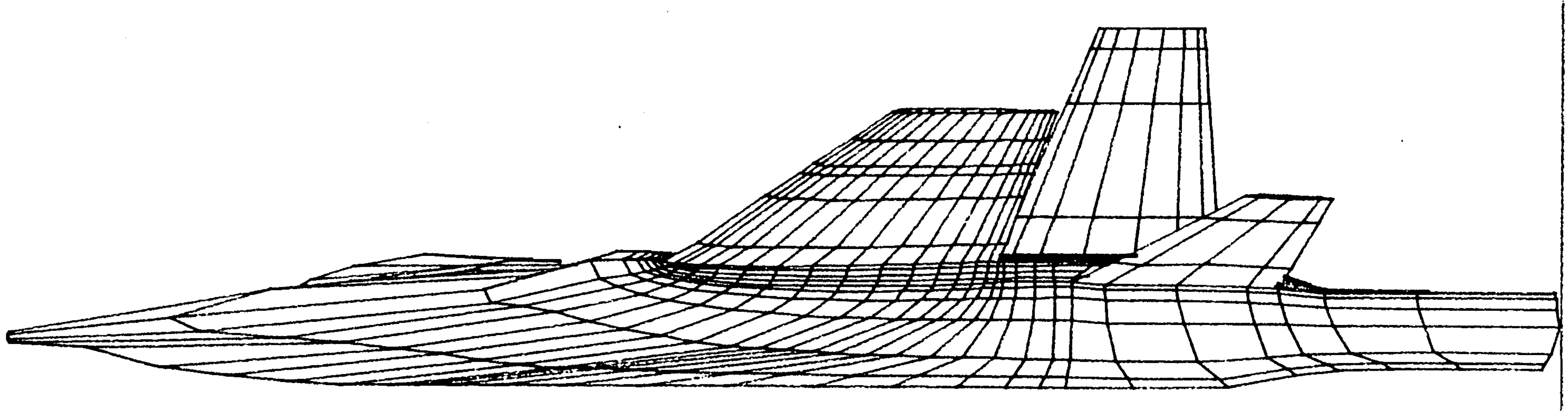
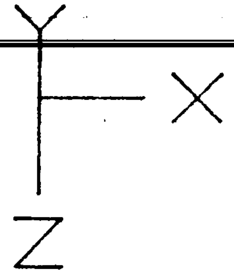


Figure 4 Generated surface grid for F-15 type configuration: bottom-side view



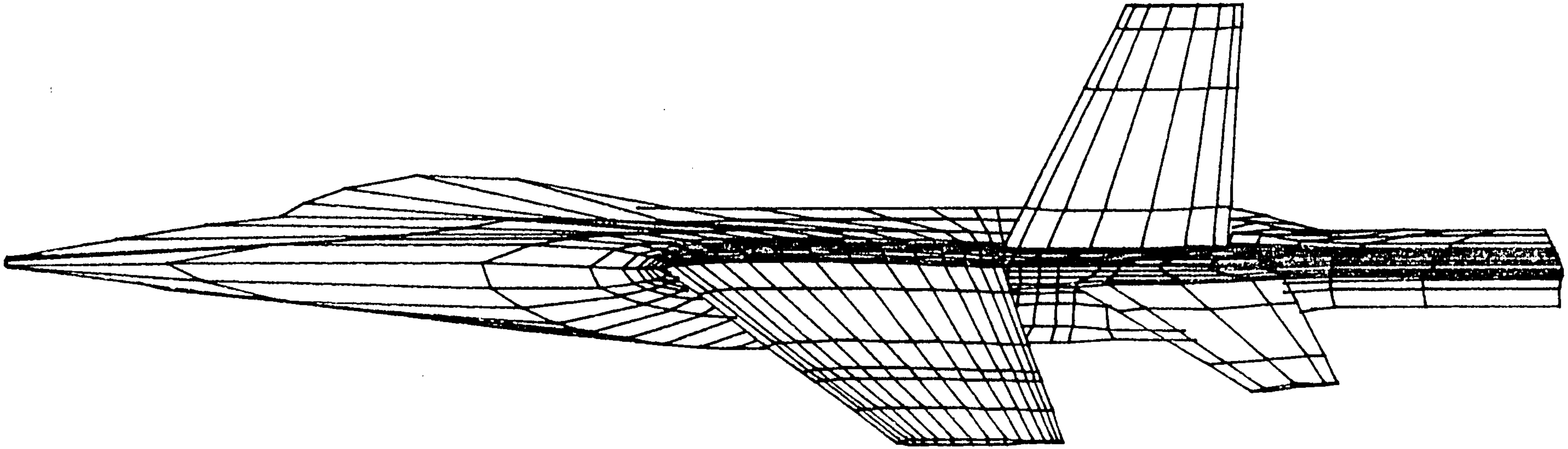
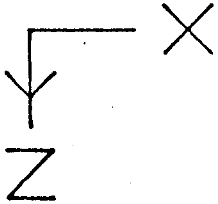


Figure 5 Generated surface grid for F-15 type configuration: side view



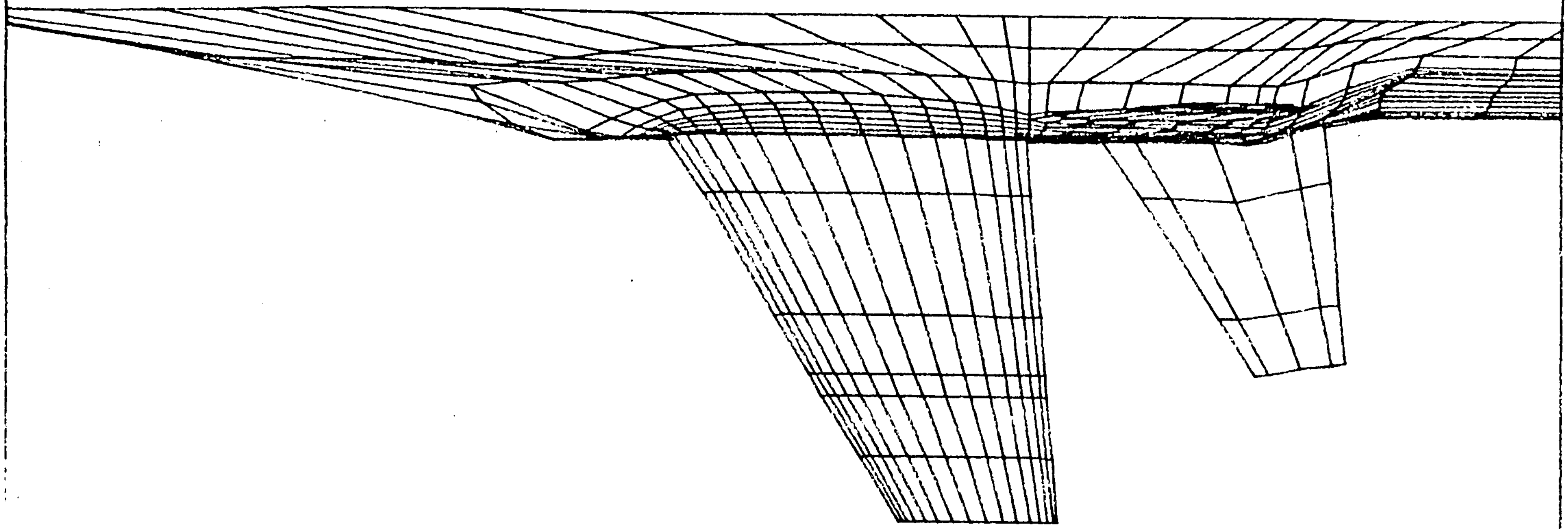
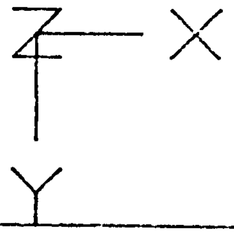


Figure 6 Generated surface grid for F-15 type configuration: top view



APPENDIX A

INPUT LISTING FOR AN F-15 TYPE CONFIGURATION

```
   F-15 TYPE CONFIGURATION, 3 LIFTING SURFACES, INPUT FOR CHCRIDC.
 $IYZGRID
      ITERMAX=25,
      MAXK=9,
      AMPKG=0.05,
      MAXY=11,
      AMP1=0.125,
      XMINF=0.0,
      XPINF=28.00,
      END=11.0,
      AMPY1=0.00,
      AMPY2=0.075,
      AMPG=0.075,
      AMPYC=0.1,
      NOB=15,
      NCELLT=80,
      AYY1=0.125,
      NLS=3,
      CFACT=.9,
      TAILCL=.FALSE.,
      VTAIL=.TRUE.,
      ISOWING=.FALSE.,
      ALPHDWN=0.0,
 $END
   2
   2
   2
  24
  12
  12
NR=031NTH=18ICON=004
  -5.50000          -5.5000
  20.459132         22.0000
   1.65              1.65
AMPUW=0.075
AMPLW=0.075
XCONL1=7.
XCONL2=23.5
XCONU1=5.
XCONU2=22.
                0         .1000000000000      -1.570796326795
       .099283290    -1.385996759
       .107302729    -1.201197191
       .111957701    -1.016397623
       .101149144     -.831598055
       .099456866     -.646798488
       .107028748     -.461998920
       .103944551     -.277199352
       .093348863     -.092399784
       .093348863      .092399784
       .103944551      .277199352
       .107028748      .461998920
       .099456866      .646798488
       .101149144      .831598055
       .111957701     1.016397623
       .107302729     1.201197191
       .099283290     1.385996759
       .100000000     1.570796327
     1.000000000000       .1000000000000      -1.570796326795
       .099283290    -1.385996759
```

```
    .107302729   -1.201197191
    .111957701   -1.016397623
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    .099456866    -.646798488
    .107028748    -.461998920
    .103944551    -.277199352
    .093348863    -.092399784
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    .107028748     .461998920
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    .101149144     .831598055
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    .107302729    1.201197191
    .099283290    1.385996759
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    .335630775    -.461998920
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    .301366617    -.092399784
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   .865315889   -1.201197191
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  1.019397109    -.646798488
   .997440627    -.461998920
   .934552911    -.277199352
   .904098187    -.092399784
   .904065859     .092399784
   .934646014     .277199352
  1.005581294     .461998920
  1.050690660     .646798488
  1.082506572     .831598055
  1.069755660    1.016397623
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   .916042935    1.385996759
   .900000000    1.570796327
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