Inverse Design of Composite Turbine Blade Circular Coolant Flow Passages

T.-L. Chiang
Graduate Assistant.

G. S. Dulikravich[1]
Assistant Professor.

Department of Aerospace Engineering
and Engineering Mechanics,
The University of Texas at Austin,
Austin, TX 78712

An inverse design and optimization method is developed to determine the proper size and location of the circular holes (coolant flow passages) in a composite turbine blade. The temperature distributions specified on the outer blade surface and on the surfaces of the inner holes can be prescribed a priori. In addition, heat flux distribution on the outer blade surface can be prescribed and iteratively enforced using optimization procedures. The prescribed heat flux distribution on the outer surface is iteratively approached by using the Sequential Unconstrained Minimization Technique (SUMT) to adjust the sizes and locations of the initially guessed circular holes. During each optimization iteration, a two-dimensional heat conduction equation is solved using direct Boundary Element Method (BEM) with linear temperature singularity distribution. For manufacturing purposes the additional constraints are enforced assuring the minimal prescribed blade wall thickness and spacing between the walls of two neighboring holes. The method is applicable to both single material (homogeneous) and coated (composite) turbine blades. Three different cases were tested to prove the feasibility and the accuracy of the method.

Introduction

The idea of using an optimization technique coupled with the panel method (a kind of indirect BEM, often used in fluid mechanics to solve Laplace's equation) to develop an inverse design method for multiholed internally cooled turbine blades was originated by Kennon and Dulikravich [1-4]. They used the panel method to solve Laplace's equation for the temperature field in the solid blade material subject to partly Cauchy-type boundary conditions. The computed temperature distibution on the initially guessed inner coolant flow passage walls, and the prescribed coolant temperature on these walls, were then iteratively approached by changing the shapes and sizes of the coolant flow passages until the procedure converged.

The present work represents an improvement over this method which can be summarized as follows:

The temperature and heat flux distributions on the Γ_1 surface (Fig. 1) of the turbine blade are specified a priori in the original method. This is now changed to the temperature distribution and heat flux distribution specified on Γ_1 and the temperature distribution specified on Γ_3 surface, thus changing boundary conditions for Laplace's equation from a partly Cauchy type to a Dirichlet type during each iterative step.

The objective function is changed to the error function defined by the differences between the calculated and specified heat flux distributions on the surface Γ_1 instead of differences in temperatures on the surface Γ_3.

In this paper the direct BEM is used instead of the panel method to solve the two-dimensional Laplace equation for the steady-state temperature field. Also, the elements used now have a linear temperature distribution instead of the constant temperature distribution.

Two constraints that might be required in the practical

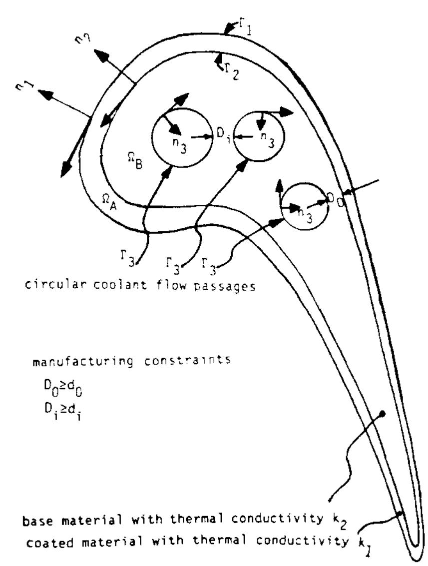


Fig. 1 Geometry and manufacturing constraints

[1]Presently Associate Professor, Aerospace Engineering Department, 233 Hammond Boulevard, Pennsylvania State University, University Park, PA 15802.

Contributed by the Gas Turbine Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS and presented at the 31st International Gas Turbine Conference and Exhibit, Düsseldorf, Federal Republic of Germany, June 8-12, 1986. Manuscript received at ASME Headquarters February 10, 1986. Paper No. 86-GT-190.

blade manufacturing process are added. They allow a minimum distance d_0 to be maintained between any hole and the Γ_2 surface and a minimum distance d_t to be maintained between the walls of any two neighboring holes (Fig. 1).

The nonhomogeneous blade design is allowed whereby a surface layer of, for example, ceramic material is used to coat the turbine surface Γ_2 (Fig. 1). This results in two coupled Laplace equations that need to be solved simultaneously.

All the inner coolant flow passages are forced to be circular, since the circular shape is more acceptable than the arbitrary shape from the manufacturing point of view.

Analytic Formulation

There are two methods of formulating the boundary-value problems of potential theory. The first method is referred to as an indirect formulation. It represents the potential function u with a single-layer or a double-layer potential generated by continuous source distibution over a surface Γ. This procedure leads to the formulation of integral equations which define the source densities. This method is mainly used in fluid mechanics where it is known as the source panel method.

However, one of the disadvantages of the indirect formulation is that the calculated source strengths usually have no obvious physical relation to the problem [5]. The other disadvantage is that the boundary surface is restricted to be a Liapunov (smooth) surface. These disadvantages can be overcome by using the direct formulation of the BEM [5].

The direct formulation can be deduced [5] from Green's third identity or the weighted residual method, since the latter permits a straightforward extension to solve more complex partial differential equations and can combine the BEM with more classical numerical methods. Therefore the latter method is usually used to formulate integral equations. The weighted residual statement can be written as

$$\int_{\Omega} (\nabla^2 u) u^* d\Omega = \int_{\Gamma_N} (q-\bar{q}) u^* d\Gamma - \int_{\Gamma_E} (u-\bar{u}) q^* d\Gamma \qquad (1)$$

where u^* is the fundamental solution of the Laplace equation on a domain Ω, that is,

$$\nabla^2 u^* + \Delta_i = 0 \qquad (2)$$

where Δ_i is the Dirac delta function. For an isotropic two-dimensional medium

$$u^* = \frac{1}{2\pi} \ln \frac{1}{r} \qquad (3)$$

where r is the distance from point i to the point under consideration. Then

$$q = \frac{\partial u}{\partial n} \qquad q^* = \frac{\partial u^*}{\partial n} \qquad (4)$$

Usually, $u = \bar{u}$ on Γ_E are called the essential conditions and $\partial u/\partial n = \bar{q}$ on Γ_N are called the natural conditions.

Integrating by parts and substituting equation (2) into the left-hand side of equation (1), the final form of the boundary integral equation is

$$c_i u_i + \int_{\Gamma} u q^* d\Gamma = \int_{\Gamma} q u^* d\Gamma \qquad (5)$$

This equation provides a functional constraint between u and q over Γ, which ensures their compatibility as boundary data. Here, c_i is the value of the scaled internal angle of the boundary Γ at the point i (Fig. 2a), that is

$$c_i = \frac{\theta}{2\pi} \qquad (6)$$

Consequently, $c_i = 1/2$ for a point on a smooth boundary where there is a continuous tangent, $c_i = 1$ for a point in the interior Ω, and $c_i = 0$ for a point exterior to Ω.

Numerical Discretization

Equation (5) can be discretized into a series of straight elements on the surface Γ with the variation of u and q assumed to be linear along each element. The points where the unknown derivatives q of the potential are considered are called nodes and are taken to be at the ends of each element (Fig. 2a).

Equation (5) can be written for the n elements as

$$c_i u_i + \sum_{j=1}^{n} \int_{\Gamma_j} u q^* d\Gamma = \sum_{j=1}^{n} \int_{\Gamma_j} q u^* d\Gamma \qquad (7)$$

The values of u and q at any point of the element can be defined in terms of their nodal values and the linear interpolation functions ϕ_1 and ϕ_2, that is

$$u(\xi) = [\phi_1 \phi_2] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (8)$$

$$q(\xi) = [\phi_1 \phi_2] \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \qquad (9)$$

where ξ is the dimensionless coordinate (Fig. 2b), $\xi = 2x/1$, $\phi_1 = (1/2)(1-\xi)$, and $\phi_2 = (1/2)(1+\xi)$. Then

$$\int_{\Gamma_j} u q^* d\Gamma = [h_{ij}^1, h_{ij}^2] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (10)$$

where

$$h_{ij}^1 = \int_{\Gamma_j} \phi_1 q^* d\Gamma \quad h_{ij}^2 = \int_{\Gamma_j} \phi_2 q^* d\Gamma$$

Hence

$$\int_{\Gamma_j} q u^* d\Gamma = [g_{ij}^1, g_{ij}^2] \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \qquad (11)$$

where

$$g_{ij}^1 = \int_{\Gamma_j} \phi_1 u^* d\Gamma \quad g_{ij}^2 = \int_{\Gamma_j} \phi_2 u^* d\Gamma$$

All coefficients h_{ij}^1, h_{ij}^2, g_{ij}^1, and g_{ij}^2 can be evaluated by using the numerical integration. When $i=j$, g_{ij}^1 and g_{ij}^2 are determined analytically [5].

Substituting these into equation (7), the equation for node i can be obtained as

$$c_i u_i + [\hat{H}_{i1}, \ldots, \hat{H}_{iN}] \begin{bmatrix} u_1 \\ \cdot \\ \cdot \\ \cdot \\ u_N \end{bmatrix} = [G_{i1}, G_{i2}, \ldots, G_{iN}] \begin{bmatrix} q_1 \\ \cdot \\ \cdot \\ \cdot \\ q_N \end{bmatrix} \qquad (12)$$

where for all $j \neq 1$

$$\hat{H}_{ij} = h_{i,j-1}^2 + h_{i,j}^1, \; G_{ij} = g_{i,j-1}^2 + g_{i,j}^1 \qquad (13)$$

and for $j = 1$

$$\hat{H}_{i,j} = h_{i,1}^1 + h_{i,N}^2, \; G_{ij} = g_{i,1}^1 + g_{i,N}^2 \qquad (14)$$

Then

$$c_i u_i + \sum_{j=1}^{N} \hat{H}_{ij} u_j = \sum_{j=1}^{N} G_{ij} q_j \tag{15}$$

or more simply

$$\sum_{j=1}^{N} H_{ij} u_j = \sum_{j=1}^{N} G_{i,j} q_j \tag{16}$$

where

$$H_{i,j} = \hat{H}_{i,j} \text{ for } i \neq j$$
$$H_{i,j} = \hat{H}_{i,j} + c_i \text{ for } i = j \tag{17}$$

Composite Blades

Turbine blades with, say, ceramic coating have two regions of considerably different thermal conductivities. Therefore, two coupled Laplace equations for temperature field need to be solved. The corresponding two matrices can be added together by using the continuity of heat fluxes and equalizing the temperatures themselves at the interface Γ_2 between the coating and the main turbine material.

Assume that there are N_i elements on the surfaces Γ_i where $i = 1, 2, 3$ and that the thermal conductivity in Ω_A is k_1 and in Ω_B is k_2. For domain Ω_A (coating material) the governing equations are then

$$\begin{bmatrix} H_1^{\Omega_A} & , & H_2^{\Omega_A} \end{bmatrix} \begin{bmatrix} u^{\Gamma_1} \\ u^{\Gamma_2} \end{bmatrix} = \begin{bmatrix} G_1^{\Omega_A} & , & G_2^{\Omega_A} \end{bmatrix} \begin{bmatrix} Q^{\Gamma_1}/k_1 \\ Q^{\Gamma_2}/k_1 \end{bmatrix} \tag{18}$$

$$\underbrace{\begin{bmatrix} H_{1,1} & H_{1,N_1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ H_{N_1+N_2,1} & H_{N_1+N_2,N_1} \end{bmatrix}}_{H_1^{\Omega_A}} \underbrace{\begin{matrix} H_{1,N_1+1}, & H_{1,N_1+N_2} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ H_{N_1+N_2,N_1+1} & H_{N_1+N_2,N_1+N_2} \end{matrix}}_{H_2^{\Omega_A}} \begin{bmatrix} u_1^{\Gamma_1} \\ \cdot \\ u_{N_1}^{\Gamma_1} \\ u_1^{\Gamma_2} \\ \cdot \\ u_{N_2}^{\Gamma_2} \end{bmatrix} =$$

$$\underbrace{\begin{bmatrix} G_{1,1} & G_{1,N_1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ G_{N_1+N_2,1} & G_{N_1+N_2,N_1} \end{bmatrix}}_{G_1^{\Omega_A}} \underbrace{\begin{matrix} G_{1,N_1+1}, & G_{1,N_1+N_2} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ G_{N_1+N_2,N_1+1} & G_{N_1+N_2,N_1+N_2} \end{matrix}}_{G_1^{\Omega_A}} \begin{bmatrix} Q_1^{\Gamma_1}/k_1 \\ \cdot \\ Q_{N_1}^{\Gamma_1}/k_1 \\ Q_1^{\Gamma_2}/k_1 \\ \cdot \\ \cdot \\ \cdot \\ Q_{N_2}^{\Gamma_2}/k_1 \end{bmatrix} \tag{18}$$

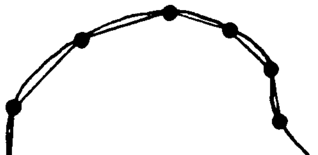
or more simply

$$\begin{bmatrix} \Omega_A & \Omega_A \\ H_1 & , H_2 \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ u \\ \Gamma_2 \\ u \end{bmatrix} = \begin{bmatrix} \Omega_A & \Omega_A \\ G_1 & , G_2 \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ Q \ /k_1 \\ \Gamma_2 \\ Q \ /k_1 \end{bmatrix} \tag{19}$$

In the same way, for domain Ω_B (main blade material)

$$\begin{bmatrix} \Omega_B & \Omega_B \\ H_1 & , H_2 \end{bmatrix} \begin{bmatrix} \Gamma_2 \\ u \\ \Gamma_3 \\ u \end{bmatrix} = \begin{bmatrix} \Omega_B & \Omega_B \\ G_1 & , G_2 \end{bmatrix} \begin{bmatrix} \Gamma_2 \\ -Q \ /k_2 \\ \Gamma_3 \\ Q \ /k_2 \end{bmatrix} \tag{20}$$

Combining equation (19) and equation (20) and moving all the unknowns to the right-hand side results in



Problem of Nonuniqueness

For a multiply connected domain, the solution of the integral equation with Dirichlet or mixed-type boundary conditions does not always have a unique solution [6]. That is, for a given curve shape Γ_1 there will always exist a particular curve magnitude, where difficulties will occur in connection with the

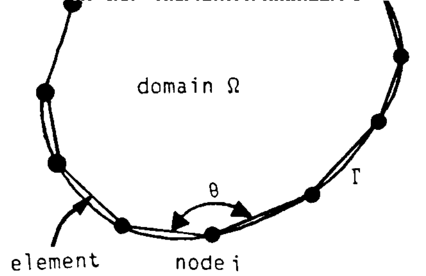


Fig. 2(a) Discretized boundary Γ

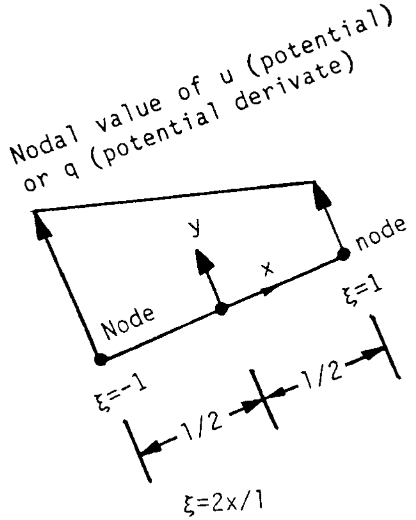


Fig. 2(b) Linear element

$$
\begin{bmatrix} \Omega_A & \\ k_1H_1 & 0 \\ & \Omega_B \\ 0 & k_2H_2 \end{bmatrix}
\begin{bmatrix} \Gamma_1 \\ u \\ \Gamma_3 \\ u \end{bmatrix}
=
\begin{bmatrix} \Omega_A & \Omega_A & \Omega_A & \\ G_1 & G_2 & -k_1H_2 & 0 \\ & \Omega_B & \Omega_B & \Omega_B \\ 0 & -G_1 & -k_2H_1 & G_2 \end{bmatrix}
\begin{bmatrix} \Gamma_1 \\ Q \\ \Gamma_2 \\ Q \\ \Gamma_2 \\ u \\ \Gamma_3 \\ Q \end{bmatrix} \quad (21)
$$

Notice the directions of the normal defined on different surfaces as shown in Fig. 1. So, for surfaces Γ_1 and Γ_2, the numbering scheme is defined in the counterclockwise direction, while for Γ_3 it is in the clockwise direction.

integral equations of the first kind with log-

In the present work, this situation is avoided by a simple change of scale—a method adopted by Symm [6], so that the maximum diameter of the domain is not greater than unity. This is sufficient to ensure that there is no possibility of nonuniqueness due to Γ_1 being a Γ contour. Alternatively, a unique solution may also be obtained by adding an appropriate auxiliary condition [7]. Detailed discussions on the nonuniqueness of the solutions of the integral equations can be found in [8, 9].

Optimization Procedures and Concept

The iterative optimization procedure used to modify the sizes and locations of the guessed coolant flow passages can be explained using the following steps:

1 Specify the potential (temperature) distributions u on surfaces Γ_1 and Γ_3.

2 Specify the heat flux distribution Q^R_j (for $j=1, \ldots, N_1$) on surface Γ_1, and the thermal conductivities k_1 and k_2.

3 Specify the manufacturing constraints: (*i*) minimum distance d_0 allowed between the holes and surface Γ_2; (*ii*) minimum distance d_i allowed between any two neighboring hole surfaces Γ_3.

4 Specify the number of holes required, and the initial guess of the radii and location of the centers of the holes. Also required is specification of the number of boundary elements to be used on surfaces Γ_1, Γ_2, and each of the holes, Γ_3.

The geometry for each circular hole can be defined by three independent variables: center coordinates x and y, and the radius r. If there are M holes, there will be $3M$ independent variables in the error function.

However, the initially guessed variables should locate the holes entirely in the feasible region of domain Ω_B, that is, the constraints in step 3 must be satisfied (Fig. 1).

5 With the BEM described earlier, solve the Laplace equation for temperature field and calculate the heat flux Q^c_j ($j=1, \ldots, N_1$) on surface Γ_1.

6 Use the Q^c_j to determine the values of the error function and the objective function (OBJ) of the optimization problem.

The nondimensional error function E_0 can be defined as

$$E_0(\mathbf{x}) = E_0(x_i, y_i; r_i) = \frac{\left[\sum_{j=1}^{N_1} (Q^c_j - Q^R_j)^2\right]^{1/2}}{\left[\sum_{j=1}^{N_1} (Q^R_j)^2\right]^{1/2}} \quad (22)$$

The purpose is to find the optimal value of $\mathbf{x}(x_i, y_i; r_i)$ for $i=1, \ldots, M$ such that E_0 is minimized. A penalty function must be added to E_0 to construct the OBJ function $E^*(\mathbf{x})$ for the two manufacturing constraints, that is,

$$E^*(\mathbf{x}) = E_0(\mathbf{x}) + \text{penalty function} \quad (23)$$

There are many different forms for the penalty function [10]. The penalty function used here is of the interior method type with the inverse barrier function proposed by Carroll [11]

$$\text{Penalty function} = R \cdot \left[\sum_{j=1}^{N_3} \frac{d_0}{(D_j - d_0)} + \sum_{k=1}^{N_4} \frac{d_i}{(D_k - d_i)}\right] \quad (24)$$

where $N_4 = M!/2!\,(M-2)!$. Here, D_j is the minimum com-

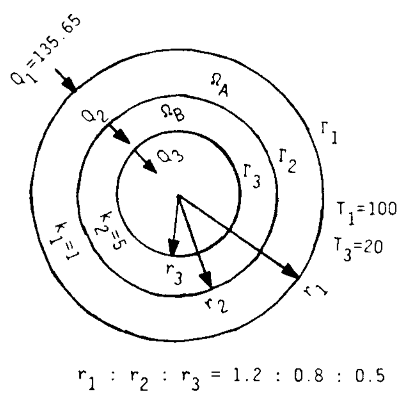


Fig. 3 Geometry and boundary conditions, test case 1

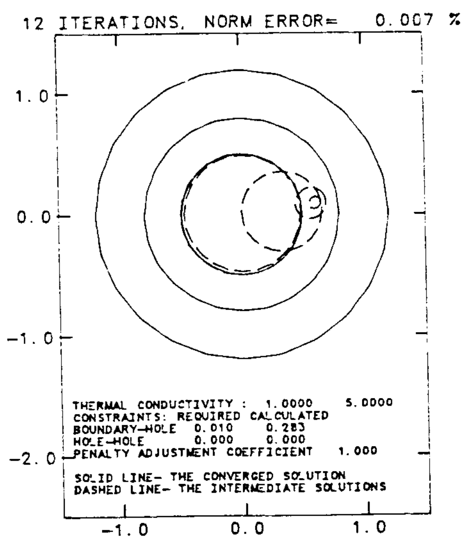


Fig. 4 Iteration sequence, test case 1

puted distance between the element j of Γ_3 and the surface Γ_2 and D_k is the computed distance between any two circular holes

$$D_k = [(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2]^{1/2} - (r_i + r_{i+1})$$

for holes i and $i+1$.

R is a positive constant which is chosen to be initially quite large during the first few optimization iterations, and then gradually reduced to near zero. $E^*(\mathbf{x})$ will then approach E_0. This method is called the Sequential Unconstrained Minimization Technique (SUMT).

A relation between the initial penalty function and the error function is defined as the Penalty Adjustment Coefficient (PAC), that is

$$\text{PAC} = \frac{\text{Penalty function}}{E_0}$$

7 Use the steepest-descent optimization technique to find the new values of the independent variables $\mathbf{x}$ until the corresponding $E^*(\mathbf{x})$ is below a satisfactory value, otherwise return to step 5.

Results and Discussion

On the basis of the preceding analysis a computer program [12] was developed and tested using the following three test cases.

The first test case was used to test the reliability of the computer program as an analysis tool. The geometry consists of a coating and a single hole (Fig. 3) with r_3:r_2:r_1 = 0.5:0.8:1.2,

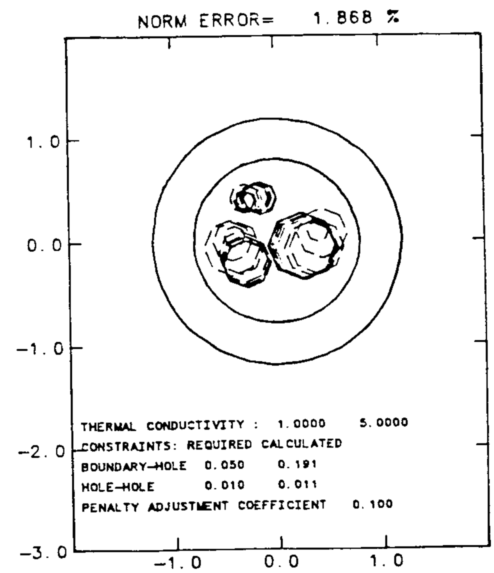


Fig. 5(*a*) Iteration sequence, test case 2

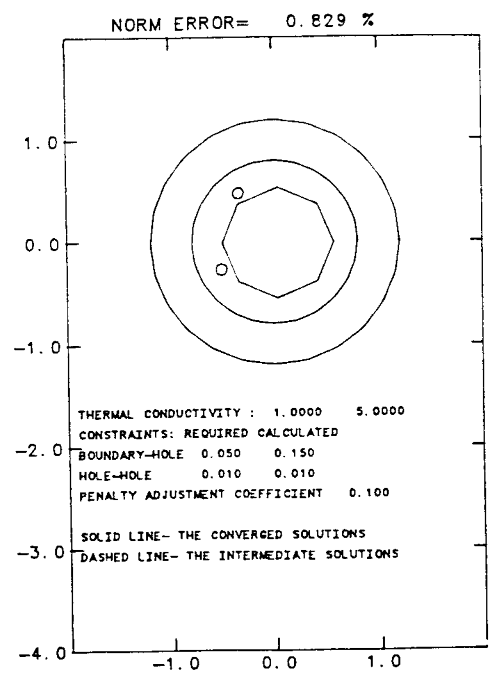


Fig. 5(*b*) Iteration sequence, test case 2

$k_1 = 1$, $k_2 = 5$, $T_1 = 100$, $T_3 = 20$ (uniform distributed). A total of 72 boundary elements were used. The results are listed in Table 1, showing that the largest error between the analytic solution and the BEM solution is about 1.63 percent in Q_1. The accuracy of the BEM can be further improved by either increasing the number of elements or using higher order elements. The inverse optimization solutions were accomplished (Table 1) by specifying the heat flux distribution on Γ_1 and temperature distribution on Γ_1 and Γ_3 surfaces. The heat flux on Γ_1 surface was then calculated by the BEM after each iteration, that is, after each adjustment of the hole shapes and their locations. The initially guessed surface Γ_3 and its iterative evolution sequence are shown in Fig. 4.

The second test case was used to test the feasibility of the inverse design concept. The same heat flux distribution on the surface Γ_1 was kept as in the first case, but the number of the circular holes was changed to three instead of one (Fig. 5*a*). Temperature distributions on Γ_1 and the holes Γ_3 did not

Table 1 Results for the first test case, using linear BEM; *—values are given at the specified conditions

Surface		Γ_1	Γ_2		Γ_3			
						hole Center		hole Radius
Method / Properties		Q_1	Q_2	T_2	Q_3	x	y	r
Analytic Solution		133.47	-200.21	35.06	-320.34	0.00*	0.00*	0.5*
BEM Approximate Solution		135.65	-200.43	35.21	-324.1	0.00*	0.00*	0.5*
Inverse Optimization Solution	P.A. Coeff							
	0.1	135.65*	-200.52	35.24	-324.74	0.00032	0.0006	0.4998
	1.0	135.65*	-200.44	35.21	-324.64	0.0001	0.0000	0.5001
	5.0	135.65*	-200.34	35.25	-324.90	0.0005	0.0003	0.4994

Table 2 Results for the third test case using a linear element; *—values are given at the specified conditions

		Total Heat Flux on Surface			Hole 1			Hole 2			L2 Norm Error x 100%
					Center		Radius	Center		Radius	
		Γ_1	Γ_2	Γ_3	x_1	y_1	r_1	x_2	y_2	r_2	
Approximate Solution		70554	-70636	-71501	0.3*	1.25*	0.08*	0.5*	1.0*	0.05*	
Inverse Optimization Solution	P.A. Coeff										
	0.1	70570	-70653	-71648	0.2978	1.2511	0.0783	0.5031	1.008	0.053	0.927
	0.5	70214	-70298	-71339	0.2953	1.2530	0.0774	0.4997	1.021	0.055	2.346
	1.0	7.0564	-70647	-71648	0.2976	1.2512	0.0782	0.5031	1.009	0.054	0.991
	1.5	70556	-70639	-71635	0.2977	1.2512	0.0783	0.5031	1.008	0.054	0.95
	5.0	70564	-70647	-71646	0.2977	1.2512	0.0782	0.5031	1.008	0.054	0.972
	8.0	70585	-70669	-71664	0.2977	1.2512	0.0783	0.5031	1.008	0.053	0.921

change, that is, it was still $T_1 = 100$ on Γ_1 and $T_3 = 20$ on Γ_3. Comparison of the calculated heat flux distribution with the specified heat flux is shown in Fig. 6. The corresponding $L2$-norm error was below 2 percent and it is distributed in the form of a sine function (see Fig 6). It can be concluded that the inverse design concept is quite feasible for multihole configurations. Note that when the error was decreased to 0.829 percent (Fig. 5*b*), one of the three holes converged to a large hole located near the center. The other two holes became negligibly small in comparison with the large hole (Fig. 5*b*).

The third test case was used to prove that for an arbitrarily shaped blade, good accuracy can be obtained between the BEM approximate solutions and the inverse optimization solutions.

The contour Γ_1 used in this case was a realistic turbine blade (Fig. 8). The variable temperature distribution specified on the surface Γ_1 can be seen in Fig. 7 and does not represent any actually measured value. The results of the inverse design procedure are listed in Table 2 and the evolutionary history of the holes can be seen in Fig. 8.

No obvious irregularity can be seen from the convergence history of E_0 except when the PAC is chosen to be too big. Then, there will be an upshoot during the first iteration (see Figs. 9 and 10). Note that in Fig. 9 for the PAC equal to 8 and in Fig. 10 for the PAC equal to 0.5, the iterative process converged to local minima.

Also, the third test case, using a PAC of 10, resulted in an infeasible solution, that is, the radius of one of the circular holes became negative. The conclusion is that too big a PAC will create a large $E^*(\mathbf{x})$ value, so the hole radius derived from the quadratic interpolation will fall below a physically meaningful value. The conclusion is that PAC should be of the order one.

The rate of convergence of any optimum search technique is highly dependent on the given function E^*. In certain problems proper scaling can be performed so as to make the contours of constant error as circular as possible. This can significantly accelerate the rate of convergence. Unfortunately, the $E^*(\mathbf{x})$ in this inverse design problem is an implicit function of $\mathbf{x}$ and the scaling technique is hard to apply.

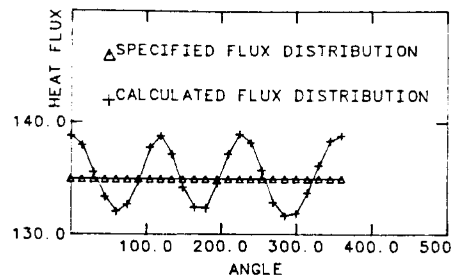


Fig. 6 Calculated and specified heat flux distributions, test case 2

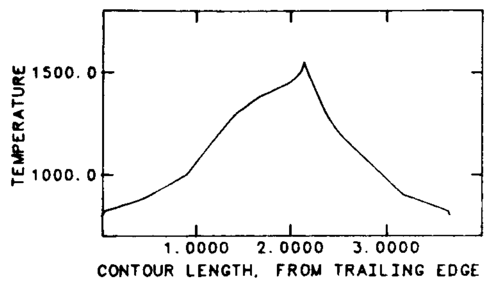


Fig. 7 Temperature distribution prescribed on Γ_1, test case 3

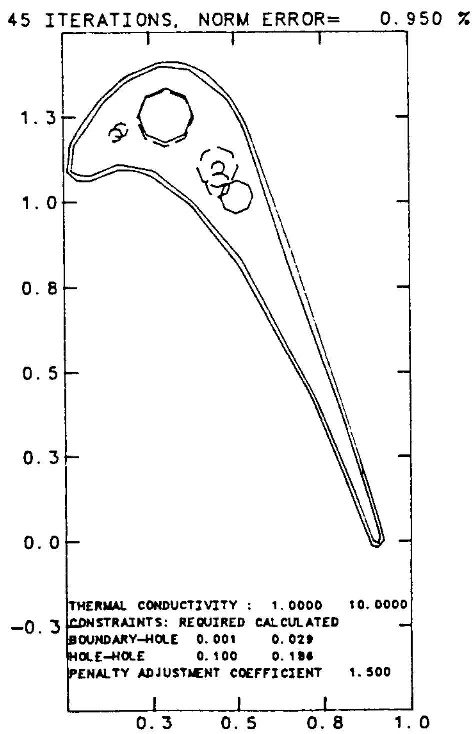


Fig. 8 Iteration sequence, test case 3

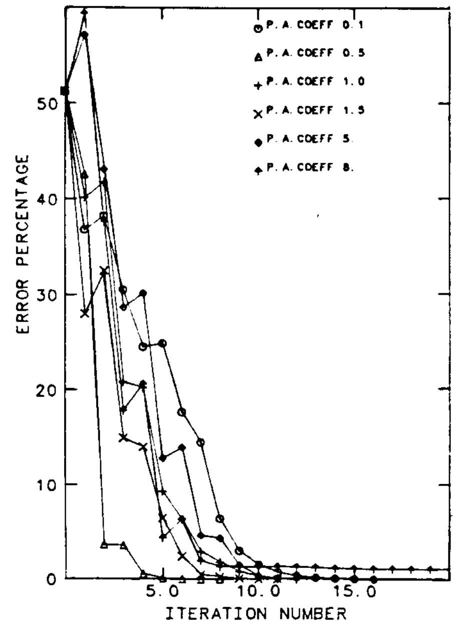


Fig. 9 Convergence history, test case 1

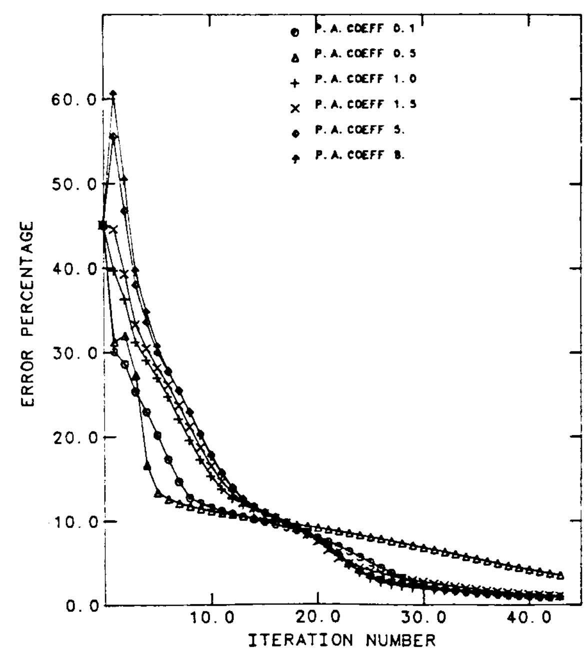


Fig. 10 Convergence history, test case 3

The problem of failing to find the global minimum might be resolved in any optimization technique by recomputing the problem with different initial guesses [13]. The initial variables should be systematically chosen for good distribution over the variable space until a sufficiently low value of E_0 has been located.

Summary

An efficient inverse design procedure for multiple circular holes (coolant flow passages) in nonhomogeneous turbine blades has been developed. The work is accomplished by coupling the direct boundary element method and the sequential unconstrained minimization technique.

The specified heat flux distribution on the outer surface of the blade is iteratively approached while satisfying the prescribed temperature distributions on the outer surface of the blade and on surfaces of the holes by a successive adjustment of the sizes and locations of the holes. Also included are two manufacturing constraints concerning the minimal allowable blade wall thickness and hole spacing.

This procedure can be successfully applied to the inverse design of coated turbine blade multiple coolant flow passage shapes. In earlier works [1–4] it was demonstrated that the coolant flow passage shapes can be changed from circular to other families of noncircular holes by adjusting the relation between the independent variables in the optimization objective function. It can also be revised to be used for the inverse design and analysis of the transient thermal problems or coupled with forced convection boundary conditions on the coolant flow passage walls if the coolant temperature and heat transfer coefficients are provided.

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