Proceedings of the Sixth International Symposium on
Finite Element Methods in Flow Problems, Antibes, France,
June 16-20, 1986.

Analysis of Unsteady Compressible Cascade Flows

Using Boundary Element and Free-Vortex Method

Tatsuo Fujinami* George S. Dulikravich**
Graduate Assistant Assistant Professor
Department of Aerospace and Engineering Mechanics
University of Texas at Austin, Austin, Texas 78712, U.S.A.
TEL (512) 471-4178, TELEX 9108741305

ABSTRACT

A fast and accurate computer program using coupled boundary element and free-vortex method is developed to calculate inviscid steady and unsteady cascade stage flows. The integral potential flow equation is discretized by linear and bi-linear finite element expansions. The unsteady vortex wake due to the interaction between static and moving cascade is simulated using free point vortices.

INTRODUCTION

The present method represents an application of boundary element methods to the unsteady separated flow simulations induced by an interaction between a moving cascade of airfoils and a static cascade of airfoils. This method provides an attractive alternative to unsteady compressible flow analysis in two and three dimensions.

ANALYSIS

Using quasi-steady approach, the mass conservation equation is expressed in terms of the potential function ϕ as

$$\nabla^2\phi = f \tag{1}$$

Here, f is a nonlinear term representing compressibility effects and can be writen as

---

* Presently with Recruit Co.,Ltd Remote Computing Services Division
** Presently Associate Professor, Pensylvania State University

$$f = (1/a^2)[V_c\cdot\nabla(V_c\cdot\nabla\phi) + \frac{1}{2}\nabla\phi\cdot\nabla(\nabla\phi\cdot\nabla\phi) + 2\nabla\phi\cdot\nabla(V_c\cdot\nabla\phi)] \qquad (2)$$

where a is the local speed of sound and Vc is the local convection velocity.

Eq.(1) is solved at each discrete time step, that is, the solution of the continuity equation is determined by the time dependent boundary conditions on the cascade surfaces. Using Green's formula, the potential inside the flow domain becomes

$$\phi = \int_{\partial\Omega}\left(\chi\frac{\partial\phi}{\partial n} - \phi\frac{\partial\chi}{\partial n}\right)dS + \int_{\Omega} f\chi\, dA \qquad (3)$$

where the first integration represents line integral along the cascade contour and the second integration is the surface integral of the flow field. Replacing the potential at infinity with $\phi = V_\infty(x\cos\alpha + y\sin\alpha)$

and using the boudary conditions $\int_B \chi\frac{\partial\phi}{\partial n}dS = 0$ on the cascade surface and $\int_W \chi\frac{\partial\phi}{\partial\phi}dS = 0$ on the wake, results in

$$\phi = \int_B \phi\frac{\partial\chi}{\partial n}dS - \int_W \Delta\phi\frac{\partial\chi}{\partial n}dS + V_\infty(x\cos\alpha + y\sin\alpha) + \int_\Omega f\frac{\partial\chi}{\partial n}dA \qquad (4)$$

Integrating eq.(4) by parts with the condition $\partial/\partial s(\Delta\phi) = 0$ along the steady cut (wake) and employing flow tangency boundary condition on the cascade surfaces results in an integral potential flow formulation [1].

$$\int \gamma\frac{\partial\phi_v}{\partial n}dS = V_\infty(\cos\alpha + \sin\alpha) + \int_\Omega f\frac{\partial\chi}{\partial n}dA \qquad (5)$$

where the left hand side term can be interpreted as bound vortex distribution along the cascade surface with unkown vortex strength γ. For cascade problems, the potential with unit vortex strength and fundamental solution with unit source stregth are obtained as

$$\phi_v = -\frac{1}{2\pi}\tan^{-1}[\coth\frac{\pi}{t}(x-\xi)\tan\frac{\pi}{t}(y-\eta)] \qquad (6)$$

$$\chi = \frac{1}{2\pi}\ln[\sinh^2\frac{\pi}{t}(x-\xi) + \sin^2\frac{\pi}{t}(y-2)] \qquad (7)$$

Piecewise linear finite element expansion for unknown bound vortex strength γ is convenient for an explicit enforcement of the Kutta-Zhukowski trailing edge condition since the trailing edge vortex strength must vanish according to the conventional thin airfoil theory.

When a relative motion between cascades of airfoils is introdued, each airfoil will experience a time variation of its circulation. This time variation of bound circulation will be transformed into a free vortex leaving the airfoil trailing edge at each time step. New free vortex strength shed from the trailing edge is determined at each time step using Kelvin's theorem about the persevation of total circulation.

$$\frac{D\Gamma}{Dt} = \frac{D\Gamma_B}{Dt} + \frac{D\bar{\Gamma}}{Dt} = 0 \qquad (8)$$

where overbar - relates to the unsteady wake of free vortices. Potential function representing the free vortices is also obtained as eq.(6).

NUMERICAL PROCEDURE

Using linear finite element expansion for boundary integral discretisation and bi-linear expansion for surface integral discrtization, eq.(5) reduces to the set of linear algebraic equations as

$$\sum_{j=1}^{N} A_{ij}\,\gamma_j = B_i \qquad 1 \le i \le N-1 \qquad (9)$$

Here, N is the number of node points discretizing the cascade surface. The right hand side of eq.(9) includes nonlinear contribution from field sources and is evaluated iteratively. Enforcing Kutta-Zhukowski condition for a single cascade analysis means that

$$\gamma_1 = \gamma_N = 0 \qquad (10)$$

Solving eq.(9) subject to eq.(10) at each discrete time step and using unsteady Bernoulli equation, usteady flow field due to a moving cascade and a fixed cascade interaction without wakes can be calculated. In order to simulate the unsteady wake of free vortices, the Kelvin's Theorem given by equation (8) can be rearranged as

$$\oint_B \vec{v}\cdot d\vec{s} + \sum_{k=1}^{N} \bar{\gamma}_k = 0 \qquad (11)$$

In order to determine each new free vortex strength, eqs.(9)-(11) must be solved simultaneously using least squares method since the system is overdetermined. Once the system is solved, the strengths of all bound vortices and new free vortex is determined. All free vortices are convected using their local convection velocities.

NUMERICAL RESULTS

An analytic solution for the incompressible potential flow through Gostelow's compressor cascade is known [2]. Sixty four boundary elements were used to obtain results shown in Figure 1. Comparison between the finite area solution [3] and the boundary element solution including field source contribution shown in Figure 2. A cascade of symmetric Zhukowski airfoils at free stream Mach number 0.5 was used. Less than ten iterations were required to get convergent results.

Figure 3 represents the unsteady wake simulation. Unsteady vortex wake from the moving cascade trailing edge is convected downstream and interacts with the static cascade. Flow history for a non-dimensional time $t^* = \frac{t}{c/V_\infty}$ varying from 0.7 to 1.2 is depicted in Figure 3.

CONCLUSIONS

A method and computer program [4] for calculating two-dimensional steady and unsteady separated lifting flows through a cascade stage was developed. Boundary element method with thin airfoil theory Kutta-Zhukowski trailing edge condition gives highly accurate results for arbitrary cascade shapes.

REFERENCES

[1] Fujinami,T.,"Computation of Unsteady Separated Compressible Flows Using Free-Vortex Method", M.Sc Thesis, Dept. of Aerospace Eng. & Eng. Mechanics, University of Texas at Austin, August, 1985.

[2] Gostelow,J.P.,"Potential Flow Through Cascades. A Comparison Between Exact and Approximate Solutions", Aeronuatical Research Council, CP 807, 1964.

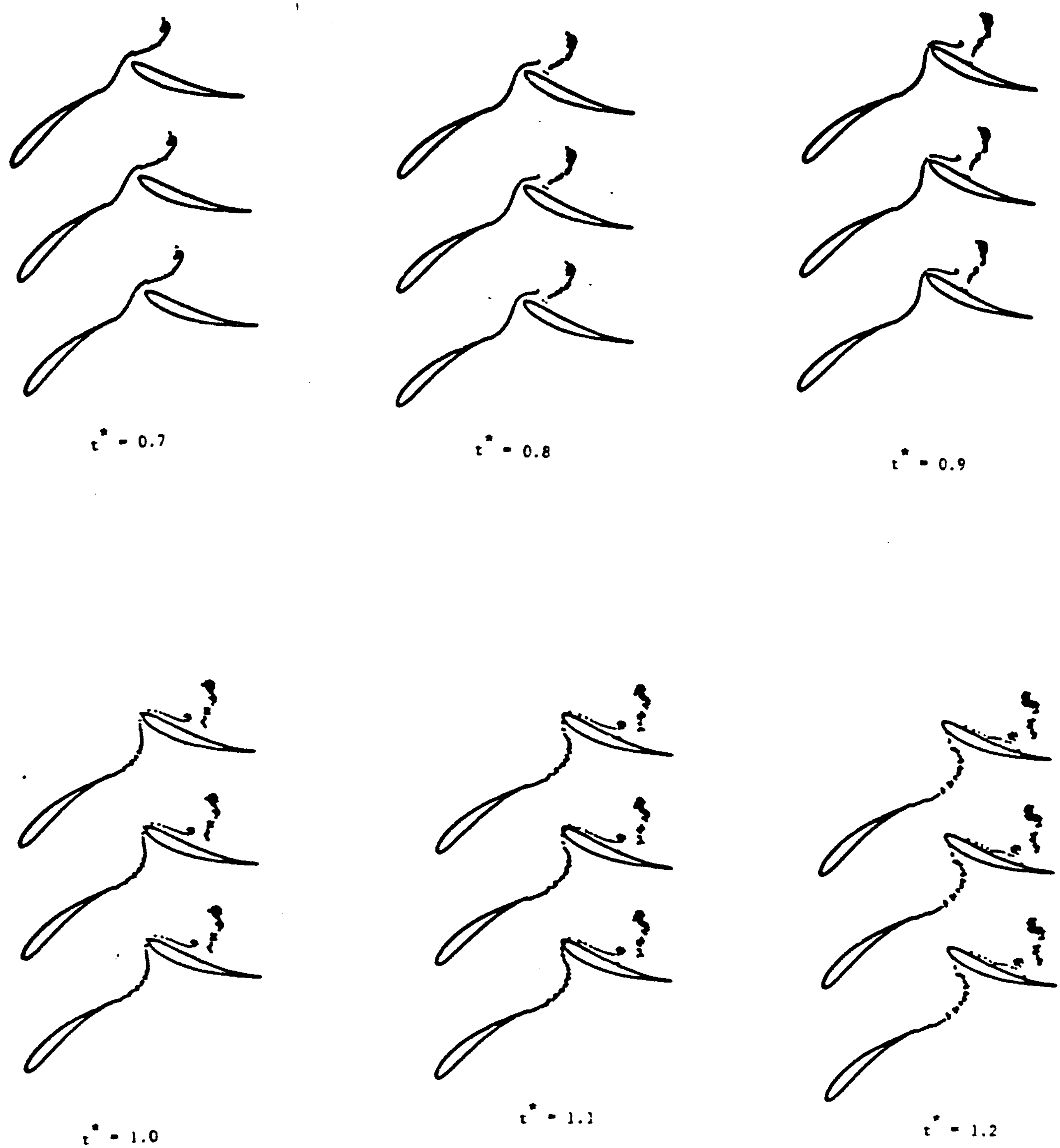

Figure 3. Wake Simulation