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COMMON MISCONCEPTIONS IN THE CALCULATION OF TRANSONIC FULL POTENTIAL FLOWS

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ABSTRACT

Reasons for certain inaccuracies in the majority of existing computer programs that numerically calculate transonic full potential flows are clearly exposed. Problems of post-shock oscillations, non-unique shock strengths and their locations, diverging solutions of choked nozzle flows and flows about sharp leading edges are shown to be related to widely accepted oversimplifications in the treatment of boundary conditions. Forms of numerical dissipation that are presently widely used are shown to be not fully understood and an alternative dissipation generation concept is proposed.

INTRODUCTION

Since the pioneering works of Murman and Cole [1], Jameson [2], Jameson and Caughey [3] and Hafez, South and Murman [4] there has been an unprecedented surge in follow-up work in the area of computational transonic aerodynamics. The transonic small perturbation equation was abandoned by the mid-seventies in favor of an exact full potential equation for homentropic and homoenergetic steady flow of a compressible perfect fluid. Numerical dissipation was for the first time [2,4] semi-analytically substantiated and the need for a fully conservative [3] numerical scheme was explained as a necessary (but not sufficient) condition for the uniqueness of a finite difference scheme.

The initial enthusiasm and belief that these numerical schemes are unquestionable in their correctness was seriously dampened for the first time during the GAMM meeting in Stockholm [5]. This was the first attempt at an open and honest discussion and comparison of results obtained by different researchers using different techniques. Since the results did not compare too well the whole affair was quietly forgotten and very little original work was subsequently performed in this area.

The intention of this paper is to point out several of the most common oversimplifications and misconceptions that the originators of the popular numerical schemes had and practically all of us followers and users never fully understood and exposed. The observations and suggestions that follow are the result of many years of work that the author of this paper has invested in developing new computer programs that are based on the original works of the others [1,2,3,4].

The numerical results presented in this paper were obtained using the finite area technique [3,6] in a mapped rectangular computational plane (fig. 1). Mapping from the physical x,y plane to the computational X,Y plane is performed using bi-linear basis functions

$$x = \frac{1}{4}\sum_{i=1}^{4}(1 - X X_i)(1 - Y Y_i) \qquad (1)$$

Therefore, certain types of errors mentioned in this work should not regularly occur in those schemes that perform numerical integration in the physical space.

BOUNDARY CONDITIONS ON SOLID SURFACES

The non-penetration or tangency boundary condition on a solid surface is that the normal component of the velocity vector must be zero at every point of that surface.

$$\frac{\partial \Phi}{\partial n} = 0 \qquad (2)$$

When working with the full velocity potential function $\Phi(x,y)$ the enforcement of boundary conditions and periodic flow conditions becomes somewhat complex.

But if the potential due to the uniform free stream is subtracted from $\emptyset(x,y)$ these conditions become very simple in terms of the reduced velocity potential [2] defined as

$$\emptyset\,(x,y) = (\,u_{-\infty}\,x + v_{-\infty}\,y\,) + G\,(x,y) \tag{3}$$

In the computational X,Y plane it is convenient to work with the contravariant components U,V of the velocity vector defined as

$$\begin{Bmatrix} U \\ V \end{Bmatrix} = \frac{1}{D} \begin{bmatrix} y_{,Y} & -x_{,Y} \\ -y_{,X} & x_{,X} \end{bmatrix} \begin{Bmatrix} u_{-\infty} + G_{,x} \\ v_{-\infty} + G_{,y} \end{Bmatrix} \tag{4}$$

where u,v are the velocity vector components in physical x,y plane and D is the determinant of the geometric transformation matrix. It can be easily shown [3,6] that the solid wall boundary condition (eq. 1) becomes in X,Y computational plane

$$V = 0 \tag{5}$$

The usual way of satisfying eq. 5 is to set the value of V explicitly to zero at the airfoil surface

$$V\,(I,2) = 0 \tag{6}$$

while at the same time evaluating V at imaginary points inside the solid body (fig. 2) using the reflection principle

$$V(I,1+1/2) = -\,V(I,2+1/2) \tag{7}$$

In order to update the values of the reduced potential at the imaginary points inside the airfoil surface after each complete iterative sweep through the flow field it is very common and economical to use simple linear extrapolation [2,6]

$$G(I,1) = 2.\ *\ G(I,2) - G(I,3) \tag{8}$$

Although it can be argued that the error introduced when using eq. 8 is negligibly small, the truth is quite the opposite especially in the case of internal transonic flows. The printout of the values of contravariant velocity V(I,2) on the airfoil surface reveals that even for an acceptably converged solution these values vary several orders of magnitude reaching their peaks at the leading and trailing edge (fig. 3). This is a clear problem of numerical leakage where an appreciable velocity can result at the stagnation points although the solution for the rest of the flowfield is relatively acceptable. The incorrectness of this oversimplified boundary condition (eqs. 7 and 8) may lead to a number of other problems, for example, an inadequate capability to determine the leading edge stagnation point location neccesary for accurate boundary layer calculation.

Therefore, an exact formula for determining the values of G(I,1) should be used instead of the approximate formula given by eq. 8. From eq. 4 it follows that since V = 0 on the solid surface

$$G_{,y} = \frac{D}{x_{,Y}^2 + y_{,Y}^2}\,(u_{-\infty}\,x_{,Y} - v_{-\infty}\,x_{,Y}) + G_{,\xi}\,\frac{x_{,\eta}\,x_{,\xi} + y_{,\eta}\,y_{,\xi}}{D} \tag{9}$$

Hence the exact value of the reduced potential at imaginary points can be obtained using central differencing in the computational X,Y plane where $\Delta X = \Delta Y$.

Note that this procedure is directly applicable to flows with surface transpiration.

BOUNDARY CONDITIONS AT INLET AND EXIT BOUNDARIES

In the case of an O-type or a C-type computational grid it is relatively hard to position the imaginary points at upstream and downstream flow boundaries. This problem is routinely dealt with by avoiding to solve the full potential equation at the grid points situated along the upstream and downstream boundaries (fig. 4). Usually, a fixed value of the reduced velocity potential is kept at these points without ever solving the governing equation there - a procedure frequently refered to as cutting out a window. This procedure leads to an error that manifests itself as a velocity defect (and sometimes an overestimate) at the upstream and downstream flow field boundaries. The error is not negligible since it strongly influences the strength and the location (fig. 5) of the isentropic discontinuities (shock waves) and causes choked nozzle flow computations to diverge. Therefore, the fully conservative form of the full potential equation should be solved at every single point of the flow field and its boundaries (fig. 4).

Nevertheless, the question of determining or updating the values of the reduced velocity potential at the imaginary points at the upstream boundary (fig. 4) can also profoundly affect the location and strength of the shock waves. If the value of the reduced velocity potential G(x,y) at these points is kept fixed throughout the computation the result will be as shown in fig. 5 since the free stream value at the upstream boundary will never be satisfied exactly despite the fact that it is properly accounted for in the formulation of the reduced velocity potential.

One suggestion to overcome this inaccuracy would be to use the distribution of imaginary points at the upstream and and downstream boundaries as shown in fig. 6 and to assign these points the following values of the reduced potential after each iterative sweep through the flow field

$$G_a = G_b = G_e = G_d = G_f \quad ; \quad G_c = G_g \tag{10}$$

At the flow field exit boundary imaginary points (fig. 4) the value of reduced potential should be reset after each iterative sweep through the flow field by enforcing the global mass flow

COMPUTATIONAL GRIDS

When generating boundary conforming computational grids it is often desirable to cluster grid points in the regions of expected strong velocity gradients. This inevitably leads to a redistribution of input points along the airfoil contour. Spline fitting and interpolation commonly used in this procedure can lead to seemingly small errors in the description of the original airfoil contour. Nevertheless, in the case of transonic flow through a closely spaced cascade of airfoils this small error in the definition of airfoil

shape can cause a significant difference in the numerical results: otherwise shockless airfoils become shocked and almost choked nozzles become fully choked [7,8].

Related to this type of error is a major misconception about the validity of any inviscid flow calculation alone. At transonic speeds the boundary layer displacement thickness must be taken carefully into account if one intends to make any meaningful comparison between numerical and experimental results [9].

An unintentional error is often committed by those who generate computational grids for transonic flow computations without fully understanding all the intricacies of the expected flow field and the numerical technique used in the flow solver. While excessivelly clustering grid points along the family of grid lines conforming with the global flow direction it is often forgotten that this leads to the creation of very elongated grid cells with large aspect ratios. The author's personal experience confirms that the grid cells with aspect ratios of about three already cause an oscillatory convergence pattern while larger aspect ratios unavoidably lead to the divergence of the iterative flow solution procedure.

The effects of airfoil surface grid clustering alone might lead not only to slightly different shock locations and sharpness but can also create oscillations behind the shock (fig. 7).

NUMERICAL DISSIPATION

The full potential equation is a second order quasilinear partial differential equation of mixed elliptic-hyperbolic type.

$$\vec{\nabla}\cdot(\rho\vec{\nabla}\phi) = \rho[(1-M^2)\phi_{,ss} + \phi_{,nn}] = 0 \tag{11}$$

When the flow is locally subsonic eq. 11 becomes elliptic and all second derivatives can be discretized using central differencing. On the other hand, when the flow is locally supersonic the full potential equation becomes hyperbolic and one-sided streamline aligned differencing should be used on the second derivative multiplying the term (1 - M*M) in eq. 11. This type-dependent [1] rotated [2,3] finite differencing has the only purpose of trying to numerically approximate an analytic upstream facing domain of dependence when the full potential equation is locally hyperbolic. The truncation errors for central and upstream differencing are not the same and this discretization error should be somehow eliminated. Since the leading term of the difference in the two truncation errors is a third derivative of the reduced potential it is common to refer to these non-physical terms as numerical dissipation: a term most probably based on the similarity of the discretized version of eq. 11 with the Burgers equation [10]

$$(\frac{1}{2}u^2)_{,x} = \epsilon\, u_{,xx} \tag{12}$$

There are presently two similar concepts that are widely used in a firm belief that they can nullify the numerical error introduced when the rotated upstream differencing is applied in locally supersonic regions. These two concepts are artificial density [4] and artificial viscosity [2] added in a fully conservative form.

Artificial density is based on an implicit modification of the value of isentropic density according to the formula [4]

$$\tilde{\rho} = \rho - \Delta s\, \mu \rho_{,s} \tag{13}$$

where the switching function is defined as

$$\mu = \mu_{JH} = 1 - \frac{1}{M^2} \tag{14}$$

A number of other attempts [11,12,13] have been made at modeling the artificial dissipation, although most of them are without any theoretical justification. Actually, it can be analytically proved [14] that even the artificial density concept as defined in eqs. 13 and 14 does not produce only a linear dissipation term (as it is widely believed) but also generates extraneous truly nonlinear terms so that the discretized full potential equation becomes

$$\tilde{\rho}\{[1-M^2)\phi_{,ss} + \phi_{,nn}] + \Delta s\, \mu M^2 \phi_{,sss} + E\} = 0 \tag{15}$$

where the unexpected and unwanted terms are [14]

$$E = C\left\{\mu\,[2\frac{M^2}{\phi_{,s}} - (2-\nu)\frac{M^4}{\phi_{,s}}](\phi_{,ss})^2 + \mu\frac{M^2}{\phi_{,s}}\phi_{,ss}\phi_{,nn} + \mu_{,s}M^2\phi_{,ss}\right\} \tag{16}$$

Instead of the artificial density concept many users prefer the artificial viscosity concept [2]. But closer inspection of the artificial density and artificial viscosity concepts reveals that they are basically the same: they both generate a number of truly nonlinear terms in the discretized version of the full potential equation. A rigorous analysis of the influence of these non-physical terms that exist for any finite grid size is still lacking. The question concerning the uniqueness of such discretized equations should be answered first before claiming that the existing numerical schemes utilizing artificial density and artificial viscosity and linearized outer boundary conditions can be used to prove the nonuniqueness [15,16] of the original partial differential equation.

Besides, it was analytically demonstrated [17,18] that the asymptotic flow pattern of a physically viscous, thermally conducting gas is qualitatively different from the pattern given by the solution of equations for an ideal gas. These differences become substantial and dissipation processes play an increasing role with a decrease of airfoil thickness ratio. Consequently, modeling of the numerical dissipation should not be as carelesly undertaken as it was regularly done in the past but should try to mimic the physics as much as possible [19].

As an alternative approach to artificial density and artificial viscosity one can try to use the artificial mass flux concept [14] where the product of entire mass flow vector is upstream differentiated

$$\vec{\nabla}\cdot\overrightarrow{(\rho\vec{\nabla}\phi)} = (\frac{\partial}{\partial s}\hat{e}_s + \frac{\partial}{\partial n}\hat{e}_n)\cdot$$

$$[(\rho\phi_{,s} - \Delta s\,\mu(\rho\phi_{,s})_{,s})\hat{e}_s + (\rho\phi_{,n})\hat{e}_n] = 0 \tag{17}$$

The value of the switching function is here determined analytically from the condition that all the nonlinear terms introduced by the upstream differencing of the mass flux vector be eliminated identically. The result is

$$\mu = \frac{1}{(\phi_{,s}^2 - 1)} (a^2)^{\frac{\gamma - 2}{\gamma - 1}} \qquad (18)$$

that should generate a linearly dissipative modified full potential equation

$$[(1 - M^2)\, \phi_{,ss} + \phi_{,nn}] + C(\frac{\gamma + 1}{2})\, \frac{1}{\rho}\, \phi_{,sss} = 0 \qquad (19)$$

This equation is similar to a well known viscous transonic equation [20,21] obtained by accounting for linear terms of a physically existent dissipation due to compressive viscosity effects and heat conduction in the gas. Since the numerical solution of this equation has been succesfully obtained [22,23] one could expect that the artificial mass flux concept is a valuable alternative to the existing approaches for the numerical solution of the non-dissipative full potential equation.

The artificial mass flux concept should be understood as only a first step in an attempt to derive analytical expresssions for artificial dissipation that is based on actual physical dissipation.

SUMMARY

A number of probable causes of errors frequently committted when numerically solving the transonic full potential equation have been identified and remedies and alternatives have been suggested. The conclusion is that the present numerical techniques use numerous oversimplifications which, combined with certain basic misconceptions, often produce misleading and erroneous results.

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REFERENCES

1. Murman, E. M., and J. Cole, "Calculation of Plane Steady Transonic Flows," AIAA Journal, Vol.9, 1971, pp.114-121.

2. Jameson, A., "Iterative Solution of Transonic Flows Over Airfoils and Wings Including Flows at Mach 1," Comm. Pure and Appl. Math., Vol.27, 1974, pp.283-309.

3. Jameson, A. and D. Caughey, "A Finite Volume Scheme for Transonic Potential Flow Calculations," Proceedings of AIAA 3rd Computational Fluid Dynamics Conference, 1977, pp.35-54.

4. Hafez, M., J. South, and E. Murman, "Artificial Compressibility Methods for Numerical Solutions of Transonic Full Potential Equation," AIAA Journal, Vol. 17, No.8, August 1979, pp.838-844.

5. Rizzi, A., editor, "GAMM Workshop on Numerical Methods for the Computation of Inviscid Transonic Flow with Shock Waves," FFA-Stockholm, September 18-19, 1979.

6. Dulikravich, G. S., "Numerical Calculation of Inviscid Transonic Flow Through Rotors and Fans," Ph.D. Thesis, Cornell University, January 1979. (Available through University Microfilms International, 300 N. Zeeb Road, Ann Arbor, Michigan 48106.)

7. Dulikravich, D. S. and H. Sobiezcky, "CAS22-FORTRAN Program for Fast Design and Analysis of Shock-Free Airfoil Cascades Using Fictitious-Gas Concept," NASA CR 3507, January 1982.

8. Dulikravich, D. S. and H. Sobieczky, "Shockless Design and Analysis of Transonic Cascade Shapes," AIAA Journal, Vol.20, No. 11, November 1982, pp.1572-1578.

9. Lee, C. S. and Van Dalsem, R. W., "Numerical Simulation of Steady Transonic Flow About Airfoils," Proceedings of AIAA 5th Computational Fluid Dynamics Conference, 1981, pp.69-76.

10. Carrier, G. F. and Pearson, C. E., "Partial Differential Equations," Academic Press, 1976, pp.233-236.

11. Roach, R. L. and N. L. Sankar, "The Strongly Implicit Procedure Applied to the Flow Field of Transonic Turbine Cascades," AIAA Paper Number 81-0211 presented at the AIAA 19th Aerospace Sciences Meeting, St. Louis, Missouri, January 12-15, 1981.

12. Chen, L.-T. and D. Caughey, "On Various Treatments of Potential Equations at Shocks," NASA CP 2201 (a symposium held at Ames Research Center, October 19-20, 1981.)

13. Doria, M. and J. South, "Transonic Flow and Coordinate Generation for Bodies in a Wind Tunnel," AIAA Paper presented at the 20th AIAA Aerospace Sciences Meeting, Orlando, Florida, January 11-14, 1982.

14. Dulikravich, G. S. and P. Niederdrenk, "Artificial Mass Concept and Transonic Viscous Flow Equation," presented at the First Army Conference on Applied Mathematics and Computing, Washington, D. C., May 9-11, 1983.

15. Steinhoff, J., and A. Jameson, "Multiple Solutions of the Transonic Potential Flow Equation," AIAA Journal, Vol.20, No. 11, November 1982, pp. 1521-1525.

16. Salas, M., A. Jameson, and R. E. Melnik, "A Comparative Study of the Nonuniqueness Problem of the Potential Equation," AIAA Paper No.83-1888 presented at the 6th AIAA Computational Fluid Dynamics Conference, Danvers, Massachusetts, July 13-15, 1983.

17. Ryzhov, O. S. and Shefter, G. M., "On The Effect of Viscosity and Thermal Conductivity on the Structure of Compressible Flows," Appl. Math. Mech. 28, 1964, pp.1206-1218.

18. Ryzhov, O. S., "Viscous Transonic Flows," Ann. Review of Fluid Mechanics, vol. 10, 1978, pp.65-92.

19. Klopfer, G. H. and Nixon, D., "Non-Isentropic Potential Formulation for Transonic Flows," AIAA Paper 83-0375 presented at the AIAA 21st Aerospace Sciences Meeting, Reno, Nevada, January 10-13, 1983.

20. Cole, J. D., "Problems in Transonic Flow," Ph.D. Thesis, California Institute of Technology, 1949.

21. Sichel, M., "Structure of Weak Non-Hugoniot Shocks," The Physics of Fluids, Vol. 6, May 1963, pp. 653-663.

22. Chin, W., "Numerical Solution for Viscous Transonic Flow," AIAA Journal, Vol. 15, No. 9, September 1977, pp.1360-1362.

23. Chin, W., "Type-Independent Solution for Mixed Compressible Flows," AIAA Journal, Vol. 16, No. 8, August 1978, pp.854-856.

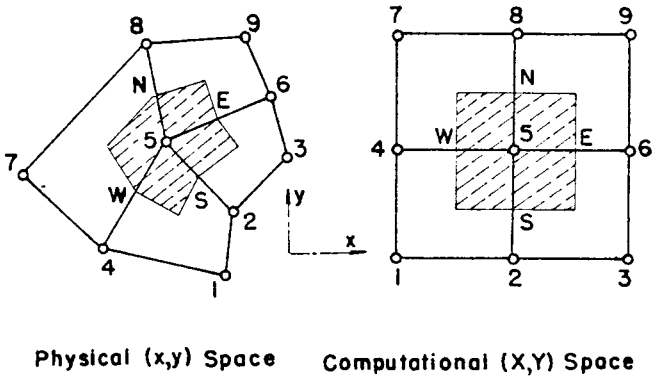

Fig. 1 Local Transformation from Physical x,y Plane to Computational X,Y Plane

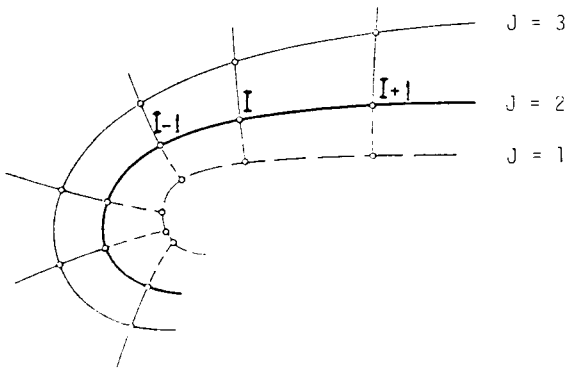

Fig. 2 Computational Grid Points at Solid Boundary

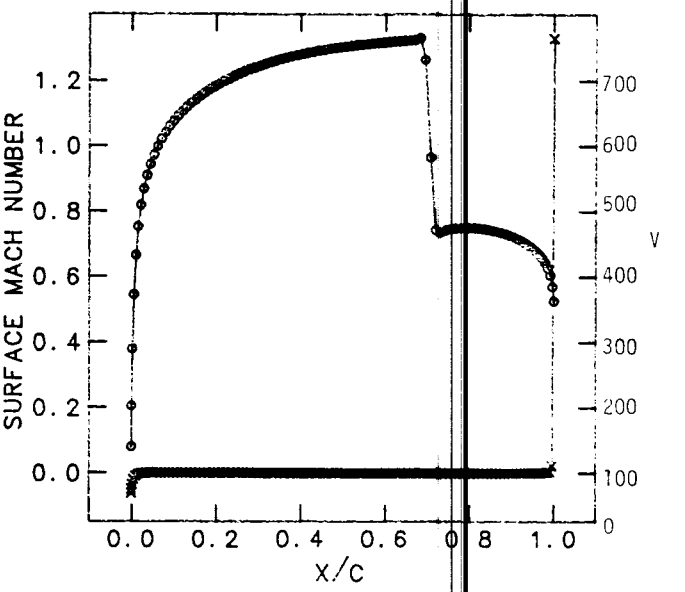

Fig. 3 Surface Mach Number (ooo) and Contravariant Velocity Vector Component V (xxx) When Using Approximate Surface Boundary Conditions (eq.8).

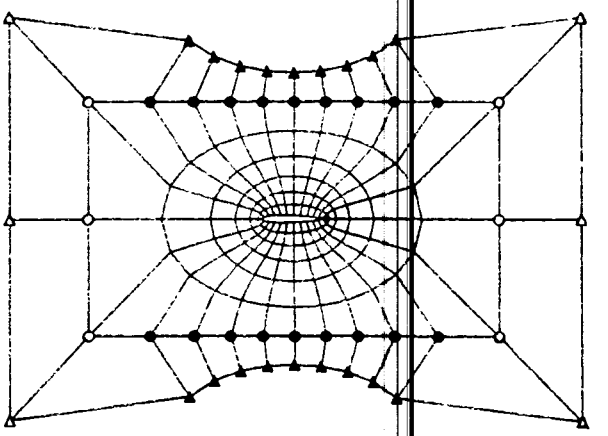

Fig. 4 Geometrically Periodic O-Type Cascade Grid: Inlet and Exit Boundary Points (o), Periodic Boundary Points (•), Inlet and Exit Imaginary Points (Δ), Periodic Overlap Points (▲). When Cutting Out Windows the Full Potential Equation is Never Actually Solved at (o,Δ).

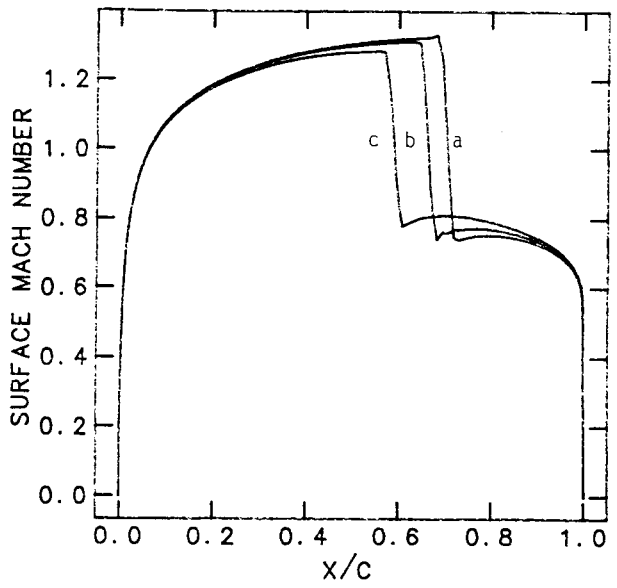

Fig. 5 Nonuniqueness of Shock Strength and Location: Correct Result (a); Using Fixed Value of the Reduced Velocity Potential at Inlet Imaginary Points (b); Cutting Out Windows at Inlet and Exit Boundaries (c).

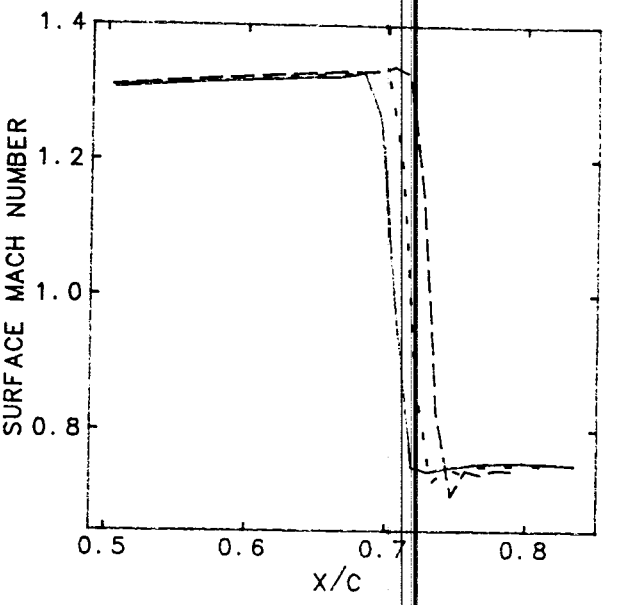

Fig. 7 Influence of Surface Grid Point Clustering: Equidistant Surface Distribution (- - -); Mild Clustering Toward Leading and Trailing Edge (---); Strong Clustering Toward Leading and Trailing Edge (———).

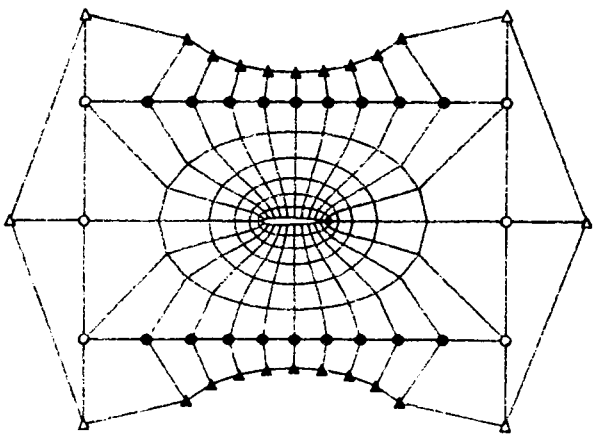

Fig. 6 Suggested Distribution of Inlet and Exit Imaginary Points