The Inverse Design of Internally Cooled Turbine Blades

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A methodology is developed for the inverse design and/or analysis of interior coolant flow passage shapes in internally cooled configurations with particular applications to turbine cascade blade design. The user of this technique may specify the temperature (or heat flux) distribution along the blade outer fixed surface shape and the unknown interior coolant/blade interface. The numerical solution of the outer gas flow field determines the remaining unspecified blade outer surface quantity — surface heat flux if temperature was originally specified or vice versa. Along the unknown coolant flow passage shape the designer has the freedom to specify the desired temperature distribution. The hollow blade wall thickness distribution is then found from the solution of Laplace's equation governing the temperature field within the solid portion of the hollow blade, while satisfying both boundary conditions of temperature and heat flux at the fixed outer blade surface, and the specified temperature boundary condition on the evolving inner surface. A first order panel method, coupled with Newton's N-dimensional interation scheme, is used for the iterative solution of the unknown coolant/blade interface shape. Results are shown for a simple eccentrical bore pipe cross section and a realistic turbine blade cross section. The inverse design procedure is shown to be efficient and stable for all configurations that have been tested.

Introduction

Due to rising fuel costs, an impetus has arisen in the past decade for the design of highly efficient turbomachinery. The cost of large-scale testing of new designs has also escalated, making preliminary design using computational methods more attractive.

Internally cooled turbine blade design has traditionally been accomplished using various approximate and empirical techniques. The exterior gas flow is often assumed to be isentropic and the effects of rotation and three-dimensionality neglected. In addition, the turbine blade designer has no direct control of the detailed blade temperature distribution and has to live with whatever temperature distribution is found from expensive and time consuming experimental tests.

This paper describes an entirely new concept for the inverse design and analysis of internally cooled turbine blades. The technique allows the blade designer to specify the desired temperature or heat flux at each point on the turbine airfoil outer surface, using whatever criteria he chooses (thermal stress considerations, coolant flow availability, aerodynamic effects on the outer flow field, etc.). Potential savings from using this technique include the possibility of having higher turbine inlet temperatures and lower coolant flow rates. It is worth noting that existing internally cooled turbine blades can also be analyzed using this technique, and can possibly be redesigned for better performance.

The Global Inverse Design Concept

The first step in this inverse design procedure is the specification of the blade outer surface temperature or heat flux distribution. The exterior flow is then solved using, say, a full potential [1] or Euler [2] code coupled with an appropriate boundary layer code [3]. Instead of this viscous/inviscid coupling, a Navier-Stokes equation solver [4] with appropriate turbulence modeling can be used. Regardless of what flow solver is used for the determination of the exterior compressible flow, the result will be the airfoil outer surface remaining quantity (temperature or heat flux) that was not specified. Alternatively, experimental data in the form of a Nusselt number distribution could be used with a specified surface temperature distribution to obtain the surface heat flux distribution. These flow field solutions can be obtained for two-dimensional cascades, quasi-three-dimensional cascades, or fully three-dimensional heat conduction problem in blade cross sections. Three-dimensional effects (radial heat flow) will be neglected, but it must be noted that this is not a limitation of the technique.

The dual boundary conditions of temperature and heat flux on the outer surface of the hollow airfoil and the temperature distribution alone on the unknown inner surface are then input to the solution of Laplace's equation for steady heat conduction in the solid portions of the hollow blade. This solution is accomplished by first determining an appropriate initial guess of the unknown coolant flow passage shape. The initial guess is based on the local one-dimensional heat conduction considerations. The temperature on the inner contour is then iteratively attained by reconfiguring the contour while at the same time satisfying the dual tem-

perature/heat flux outer surface boundary conditions. Note that the temperature and heat flux on the outer surface are satisfied separately; in other words, the boundary condition is not a standard linear combination of the two quantities. The calculated temperature distribution on the coolant/blade interface generally differs from the specified temperature, and Newton's iterative scheme is used to modify the inner contour and drive the temperature difference to within a given tolerance.

All two-dimensional solutions for the blade spanwise cross-sections can then be stacked up to determine the three-dimensional coolant flow passage shape. The coolant flow passage will now have a specified temperature and the obtained heat flux distribution on its surface. The heat flux alone will serve as an input to the solution of the coolant flow field. The solution of the coolant flow field using the heat flux boundary condition generally produces a different temperature distribution on the blade inner surface than the one that was initially specified. Thus, using the newly found temperature distribution on the coolant passage surface (or alternatively some combination of the new temperature and the one originally prescribed), a new coolant flow passage shape is found which satisfies the fixed blade outer surface dual boundary conditions. The process is repeated until a point is reached in which the blade inner surface temperature distribution calculated from the coolant flow field will match the temperature from the previous global iteration. Finally, the blade can be analyzed from structural considerations, including thermal stresses, and the entire process is repeated if the blade's structural integrity is not acceptable.

Coolant Flow Passage Shape Determination

Given the temperature and heat flux distributions on the outer surface, Ω_s, of a given turbine blade cross section (Fig. 1), the problem is to find the shape of the inner contour, Ω_c, that satisfies three specified boundary conditions: (i) the blade outer surface temperature, T_s; (ii) the blade outer surface heat flux, q_s; and (iii) the coolant/blade interface (inner contour) temperature, I_c. The solid portions of the hollow blade are assumed to be homogeneous and made of a material with a constant coefficient of heat conduction, λ. The heat flow is assumed to be steady and the temperature field in the material satisfies Laplace's equation

$$\lambda \nabla^2 \phi = 0 \tag{1}$$

where ϕ is the temperature. From potential theory, it is known that a solution to Laplace's equation can be found by superimposing a series of fundamental solutions. Since the final shape of the coolant flow passage can be highly irregular, depending on the temperature and heat flux boundary conditions, we decided to solve Laplace's equation using simple surface singularity distribution method. The technique described here utilizes straight panels with constant
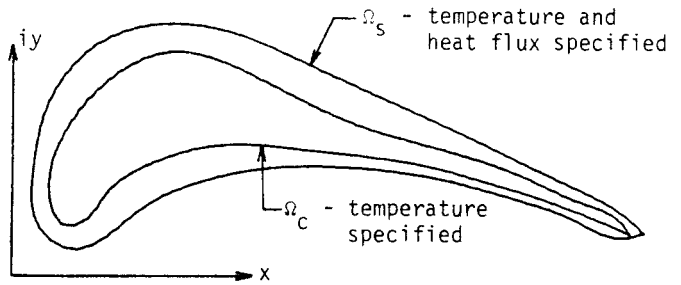

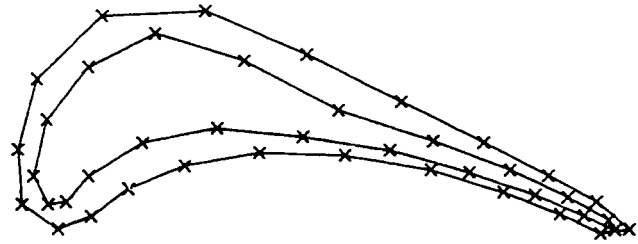Fig. 1 Geometry and boundary conditions

Fig. 2 Inner and outer contours discretized with panels

source (or sink) strength distributions to represent the heat flow between the outer, Ω_s, and inner, Ω_c, contours (Fig. 2). An initial guess for Ω_c is determined from one-dimiensional heat conduction using the temperature, T_s, and the heat flux, q_s, on the airfoil outer surface and the temperature, T_c, on the inner contour. The temperature induced at a point, z_o, by a panel of strength, K, is given by

$$\phi(z_o) = k \int \ln |z_o - z| \, ds \tag{2}$$

where the integration is performed along the panel. By replacing the outer (fixed) contour, Ω_s, with an equal number of straight panels (line segments), and denoting the strengths of these panels by k_{s_i} and k_{c_i}, respectively,

$$\phi(z_o) = \sum_{i=1}^{N} \{ k_{s_i} \int \ln |z_o - z| \, ds_i$$

$$+ k_{c_i} \int \ln |z_o - z| \, ds_i \} \tag{3}$$

for the temperature, ϕ, at any point, z_o, in the z-plane, where N is the number of panels on each contour. Also, from Fourier's heat conduction law the heat flux in the direction n at any point z_o is given by

$$-\lambda \frac{\partial \phi(z_o)}{\partial n} = -\lambda \sum_{i=1}^{N} \left\{ k_{s_i} \int \frac{\partial}{\partial n} \ln |z_o - z| \, ds_i \right.$$

$$+ k_{c_i} \int \frac{\partial}{\partial n} \ln |z - z_o| \, ds_i \left. \right\} \tag{4}$$

Nomenclature

i = complex number, $i = \sqrt{-1}$
I = temperature influence coefficient
J = heat flux influence coefficient
k = strength of a source or sink
N = number of panels on the inner and outer contours
q = specified heat flux
R = radius of curvature
s = arc length along a panel
T = specified temperature
x = x-coordinate of the z-plane

y = y-coordinate of the z-plane
z = coordinate in the complex plane $z = x + iy$
$\hat{z}$ = panel control point coordinate
$\check{z}$ = panel end point coordinate
i,j,m,p = indices

Greek Symbols

ϕ = temperature
λ = coefficient of heat conduction
δ = distance measure

Ω = closed contour in the z-plane
ω = relaxation factor
α = optimal relaxation factor

Subscripts

s = blade surface
c = coolant/blade interface
o = particular point

Superscripts

n = iteration number
$\bar{}$ = panel control point
$\hat{}$ = panel end point

Satisfying the dual boundary conditions on Ω_s will give the values of k_{s_i} and k_{c_i}. The outer contour boundary conditions of temperature and heat flux are satisfied at the control points of the panels, $\bar{z}_{s_i}$, defined as the average of the panel end points, $\bar{z}_{s_i}$ and $\bar{z}_{s_{i+1}}$. This procedure produces four $N \times N$ influence coefficient matrices that multiply the source and sink strengths as

$$\left[I_{s_{ij}}\right]\left\{k_{s_i}\right\} + \left[I_{c_{ij}}\right]\left\{k_{c_i}\right\} = \left\{T_{s_j}\right\} \tag{5}$$

$$\left[J_{s_{ij}}\right]\left\{k_{s_i}\right\} + \left[J_{c_{ij}}\right]\left\{k_{c_i}\right\} = -\frac{1}{\lambda}\left\{q_{s_j}\right\} \tag{6}$$

which can also be written as the $2N \times 2N$ partitioned matrix

$$\begin{bmatrix} I_{s_{ij}} & \vdots & I_{c_{ij}} \\ ---- & \vdots & ---- \\ J_{s_{ij}} & \vdots & J_{c_{ij}} \end{bmatrix} \begin{Bmatrix} k_{s_i} \\ ---- \\ k_{c_i} \end{Bmatrix} = \begin{Bmatrix} T_{s_j} \\ ------ \\ \dfrac{-q_{s_j}}{\lambda} \end{Bmatrix} \tag{7}$$

Here, $I_{s_{ij}}$ and $I_{c_{ij}}$ denote the influence of the ith outer or inner panel on the control point of the jth outer panel, and $J_{s_{ij}}$ and $J_{c_{ij}}$ denote the influence of the ith outer or inner panel on the heat flux at the control point of the jth outer panel. Inverting equation (7) gives the values of k_{s_i} and k_{c_i}, which satisfy the dual boundary conditions on the outer contour, Ω_s. The inversion of equation (7) is performed using the method of Cholesky [5]. The temperature on the inner contour, ϕ_{c_j}, is then calculated using equation (2). This temperature will in general be different from the specified temperature T_{c_j}; therefore, a correction to the inner contour is calculated from Newton's method and is given by

$$\left[\frac{\partial \phi(\bar{z}_{c_j}^n)}{\partial \bar{z}_{c_i}^n}\right]\left\{\Delta \bar{z}_{c_i}^n\right\} = \left\{T_{c_j} - \phi(\bar{z}_{c_j}^n)\right\} \tag{8}$$

where the correction is defined by

$$\bar{z}_{c_i}^{n+1} = \bar{z}_{c_i}^n + \omega \Delta \bar{z}_{c_i}^n \tag{9}$$

and ω is the relaxation factor. The differentiation in Newton's method is determined analytically and is performed in the direction of the normal to the ith inner panel. The calculation of the derivative matrix requires the inversion of the $2N \times 2N$ matrix of equation (7); however, the LU decomposition of the matrix is stored from the solution for the panel strengths and thus can be used to determine the derivative matrix with negligible extra computation.

The relaxation factor, ω, is set equal to 1 for most of the iteration steps, but the existence of an optimal relaxation factor, α, is checked for at each iteration. After each iteration, the coordinates of the control points of the inner panels are changed by the calculated corrections, and so the new coordinates of the end points must be found. This is accomplished by noting that the contour is closed and that the control points are halfway between the end points with the result that

$$\bar{z}_{c_j} = (-1)^{j+1}\left\{2\sum_{i=1}^{j-1}(-1)^i \bar{z}_{c_i} - \sum_{i=1}^{N}(-1)^i \bar{z}_{c_i}\right\} \tag{10}$$

is the expression for the end points as a function of the control points. The existence of an optimal relaxation factor, α, can be shown by noting that certain positions of the control points produce a highly irregular contour, since the end points are calculated from a strict constraint that the control points are halfway between the end points. Thus α is the value of ω that minimizes this irregularity as ω varies from 0 to 1 and is given by

$$\alpha = \frac{-\dfrac{1}{2}\displaystyle\sum_{j}^{N}\sum_{m}^{N}\sum_{p}^{N}(\Delta \bar{x}_m^n \bar{x}_p^n + \Delta \bar{x}_p^n \bar{x}_m^n + \Delta \bar{y}_m^n \bar{y}_p^n + \Delta \bar{y}_p^n \bar{y}_m^n)\gamma_{mj}\gamma_{pj}}{\displaystyle\sum_{j}^{N}\sum_{m}^{N}\sum_{p}^{N}(\Delta \bar{x}_m^n \Delta \bar{x}_p^n + \Delta \bar{y}_m^n \Delta \bar{y}_p^n)\gamma_{mj}\gamma_{pj}} \tag{11}$$

where

$$\gamma_{ij} = \begin{cases} (-1)^{i+j} & \text{if } i<j \\ (-1)^{i+j+1} & \text{if } i>j \\ (-1)^{i+j+1}+1 & \text{if } i=j \end{cases} \tag{12}$$

The nonexistence of an optimal relaxation factor is evident when the α calculated from equation (11) is outside an acceptable range (α less than 0.2 or greater than 1.0).

The iterative procedure is concluded when the maximum error in satisfying the temperature boundary condition on the inner contour is below a certain tolerance. In the present work, 0.1 percent of the average specified temperature, T_{c_j}, was used as the convergence criterion.

Results

A computer program was developed to implement the inverse design procedure. Input to the program includes the airfoil outer contour coordinates, the outer surface temperature and heat flux distribution, and the inner contour temperature distribution. The program then calculates an initial guess for the wall thickness distribution from the one-dimensional heat conduction relations given by

$$\delta_i = R\left[1 - \exp\left(\frac{\lambda T_{s_i} - T_{c_i}}{R\, q_{s_i}}\right)\right] \tag{13}$$

where δ_i is the normal distance from the ith outer panel, and R is the radius of curvature of the outer contour at the ith control point. As R goes to infinity, δ_i is calculated from the plane wall one-dimensional heat conduction equation.

$$\delta_i = \frac{-\lambda(T_{s_i} - T_{c_i})}{q_{s_i}} \tag{14}$$

The shape of the inner contour of the hollow airfoil is found iteratively.

Results for two cases are presented in this paper: a simple eccentrical bore pipe and a turbine cascade [6]. The eccentrical bore pipe is analogous to the buried cable problem and has an analytic solution [7] in which the isotherms are nonconcentric circles. Using the analytic solution to determine the heat flux on the outer surface and specifying constant temperatures on the outer surface on the pipe and on the unknown inner contour, the initial guess is postulated as shown in Fig. 3. After four iterations, the inner contour converged to the shape shown in Fig. 3.

The second test case is a transonic turbine cascade for which experimental data [6], in the form of Nusselt number distributions, were available (Fig. 4). Specified conditions were: constant 1500°F outer surface temperature; constant 1300°F coolant/blade interface temperature; turbine inlet temperature of 3100°F; and blade outer surface heat fluxes calculated from the given Nusselt number distribution. The blade was assumed to be made of a material with $\lambda = 70$ Btu/ft-°F. The initial guess calculated by our computer program is shown in Fig. 5 along with the final converged solution. The solution took four iterations to converge to within 0.1 percent of the specified inner wall temperature. Both test cases utilized 21 straight panels on both the inner and outer contours. The small iteration count for each case is due to the fact that the initial guesses were fairly close to the correct solution and that the optimum relaxation factor was
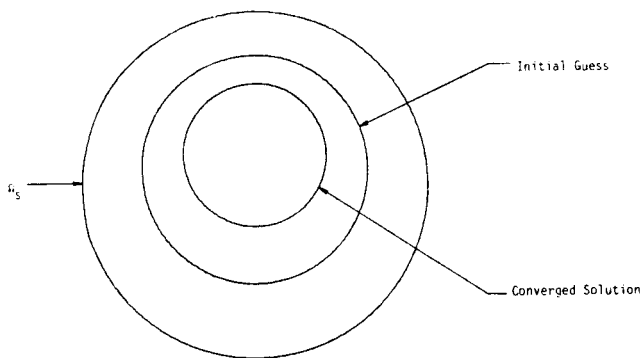
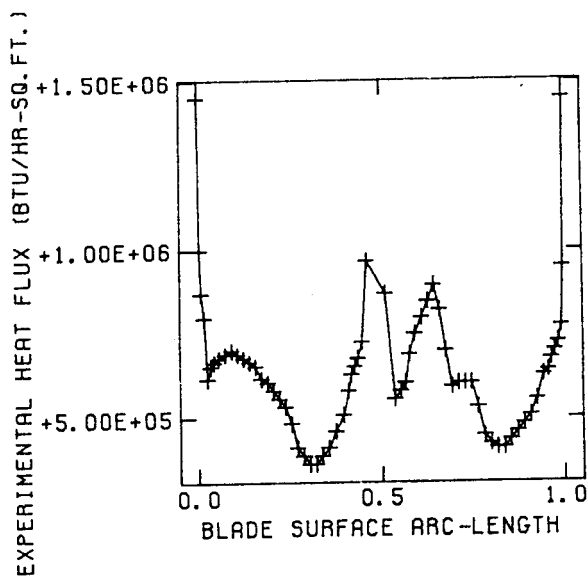Fig. 3 Eccentrical bore pipe inverse design

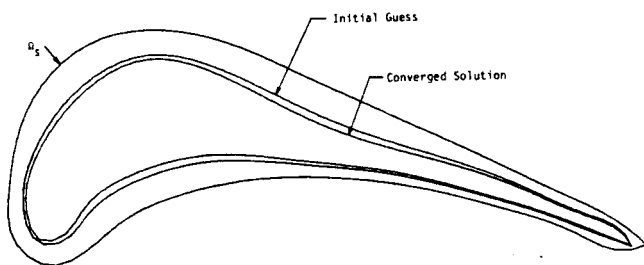Fig. 4 Experimental heat flux distribution for MIT turbine blade [6]

Fig. 5 MIT turbine blade inverse design [6]

used. Convergence displayed the usually expected quadratic convergence of Newton-type schemes. Approximately 10.0 cpu s on the CDC Cyber 170/750 were required for each iteration.

Future Research

There are many possibilities for extending the present method to more complex problems with increased accuracy. The panel method could be improved by using higher order curved panels and nonconstant panel singularity distributions [8]. A major extension of the method would be to make it fully three-dimensional by using quadrilateral elements in place of the outer blade surface (from hub to tip) and the unknown inner surface [9]. Most present turbine coolant flow passage designs include multiple holes and trailing edge coolant flow ejection; therefore, the present method could be extended to design and analyze these configurations. Tran-spiration cooled configurations could be designed and

analyzed also since the coolant and heat flow through a porous medium has been shown by Siegel [10] to be derivable from a potential. One of the most intriguing applications for this method would be to attempt a multiple-point design of an entirely shock-free outer flow field [11] that could be maintained by varying the coolant flow rate. This shock-free design procedure would not entail any modification of the overall shape of the turbine blade. In addition, the method could be used to delay the onset of boundary-layer transition [12].

Summary

A procedure has been developed for the efficient design and analysis of coolant flow passage shapes in internally cooled configurations. The method is particularly applicable to turbine blade design but can also be used for the design of other configurations that utilize internal or external cooling such as missile cone tips, rocket nozzles, and internal combustion engines. The designer is able to specify both the temperature and heat flux at the turbine blade outer surface and the coolant/blade interface surface. The result of the design procedure is the shape of the interior coolant flow passage contour that satisfies all three of the above boundary conditions. When coupled with an appropriate flow solver, the method provides the gas turbine engine designer with an efficient tool for the preliminary design of coolant flow passages. The method is not limited to cascade design, but can be used for the fully three-dimensional design of coolant flow passages.

Acknowledgments

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